Z. Phys. B - Condensed Matter 85, 395-403 (1991)

Condensed Matter
Zeitschrift für Physik B

Coulomb blockade in single tunnel-junctions: quantum mechanical effects of the electromagnetic environment

K. Flensberg[*,1,2], **S.M. Girvin**[3], **M. Jonson**[4], **D.R. Penn**[5], **and M.D. Stiles**[5]

[1] Department of Physics, University of Tennessee, Knoxville, TN 37996, USA,
[2] Solid State Division, Oak Ridge National Laboratory, Oak Ridge, TN 38731, USA
[3] Department of Physics, Indiana University, Bloomington, IN 47405, USA
[4] Institute of Theoretical Physics, Chalmers University of Technology, S-41296 Göteborg, Sweden
[5] National Institute of Standards and Technology, Gaithersburg, MD 20899, USA

Received July 3, 1991; revised version September 3, 1991

We discuss the interaction of a tunneling electron with its equilibrium electromagnetic environment. The environment of an isolated tunnel junction is modeled by a set of harmonic oscillators that are suddenly displaced when an electron tunnels across the junction. We treat these displaced oscillators quantum mechanically, predicting behavior that is very different than that predicted by a semiclassical treatment. In particular, the shape of the zero-bias anomaly caused by the Coulomb blockade (a single-electron charging effect), is found to be strongly dependent on the impedance, $Z(\omega)$, of the leads connected to the junction. Comparison with three recent experiments demonstrates that the quantum mechanical treatment of this model correctly describes the essential physics in these systems.

1. Introduction

Recent advances in microfabrication techniques have allowed the study of tunnel junctions with capacitance so low that the charging energy associated with a single electron can be several meV [1, 2]. Under appropriate conditions, this charging energy can cause a suppression of tunneling, called a Coulomb blockade. This suppression has interesting consequences both for normal and superconducting tunnel junctions at low temperatures, including the Coulomb blockade of tunneling [3-6] Coulomb staircase [7-9], and various oscillatory and dynamic effects such as single-electron tunneling (SET) oscillations and Bloch oscillations [10-15]. Because of the difficulties associated with stray capacitance the earliest and clearest observations of these effects have been in double- or multijunction systems. Recently, however, several groups [16-18] have reported the observation of a partial Coulomb blockade in a single junction which lives within an electromagnetic "environment" controlled by parasitic capacitance and inductance in the wires (transmission lines) leading from the measurement apparatus to the junction. The strength and line shape of the blockade appear to be mainly controlled by the (frequency dependent) impedance, $Z(\omega)$, of the electromagnetic environment, with some additional dependence on the impedance of the junction itself. The main purpose of this paper [19] is to enlarge upon earlier discussions [20-22] of the role of quantum fluctuations of the environment on the electrical characteristics of the tunnel junction. We make a quantitative comparison between our theoretical results and the experimental findings of Delsing et al. [16], Cleland et al. [18] and Gregory [23] and find that the measurements can be correctly accounted for. In particular, the shape of the zero-bias anomaly caused by the Coulomb blockade (a single-electron charging effect), is found to be strongly dependent on the impedance of the leads connected to the junction. In addition, we discuss the effects of quantum charge fluctuations across the tunnel junction which become important when the junction resistance becomes as small as the quantum resistance $R_H = h/e^2$.

* *Present address:* NORDITA, Blegdamsvej 17, DK-2100 Copenhagen, Denmark

2. Model of the environment

In contrast to the standard semi-classical picture of the Coulomb blockade, in which the environment is treated classically, we treat the environment quantum mechanically. In both approaches, the tunneling event is treated quantum mechanically, as it must be. In the semi-classical picture the interesting physics of this system is that the charge state of the junction may be viewed as continuously variable (the bias voltage is continuously adjustable) but the discharge process is discrete - only an integer number (one) of electrons crosses the junction during each tunnel event. The energy cost of the tunneling event due to the charging of the junction is fixed to be

$$\Delta U = [(q-e)^2 - q^2]/2\,C_0 = -e V_0 + e^2/2\,C_0, \quad (1)$$

where C_0 is the junction capacitance, and $V_0 \equiv q/C_0$ is the initial bias voltage, which due to fluctuations, can be different from the externally applied voltage, V. In the semi-classical model, the only fluctuations are thermal fluctuations. If V_0 exceeds the blockade voltage, $e/2C_0$, ΔU is negative and the tunneling process is "down hill" in energy and hence allowed. Energy conservation is satisfied by the kinetic energy increase of the tunneling electron which ends up above the Fermi level on the other side of the junction.

Our quantum treatment is motivated by the fact that after an electron tunnels across an *isolated* tunnel junction a charge wave is launched down the leads that connect to the junction. Classically, the energy in this charge wave is the same as the energy due to the initial charging of the junction as in (1). However, since the charge wave is moving, it cannot be an eigenstate of the Hamiltonian, and cannot have a quantum mechanically well defined energy. Because the energy in the charge wave is uncertain, a quantum mechanical treatment allows tunneling at energies below the classical barrier. Detailed consideration of the energy distribution of the charge wave (excitation spectrum) is crucial to a proper analysis of the current-voltage (I-V) characteristics of the junction.

We begin our discussion by defining a model Hamiltonian. The present model [22], contains the same physics as the work of Nazarov [20] and is essentially identical to the model of Devoret et al. [21]. For a more detailed discussion of the derivation see [19].

Our major physical assumption is that the degrees of freedom of the many-body system (electrons plus electromagnetic fields) separate into microscopic single-particle modes and macroscopic collective modes. The total charge on the junction is viewed as arising from two contributions

$$q = ne + q_0, \tag{2}$$

where the "fermionic charge", ne, represents the (integer) number of electrons which have tunneled and q_0 is the "bosonic" collective polarization charge supplied by the transmission line. In a fully microscopic theory, one would have to calculate tunneling matrix elements between many body states that include all of the electronic degrees of freedom as well as the electromagnetic degrees of freedom. Our approximation involves treating the electrons (the single-particle modes) as non-interacting particles that only couple to the electromagnetic modes (macroscopic collective modes) when they transfer a charge from one side of the junction to the other. This approximation allows us to include only single-particle states for the electrons in the tunneling processes. The potential energy in the junction couples the single-particle electron states to the electromagnetic modes, and is

$$U = (ne + q_0)^2/2C_0. \tag{3}$$

As additional electrons tunnel, the "bosonic charge" adjusts itself to keep the net charge on the junction small. This adjustment comes about as the electromagnetic modes are suddenly displaced out of equilibrium due to the fast tunneling process. Below, we discuss the excitation that results from this adjustment, as it determines the details of the Coulomb blockade in these systems.

Degrees of freedom representing bulk and surface plasmons are excluded from the model, because the energy of these modes $\hbar\omega \gg eV$ greatly exceeds typical bias energies. However, these modes are important for the model we use because they allow the separation of degrees of freedom discussed above. After an electron tunnels, it is rapidly screened by the plasmons so that the physical location of the excess charge is independent of the physical location of the tunneled electron. The virtual excitation of these modes does renormalize the tunneling matrix element (by "Franck-Condon factors"), but this merely serves to renormalize what we mean by the "bare" junction impedance.

Furthermore, we neglect other inelastic effects such as coupling to phonons. While electron-phonon scattering will lead to destruction of the phase coherence of the elctronic degrees of freedom, no amount of microscopic upheaval in the Fermi sea can change the fact that an extra unit of charge has appeared on the capacitor and can be discharged down the transmission line *only* by collective displacement of those environmental degrees of freedom which we are keeping in the model. We do include the effect of phonon scattering from the electromagnetic modes in so far as that coupling affects the impedance of the transmission lines. We will see that the impedance determines the degree to which the electromagnetic modes are excited by the tunneling electron.

The interaction of the tunneling event with the environment is dominated by the leads that connect to the junction and carry the charge to and from it. We model the leads as transmission lines; for an ideal transmission line the Hamiltonian is the continuum limit of a lumped circuit model,

$$H = \frac{1}{2C_0} q_0^2 + \int_0^\infty \mathrm{d}x \, \frac{1}{2c} q(x)^2 + \int_0^\infty \mathrm{d}x \, \frac{l}{2} j(x)^2, \tag{4}$$

where $q(x)$ and $j(x)$ are the excess charge and the current density, respectively; c is the specific capacitance of the transmission line, and l is its specific inductance. The impedance of the transmission lines described by this Hamiltonian is a constant, $Z = \sqrt{l/c}$, independent of frequency. This Hamiltonian is a set of harmonic oscillators, which can be readily diagonalized to find the normal modes. It is straightforward to extend this model to non-ideal leads, following the ideas of Caldeira and Leggett [24]. For general leads the low energy electromagnetic degrees of freedom are modeled as a general set of harmonic oscillators chosen to give the correct impedance, $Z(\omega)$.

After an electron has tunneled, there is an excess charge of the junction capacitor, which we treat as having been created instantaneously. The charge on the junction decays as a function of time, with a characteristic discharge time τ_d, and a wave of charge is launched down the transmission line. The wave traveling down the line is a superposition of the normal modes of the transmission line, which have been displaced from their ground state by the tunneling event. Note that for an ideal transmission line, which is dispersionless, the shape of the wave does not change as a function of time. Classically, the wave has a well-defined energy, which is just the charging energy of the junction. On the other hand, a displaced quantum mechanical oscillator does not have a well defined energy as does a classical oscillator. Understanding the quantum mechanical behavior of the transmission line depends on understanding the distribution of energies in the charge wave that has been launched down the transmission line. To find this distribution we need to know how much the normal modes of the transmission line are displaced when a charge tunnels across the junction.

A normal-mode analysis [19] of the ideal transmission line described by the Hamiltonian (4) gives dispersionless modes, that is, the frequency of a mode is proportional to its wave vector, $\omega_k = k/\sqrt{lc}$. The analysis also gives the projection of a charge at the junction onto each of the normal modes. When an electron is suddenly placed on the junction capacitor each of the harmonic oscillators is displaced by an amount δq_k equal to the charge transferred, e, times the projection of the junction charge onto that mode. The mean excitation, λ_k, of each mode is related to the ratio of the square of this *classical* oscillator displacement to its *quantum mechanical* mean square fluctuations, $\langle q_k^2 \rangle$.

For the ideal transmission line we can use the dispersion, ω_k, the discharge time, $\tau_d = C_0 Z$, and a coupling constant defined by, $g = 2Z/R_H$, to write the mean excitation of each mode as,

$$\lambda_k = \frac{(\delta q_k)^2}{4\langle q_k^2 \rangle} = g \, \frac{1}{1+\omega_k^2 \tau_d^2} \, \frac{\Delta\omega_k}{\omega_k}, \tag{5}$$

where for a transmission line of finite length, L, $\Delta\omega_k = (2\pi/L)/\sqrt{lc}$, is the mode spacing. This result gives the average number of excited quanta of a particular mode. The probability of exciting n_k quanta is Poisson distributed,

$$P_{n_k} = \mathrm{e}^{-\lambda_k} \frac{\lambda_k^{n_k}}{n_k!}. \tag{6}$$

In the limit that the system size goes to infinity, the mode spacing goes to zero, $\Delta\omega_k \to \mathrm{d}\omega_k$ in (5), and hence λ_k and the probability of exciting any particular mode also goes to zero. On the other hand, the sum over all the modes of the mean excitation numbers diverges, and the probability of *not* exciting any mode, $\Pi_k P_{n_k=0}$, also goes to zero. Because of the infrared divergence in (5), an infinite number of low frequency electromagnetic modes are excited, and it follows that the probability to tunnel elastically (i.e. at zero bias) is zero. The junction is insulating because the state of the environment is orthogonal to its new displaced ground state. This "orthogonality catastrophe" is the origin of the power-law zero-bias anomaly in the differential conductance, $\mathrm{d}I/\mathrm{d}V \sim V^g$, to be discussed below.

The distribution of energies excited in the transmission line, $A(\omega)$, is given by the sum of all possible sets of excitation weighted by the probability of that set times a delta function of its energy [19],

$$A(\omega) = 2\pi \sum_{\{n_k\}} \prod_k P_{n_k} \delta\left(\omega - \sum_k n_k \omega_k\right). \tag{7}$$

After taking the limit that the system size goes to infinity, converting sums to integrals, and carrying out a bit of algebra, this expression can be written in terms of the spectral density, $a(\omega)$, which is the Fourier transform of the classical time dependence of the charge on the junction,

$$A(\omega) = \int_{-\infty}^{\infty} \mathrm{d}t \, \mathrm{e}^{i\omega t} \times \exp\left[\frac{e^2}{2C_0} \int_0^{\infty} \frac{\mathrm{d}\nu}{2\pi} a(\nu) \frac{(\mathrm{e}^{-i\nu t} - 1)}{\hbar\nu}\right]. \tag{8}$$

For an ideal transmission line, the excess charge on the junction decays exponentially, and the spectral density of transmission line modes displaced by the tunneling charge is a Lorentzian,

$$a(\omega) = \left(\frac{g}{1+\omega^2\tau_d^2}\right)(2\pi\hbar)\frac{2C_0}{e^2}. \tag{9}$$

For a more general transmission line (9) generalizes [22] to

$$a(\omega) = \mathrm{Re}\left[\frac{4}{-i\omega + 1/C_0 Z^*(\omega)}\right] \tag{10}$$

where $Z(\omega)$ is the impedance of the transmission line [25].

If we assume that the tunneling rate is energy independent in the small interval eV above the Fermi-level, the following simple phase-space arguments give the current at zero temperature. For a junction with an applied voltage, V, an electron can start with an energy, E, in the range 0 to eV. The tunneling event deposits an energy $\hbar\omega$ into the electromagnetic modes with probability density $A(\omega)$. If the final energy of the elctron, $E - \hbar\omega$ is below the Fermi level on the other side, then the tunneling is blocked. Integrating over all combinations that contribute to the current gives:

$$I = \frac{1}{eR_0} \int_0^{eV} \mathrm{d}E \int_0^{E/\hbar} \frac{\mathrm{d}\omega}{2\pi} A(\omega), \tag{11}$$

and the differential conductance thus obeys

$$\frac{\mathrm{d}I}{\mathrm{d}V} = \frac{1}{R_0} \int_0^{eV/\hbar} \frac{\mathrm{d}\omega}{2\pi} A(\omega). \tag{12}$$

At zero temperature the conductivity is proportional to the probability that the energy excited in the transmission line is smaller than the difference in the Fermi levels of the two sides of the junction. Although an infinite number of low frequency transmission line modes are excited, the average energy in these modes is finite and equal to the charging energy, $e^2/2C_0$. This guarantees that the conductance will obey the usual Coulomb offset at large voltages, where

$$I(V)=\frac{V-e/2C_0}{R_0}. \tag{13}$$

At small voltages the infrared divergence of the shake-up spectrum leads to a power-law zero-bias anomaly for the conductance [20–22, 26]

$$\frac{dI}{dV}\sim V^g. \tag{14}$$

The exponent, $g=2Z(0)/R_H$, of the power law depends on the low-frequency limit of the impedance and – emphasizing the quantum mechanical origin of the anomaly – the quantum of resistance, $R_H=h/e^2$.

At finite temperatures a formal calculation [27, 28] confirms the "obvious" generalization of the zero temperature expression, (8), for the shake-up excitation spectrum. The argument of the exponential becomes,

$$(e^{-i\omega t}-1)\rightarrow\{[n_B(\omega)+1](e^{-i\omega t}-1) + n_B(\omega)(e^{i\omega t}-1)\} \tag{15}$$

where $n_B(\omega)=1/(\exp(\hbar\omega/k_BT)-1)$ is the Bose factor, and the two terms correspond to stimulated photon emission and photon absorption. Using the generalized excitation spectrum we write the tunneling current from left to right as

$$I_{l\rightarrow r}=\frac{1}{eR_0}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}A(\omega)\int_{-\infty}^{\infty}d\varepsilon\, n_l(\varepsilon) \times[1-n_r(\varepsilon-\hbar\omega)]. \tag{16}$$

Here $n_l(\varepsilon)$ is the probability that the state of energy ε on the left side is occupied and $1-n_r(\varepsilon-\hbar\omega)$ is the probability that the state of energy $\varepsilon-\hbar\omega$ on the right side is empty. Hence we have taken the Pauli principle into account. Note that energy is conserved in the tunneling process. A similar expression describes tunneling from right to left, and the total current is just the sum of the two. The integration over energy, ε, can be done analytically and one finds that the net tunnel current is

$$I=\frac{1}{eR_0}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}A(\omega)[(\hbar\omega-eV)n_B(\hbar\omega-eV) - (\hbar\omega+eV)n_B(\hbar\omega+eV)]. \tag{17}$$

We have assumed that there is a potential difference between the left and right sides of the tunnel junction, each in equilibrium, $n_l(\varepsilon)=n_F(\varepsilon)$ and $n_r(\varepsilon)=n_F(\varepsilon-eV)$ with $n_F(\varepsilon)=1/(\exp(\varepsilon/k_BT)+1)$.

3. Beyond the transmission line model

In addition to the assumptions we discuss above, the separation of degrees of freedom, and the neglect of high energy plasmon modes, there are several other approximations that are implicit in the present model. We assume that the tunneling process is instantaneous so that the elctromagnetic modes behave as if they have been suddenly displaced. We also assume that higher order processes in perturbation theory in the tunneling matrix element are not important. This assumption manifest itself in several ways. First, we assume that the tunneling events are far enough apart in time that they do not affect each other; second, we assume that virtual tunneling events do not strongly renormalize these results, or wash them out. We further assume that the electrons in the leads form a good Fermi liquid. Here, we discuss some of these assumptions.

3.1. Finite traversal time

The chief assumption that has been made is that the environment oscillators are displaced suddenly by the "instantaneous" tunneling. We turn now to the question of the duration of the tunneling event. If we use the Landauer-Büttiker picture as a characteristic time scale for the very short but finite traversal time τ_T of the electon, then we see that the sudden approximation breaks down [22, 29] for short-wavelength modes k with frequencies $\omega_k\tau_T>1$. For the highest frequencies, it is more appropriate to use the *adiabatic* approximation in which the displaced oscillator gradually moves to its new ground state with essentially unit probability. Physically this mean that as the electron tunnels, the shortest wavelength transmission line modes "see it coming" and begin to transfer charge down the line away from the approaching electron. Given the $\omega_k\tau_T=1$ dividing line (which is of course not perfectly sharp) between sudden and adiabatic, we see that the adiabatic displacement will affect the transmission line out to the "horizon" distance τ_Tc. However, the only significant effect of this is to slightly reduce the mean shake-up energy; i.e., renormalize the effective junction capacitance upwards by the amount of transmission line capacitance distributed in the distance τ_Tc. The exponent g of the zero-bias anomaly is unaffected since it is independent of C_0. The "offset voltage" $e/2C_0$ as well as the range, of the non-linear region, on the other hand, will be reduced. For superconducting junctions the effective tunnel barrier for the "phase particle" can be quite small τ_Tc can be large, and τ_T can in fact be measured [30]. However for normal junctions the oxide barrier height is so large that τ_T is of order 1 fs and τ_Tc probably no more than a few tenths of a micron. While the existence of a single traversal time may be problematical, for these systems the appropriate times are almost certainly very short.

3.2. Finite junction resistance

From our exact solution of the general transmission line problem, we see that there is an infrared divergence associated with the excitation of an infinite number of low energy quanta. As a result, a displaced state of the oscillators is orthogonal to the ground state. This "orthogonality catastrophe" means that the probability to tunnel elastically (with no shake-up) is zero, and there is thus a singular zero-bias anomaly in which dI/dV vanishes as a power law at low bias.

The solution we have obtained is the exact result to leading order in an expansion in $1/R_0$, the "bare" conductance of the junction. For finite values of R_0 we must consider the effects of interference and correlation among multiple tunneling events [31–33]. Of particular interest is the question of the effect of a finite value of R_0 on the zero-bias anomaly.

The excitation spectrum $A(\omega)$ of the electro-magnetic modes given by (8) is exact for harmonic oscillators. In order to treat higher order effects in $1/R_0$ a lowest-order cumulant expansion in the fermion operators was done in [28] using a path integral formulation. This is an *approximation*, which allows for multiple correlated hops but treats the microscopic electron tunnel events using the equilibrium fermion Green's functions. Viewing the path integral as a statistical mechanics problem, tunnel events appear as positive and negative "charges" which interact logarithmically and are therefore correlated. Charges of different sign are attracted to each other and hence when few charges are present the most likely event after an electron tunnels forward across the junction (positive charge) is a tunneling backwards (negative charge), which tends to suppress the current. The chemical potential for the charges is related to the junction resistance, R_0. If R_0 is low the energy cost for creating charges (tunnel events) is small, and for sufficiently low junction resistance – with many charges present – screening effects may lead to an unbinding of the positive and negative charges and to a crossover or transition from a blockade state to a conducting state.

If we ignore the extra factors of the fermion Green's functions, the equivalent statistical mechanics problem is closely related [34] to the problem of a superconducting phase "particle" moving in a frictional medium [35–37]. It is known that there is a phase transition (at zero temperature) which, when translated into the present problem, would imply a finite conductance at zero bias for $g<1$ and a blockade for $g>1$. The transition is controlled however by the coupling constant g and *not* the value of the junction resistance R_0 relative to R_H. If this analogy holds, there is presumably a temperature below which the difference between this new conducting state and the blockade state is significant and above which it is not. While the existence of a transition to a renormalized conducting state depends on the coupling constant, g, rather than the junction resistance, this crossover temperature must depend on the junction resistance. Clearly this is an interesting question that deserves further study.

In recent theoretical work on ground state properties (as opposed to the conductance) of normal tunnel junctions indications of a transition from a blockade state to a conducting state controlled either by the junction resistance [32, 33, 38] or the resistance of the external circuit [31] have been found.

For small enough R_0 we presumably cannot make the lowest-order cumulant expansion which uses equilibrium fermion Green's functions. Particles tunnel back and forth quickly and, as in the Kondo problem, the single-particle occupation numbers "remember" the history of tunnel events. It is possible that there is a crossover (as in the Kondo problem) to a "Fermi-liquid-like" state with renormalized, but finite conductance.

There is an important distinction between the *continuous* charge fluctuations in the transmission line and the *discrete* fluctuations (in units of e) across the junction. In a Gaussian fluctuation or "spin-wave" approximation [28, 39, 40] this distinction is ignored. However, one obtains a tractable model in which the essential effect is a simple renormalization of the effective impedance [28],

$$Z^{-1}(\omega)\rightarrow Z^{-1}(\omega)+R_0^{-1}, \qquad (18)$$

due to the addition of the fluctuations across the junction and in the leads. This approximation correctly weakens the blockade for small R_0/R_H, but presumably cannot capture all of the physics discussed above, associated with the discreteness of the charge.

3.3. Electron-hole pair excitations

In a transmission line there are both electromagnetic (photon-like) boson modes and electron-hole pairs which behave as bosons. Since the electron-hole pair excitations create a charge disturbance, these modes are coupled. In the present model we have assumed that the important modes for low-energy losses are the combined modes that are derived in some adiabatic sense from the electromagnetic modes. We ignored the rest of the electron-hole pair derived modes. In contrast Ueda and Kurihara [41] have suggested that these modes *also* give rise to an infrared divergence and, therefore, should be important. We strongly doubt that this conclusion is correct.

Although Ueda and Kurihara's theory is formally analogous to that used for the X-ray edge problem, there are important differences between that problem and the tunneling problem. In the X-ray edge problem a high energy interaction suddenly creates a localized, immobile core-hole. In the tunneling problem a low energy tunneling event suddenly creates a charge disturbance that is neither localized, nor immobile on the *relevant length scale* for elecron-hole pairs. Since the charge disturbance is spread out spatially, the potential that scatters the electron-hole pairs, $V_{kk'}$, is appreciable only for $q=k-k'$ small. The restriction to small q substantially reduces the phase space available for exciting electron-hole pairs.

Without a spatially localized charge disturbance (like the core hole in the X-ray edge problem) Fermi liquid theory guarantees that there will not be any singularities due to electron-hole pairs. If the potential is not localized on an atomic scale, the potential only has low wave vector components, but in a Fermi liquid all of the low wave vector spectral weight is in the plasmons and not the electron-hole pairs. In addition, the charge disturbance is extremely mobile as it moves down the transmission line at the speed of light. This means that the scattering potential has to be time-dependent on the scale of electron-hole pair energies and cannot be treated as static as it is for a core-hole excitation. Hence the effect of the electron-hole pair modes should be much smaller than the effect of the electromagnetic modes contained in the present model. In particular the excitation spectrum for the electron-hole pair excitations should not be singular at low frequencies as is the excitation spectrum of the electromagnetic modes.

4. Comparison with experiments

Here we compare the results of the transmission line model with three measurements. We find that the model agrees well with the size of the zero-bias anomalies measured for isolated single junctions. Quantitative comparisons between theory and experiment are complicated by the difficulty in independently measuring the details of the systems. However, based on the ability to fit most aspects of the experimental data with parameters consistent with the experimentally estimated parameters, we are confident that the present model correctly describes the effect of the electromagnetic environment in these systems.

First we consider the experiment of Delsing et al. [16]. Figure 1 shows meaured and calculated conductivities of an isolated junction in a low impedance electromagnetic environment. The Coulomb blockade is almost completely washed out and the theory qualitatively accounts for this. The difference between the two calculated curves illustrates the difficulty in comparing theory with experiment. The dotted curve, which does not agree well at all, was calculated using the parameters estimated in the experimental paper. The solid curve was calculated by adjusting those parameters to get the size of the zero-bias anomaly approximately correct and the period of the oscillations approximately correct (see below). The adjustments necessary just to get the zero bias anomaly to be the right size are much smaller and well within experimental uncertainty. An obvious difference between the calculated and the measured curves is the long tails seen in the calculated curves. We believe that these tails are not found in the experimental curves just because of the difficulty in knowing experimentally what the asymptotic value of the resistance is.

One of the interesting features of the data is the appearance of small oscillations in dI/dV in the wings of the curve. Nazarov [42] has attributed these oscillations to random features in a universal conductance fluctuation type of model. It is not clear that random fluctuations of this type can explain the rather periodic oscillations in the data. We have considered the possibility that they are due to wave reflection from the discontinuity in the structure at the contact pad which is located [43] 1.5 mm away from the junction. The reflected waves produce periodic resonances in $Z(\omega)$ and hence modulation in the conductance, which by suitable choice of parameters can be made qualitatively consistent with the data. However, we find that the oscillations wash out very rapidly with temperature. Also, as the shake-up excitation spectrum $A(\omega = eV) \sim d^2I/dV^2$ is positive definite, the differential conductance is necessarily a monotonic function of voltage in this model, while in the experiment it seems not to be. As the data is not symmetric in voltage, it is conceivable that variations in the electron density of states

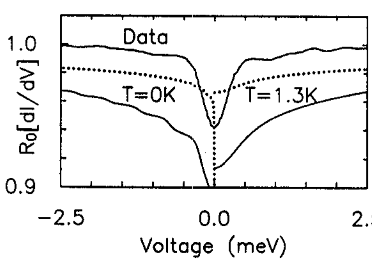

Fig. 1. Comparison of theory and the measurements of Delsing et al. [16]. The conductivity of an isolated junction is shown from both measurement (curve labeled data) and calculation. The calculated curves are shown at 0 K for negative voltages and 1.3 K for positive voltages. The solid curves were calculated using a specific inductance of, $l = 700$ fH/μm, and a specific capacitance of $c = 0.01$ fF/μm; the dotted curves were calculated using $l = 600$ fH/μm and $c = 0.1$ fF/μm. The structure in the zero temperature curves is due to structure in the impedance due to a discontinuity in the leads 1.5 mm from the junction. This structure is washed out at higher temperatures. The rest of the parameters for the calculations are discussed in the text

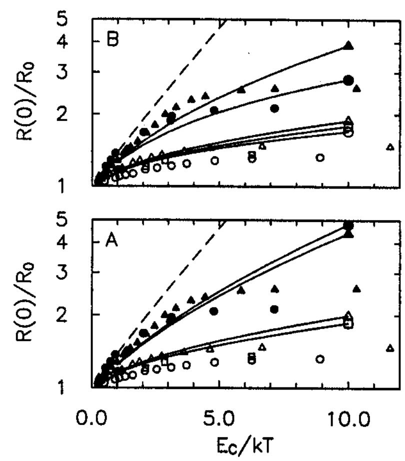

Fig. 2A, B. Comparison of theory and the measurements of Cleland et al. [18]. The zero bias resistivity is plotted vs. the charging energy divided by the temperature for a series of temperatures and a set of junctions. The data for each junction are shown using a different symbol. Calculated curves for each junction are labeled by attaching the appropriate symbol to the end of the theoretical curve. **A** shows the results of calculations that ignore the finite junction resistance; **B** includes the finite junction resistance in the spin-wave approximation (see text). The dotted line gives the semiclassical prediction. The solid symbols are for junctions connected to high resistivity NiCr leads, $r = 30\ \Omega/\mu m$, and the open symbols are for lower resistivity CuAu leads, $r = 2\ \Omega/\mu m$. All of the leads were $d = 12000$ μm long with a specific capacitance of $c = 0.0098$ fF/μm, and a specific inductance of $l = 600$ fH/μm [44]. The junction capacitances and resistances were as follows: (filled triangles) $C_0 = 5$ fF, $R_0 = 29$ kΩ, (filled circles) $C_0 = 6.5$ fF, $R_0 = 8.8$ kΩ, (open triangles) $C_0 = 4$ fF, $R_0 = 23$ kΩ, (open squares) $C_0 = 3$ fF, $R_0 = 27$ kΩ, (open circles) $C_0 = 3$ fF, $R_0 = 11$ kΩ

– assumed constant in the present model – is an important factor. Alternatively, resonant tunneling through localized states in the barrier might play a role. At present the origin of the oscillations is not properly understood.

In Fig. 2, a comparison is made with the experiments of Cleland et al. [18], where a single junction is connected by resistive transmission lines to the measuring apparatus and the current source. To compare a large range of experimental conditions it is useful to plot the differential resistance at zero bias under the different conditions. Here we plot the differential resistance at zero bias as a function of the ratio of the temperature to the charging energy of each junction. In the semiclassical model all of the results would fall along the same curve. The deviations of the data – due to quantum fluctuations – from the semiclassical result are well accounted for by the model, in particular at high temperatures. The model also reproduces the largest differences between the different sets of experimental data, which are due to the use of leads with different resistivities. The agreement gets worse as the temperature gets lower; the experimental data saturates while the calculated results continue to increase. The lack of agreement is due to the power-law divergence of the resistivity, $1/V^g$, at zero temperature in the present model. We speculate that including quantum fluctuations across the tunnel junction may account for the saturation. If these are included in the "spin-wave" approximation discussed above, a slightly better agreement with the data follows, but the power-law divergence persists. It is possible that to get saturation it is necessary to use a theory which keeps the discrete nature of the charge fluctuations (in units of e) across the junction.

In an interesting experiment of a different type, Gregory [23] measured the differential conductance between a series of two crossed platinum wires, separated by an adjustable-thickness (frozen) helium film. In the series, the junction resistance is found to vary as would be expected if the distance between the wires varied. He observes a zero-bias anomaly and at larger voltages a quadratic contribution, presumably due to the voltage dependence of the tunneling matrix element, which was subtracted off. The zero-bias anomaly decreases and eventually disappears as the junction resistance approaches, from above, a resistance close to the resistance quantum. There are indications that this behavior is to be expected [39].

The comparison of the present model with this experiment is more speculative than it is for the other experiments, mainly because less is known about these systems than the other systems. In particular, the capacitances of the junctions are not independently measured, and the impedance of the environment near the tunneling path is not known. Gregory assumes that the junction capacitances are quite small, $\sim 10^{-18} - 10^{-17}$ F, and that the impedance in the neighborhood of the junction is on the order of the resistance quantum. He attributes the differences between the measurements made for different junctions as due to the variation of the junction resistance between the different measurements, assuming that the capacitances of the junctions do not vary. He also assumes that almost the full blockade is observed due to

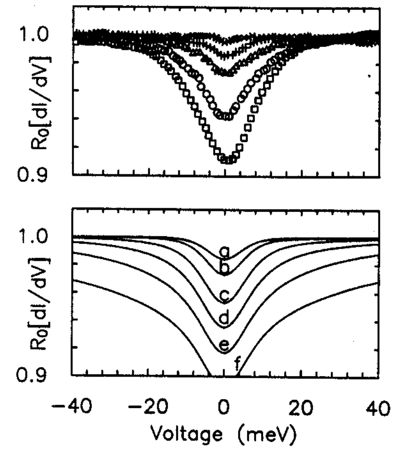

Fig. 3. Comparison of theory and the measurements of Gregory [23]. The bottom panel shows the calculated conductivity of isolated junctions with a series of different charging energies (E_C = (*a*) 0.1, (*b*) 0.2, (*c*) 0.5, (*d*) 1.0, (*e*) 2.0, (*f*) 5.0 meV) plotted as a function of voltage. The details of the calculation are discussed in the text. The top panel shows the measured conductivity for five different junctions

the large impedance he assumes for the immediate environment of the junction.

While the present model cannot accurately treat junctions with resistances close to the resistance quantum, we think that it can explain some aspects of the experimental data. In particular, we assume that the impedance of the environment is just due to the impedance of the platinum wires, which would be close to the impedance of free space, and we assume that the capacitances of the junctions are larger, in the range from 1.6×10^{-17} F to 8×10^{-15} F. In Fig. 3 we show how the present model might describe these measurements based on these assumptions. The coupling constant is chosen to be twice that of free space, 2.9×10^{-2}. To improve the agreement between the model and the data, we have artificially increased the temperature from the experimental value of 4.2 K to 20 K.

These assumptions have some strengths and weaknesses. The main strength is having the impedance of the leads be close to the impedance of free space rather than the resistance quantum. In this case we do not need to invoke a resistance that is not measured. This low impedance also gives zero-bias anomalies of about the right size. In the present model at least, much larger anomalies would be expected for the larger impedance environment. The energy scales in the model are the temperature and the energy at which the low-temperature power-law behavior crosses over to a Lorentzian behavior. For low impedance leads this is *much* larger than the charging energy. For charging energies between 1 and 50 meV, the temperature seems to be the only important energy scale, but for lower charging energies 0.1 to 0.5 meV both the temperature and the "Lorentzian" scales are important. By decreasing the charging energy it is possible to make the blockade go away as it does in the experiment as the wires get pushed closer together.

One obvious weakness is the larger temperature required to reproduce the observed widths of the zero-bias anomalies. We do not believe that heating is important in the measurement, but if the present model approximates the important physics of the measurement there must be something we have not included that would broaden the spectra. One possible broadening mechanism would be the finite junction resistance that we have otherwise ignored. In the spin-wave model the finite junction resistance would be irrelevant as it is much larger than the impedance of the leads. In an improved model which correctly describes the discrete nature of the quantum charge fluctuations between the wires, a transition to Fermi liquid-like behavior might well occur as the junction resistance approaches R_H from above.

Another weakness is that the range of capacitances is larger than would be expected for two crossed cylinders that are much closer than their radii. One possible explanation for this range is that the wires are close enough that the roughness of the surfaces might strongly affect the junction properties. Another possible explanation is that the finite junction resistance effects that we have not included decrease the size of the blockade as they become more important.

Finally the long tails that are seen in the results of the model calculation are not seen in the experimental data. We suspect that the tails that would be present in the data are removed by the subtraction of the quadratic background.

5. Summary

We have discussed the effect of the electromagnetic environment on tunneling in isolated tunnel junctions. We find that there is a big difference between what would be expected from a semiclassical analysis and what we find from our quantum mechanical analysis. In particular, if the leads attached to the junction have low impedances, then the Coulomb blockade of the tunneling is smeared out. These expectations are born out by comparison with three recent experiments.

By considering a detailed model of a transmission line, we have shown that a tunneling electron excites an infinite number of low energy electromagnetic bosons. The excitation of an infinite number of bosons leads to power-law behavior, $\mathrm{d}I/\mathrm{d}V \sim V^g$ for the differential conductivity, where the coupling constant, $g=2Z/R_H$, the ratio of the impedance of the leads to the quantum resistance, $R_H=h/e^2$. Thus, the behavior of the tunnel junctions at small voltages depends crucially on the impedance of the environment.

We have extended the results to finite temperatures, and to more complicated leads, including resistive leads, and leads with discontinuities in their properties. We have discussed the important approximations made in this calculation and the possible consequences of relaxing them. These approximations are: *1)* the degrees of freedom can be separated into microscopic single-particle modes and macroscopic collective modes, *2)* the tunneling event is instantaneous on the important time scales, and *3)* that the "bare" junction resistance is high enough that tunneling events are uncorrelated.

This model was used to analyze experiments on single aluminum/aluminum-oxide tunnel-junctions by Delsing et al. [16] and Cleland et al. [18], as well as on junctions formed by crossed platinum wires (Gregory [23]) separated by a frozen helium. The model qualitatively accounted for the zero-bias anomalies found in these measurements, particularly the size of the anomaly. We suggested reasons why the agreement was not perfect in all cases. The agreement between theory and experiment demonstrates that this model contains the essential physics necessary to explain the effect of the environment on tunneling.

We acknowledge research support from the US Department of Energy through contract DE-AC05-84OR21400 administered by Martin Marietta Energy Systems, Inc., and from the University of Tennessee (KF) and from the National Science Foundation through contract DMR-8802383 (SMG). We would also like to thank S. Gregory for permission to include his (yet) unpublished data.

References

1. For a review see, Averin, D.V., Likharev, K.K.: to appear in: Mesoscopic phenomena in solids. Altshuler, B.L., Lee, P.A., Webb, R.A. (eds.). Amsterdam: Elsevier 1991
2. For a review see Schön, G., Zaikin, A.D.: Phys. Rep. **198**, 238 (1990)
3. Ho, T-L.: Phys. Rev. Lett. **22**, 2060 (1983)
4. Ben-Jacob, E., Mottola, E., Schön, G.: Phys. Rev. Lett. **22**, 2064 (1983)
5. Kuzmin, L.S., Likharev, K.K.: JETP Lett. **45**, 495 (1987)
6. Fulton, T.A., Dolan, G.J.: Phys. Rev. Lett. **59**, 109 (1987)
7. Barner, J.B., Ruggerio, S.T.: Phys. Rev. Lett. **59**, 807 (1987)
8. Kuzmin, L.S., Likharev, K.K.: Jpn. J. Appl. Phys. [Suppl. 3] **26**, 1387 (1987)
9. Mullen, K., Ben-Jacob, E., Jaklevic, R.C., Schuss, Z.: Phys. Rev. B**37**, 98 (1988)
10. Kulik, I.O., Shekhter, R.I.: JETP **41**, 308 (1975)
11. Ben-Jacob, E., Gefen, Y.: Phys. Lett. A**108**, 289 (1985)
12. Averin, A.V., Likharev, K.K.: J. Low Temp. Phys. **62**, 345 (1986)
13. Delsing, P., Likharev, K.K., Kuzmin, L.S., Claeson, T.: Phys. Rev. Lett. **63**, 1861 (1989)
14. Geerligs, L.J., Anderegg, V.F., Holweg, P.A.M., Mooij, J.E., Pothier, H., Esteve, D., Urbina, C., Devoret, M.H.: Phys. Rev. Lett. **64**, 2691 (1990)
15. Kuzmin, L.S., Haviland, D.B.: Proceedings of Santa Fe Conference, May 1991. (submitted to Phys. Rev. Lett.)
16. Delsing, P., Likharev, K.K., Kuzmin, L.S., Claeson, T.: Phys. Rev. Lett. **63**, 1180 (1989)
17. Geerligs, L.J., Anderegg, V.F., Jeugd, C.A. van der, Rojmin, J., Mooij, J.E.: Europhys. Lett. **10**, 79 (1989)
18. Cleland, A.N., Schmidt, J.M., Clarke, J.: Phys. Rev. Lett. **64**, 1565 (1990)
19. See also Flensberg, K., Girvin, S.M., Jonson, M., Penn, D.R., Stiles, M.D.: (to be published)
20. Nazarov, Yu.V.: Zh. Eksp. Teor. Fiz. **95**, 975 (1989) [Sov. Phys. JETP **68**, 561 (1989)]; Pis'ma Zh. Eksp. Teor. Fiz. **49**, 1349 (1989) [JETP Lett. **49**, 126 (1989)]
21. Devoret, M.H., Esteve, D., Grabert, H., Ingold, G.-L., Pothier, H., Urbina, C.: Phys. Rev. Lett. **64**, 1824 (1990)
22. Girvin, S.M., Glazman, L.I., Jonson, M., Penn, D.R., Stiles, M.D.: Phys. Rev. Lett. **64**, 3183 (1990)
23. Gregory, S.: (to be published)

24. Caldeira, A.O., Leggett, A.J.: Phys. Rev. Lett. **46**, 211 (1981); Ann. Phys. **149**, 374 (1983); **153**, 445 (E)
25. We use the quantum mechanical convention for the time-dependence, $\exp(-i\omega t)$, rather than the engineering convention, $\exp(+i\omega t)$. However, $Z(\omega)$ refers to the impedance calculated using the engineering convention
26. Panyukov, S.V., Zaikin, A.D.: J. Low Temp. Phys. **73**, 1 (1988)
27. Ingold, G.-L., Grabert, H.: Europhys. Lett. **14**, 371 (1991)
28. Flensberg, K., Jonson, M.: Phys. Rev. B**43**, 7586 (1991)
29. Nazarov, Yu.V.: Phys. Rev. B**43**, 6220 (1991)
30. Esteve, D., Martinis, J.M., Urbina, C., Turlot, E., Devoret, M.H., Grabert, H., Linkwitz, S.: Phys. Scr. T**29**, 121 (1989)
31. Panyukov, S.V., Zaikin, A.D.: Moscow preprint (1991)
32. Zwerger, W.: Göttingen preprint (1991)
33. Scalia, V., Falci, G., Fazio, R., Giaquinta, G.: Physica B**165/166**, 975 (1990), and preprints (1991)
34. Fisher, M.P.A.: Private discussions
35. Guinea, F., Hakim, V., Muramatsu, A.: Phys. Rev. Lett. **54**, 262 (1985)
36. Fisher, M.P.A., Zwerger, W.: Phys. Rev. B**32**, 6190 (1985)
37. Leggett, A.J., Chakravarty, S., Dorsey, A.T., Fisher, M.P.A., Garg, A., Zwerger, W.: Rev.Mod. Phys. **59**, 1 (1987)
38. Brown, R., Šimánek, E.: Phys. Rev. B**38**, 9264 (1988)
39. Brown, R., Šimánek, E.: Phys. Rev. B**34**, 2957 (1986)
40. Odintsov, A.A.: Zh. Eksp. Teor. Fiz. **94**, 312 (1988) [Sov. Phys. JETP **67**, 1265 (1988)]
41. Ueda, M., Kurihara, S.: NTT Basic Research Laboratories preprint (1991)
42. Nazarov, Yu.V.: Zh. Eksp. Teor. Fiz. **98**, 306 (1990) [JRTP **71**, 171 (1990)]
43. Delsing, P.: Private communication
44. Cleland, A.N.: Private communication