PHYSICAL REVIEW B VOLUME 42, NUMBER 13 1 NOVEMBER 1990

Weak- versus strong-coupling theory for $Bi_2Sr_2CaCu_2O_8$

Marvin L. Cohen
Department of Physics, University of California at Berkeley, and Materials and Chemical Sciences Division, Lawrence Berkeley Laboratory, Berkeley, California 94720

David R. Penn
National Institute of Standards and Technology, Gaithersburg, Maryland 20899

(Received 16 July 1990)

The lastest optical determination of the superconducting-gap function for $Bi_2Si_2CaCu_2O_8$ is consistent with weak-coupling theory. In contrast, a large discontinuity in heat-capacity data suggest strong-coupling superconductivity, and appears to exceed the theoretical strong-coupling limit. A possible resolution to the apparent incompatibility of the two measurements is presented. The apparent disagreement with the results of strong-coupling theory is also resolved.

Because there is no consensus on the underlying mechanism responsible for high-temperature superconductivity (HTS) in the oxides, it is important to use experimental results to test the feasibility of various theoretical proposals. One important distinction between various theories is the classification into weak- or strong-coupling theories within a BCS framework.[1]

For most theories involving boson exchange pairing of electrons, the transition temperature T_c scales with the energy of the boson exchange ω_0 and is a monatomically increasing function of the coupling constant λ. For strong-coupling theories ω_0 is small and λ is large. For HTS, phonon mechanisms usually fall into this category. For large ω_0 such as those encountered in electronic mechanisms, λ is necessarily small to obtain a $T_c \sim 100$ K. Hence, most high-energy mechanisms are likely to be describable by weak-coupling theory.

Two experimentally measured ratios which signal the strength of the coupling are the zero-temperature gap $\Delta(0)$ to T_c, $2\Delta(0)/kT_c$ (where k is Boltzmann's constant), and the normalized discontinuity in the electronic heat capacity at T_c, $\Delta C/C$. In the past, these ratios have been regarded as definitive when used to ascertain whether strong or weak coupling is present. The BCS weak-coupling values $2\Delta(0)/kT_c = 3.52$ and $\Delta C/C = 1.43$ are consistent with measurements[2] on metals with small ratios of T_c to the Debye temperature. In contrast for Pb and Hg, which are strongly coupled superconductors, $2\Delta(0)/kT_c = 4.3$ and 4.6 and $\Delta C/C = 2.7$ and 2.4, respectively.[2]

Measurements of the gap for the HTS oxides vary considerably. Optical, tunneling, and photoemission values have not been consistent for the Y-Ba-Cu-O or the Bi-Sr-Ca-Cu-O systems. Variations arising because of gap anisotropy, impurity content, alignment, surface properties, and other caveats are stated as attempts to reconcile the different results. Here we consider the Bi-Sr-Ca-Cu-O (2:2:1:2) system where tunneling,[3] photoemission,[4] and optical[5] measurements yield gap ratios of 6,8.2, and 3.3, respectively.

In this study, we focus on the optical determination since it is the most detailed study of $\Delta(T)$. If the optically determined ratio is assumed, the application of conventional BCS theory suggests weak coupling for the 2:2:1:2 system. However, the heat-capacity discontinuity ratio[6–9] for the HTS oxides is in the neighborhood of or greater than 6, and these values suggests strong coupling. In fact, these ratios are beyond the theoretical limit of 3.73 obtained[10] for a standard isotropic Eliashberg superconductor. The data for the 1:2:3 oxides[6] are more definitive than for the 2:2:1:2 system at this time. The value of the electronic density of states is difficult to obtain for the latter case, and theoretical values are used. Although fluctuations and lattice contributions also contribute to obscure some of the results, the inconsistency between the experimental results remains. Below we account for the apparent conflict between these results by including effects arising from the temperature dependence of the gap. Large values of the heat-capacity ratio are obtained without including strong-coupling effects.

Following BCS, the discontinuity in the heat capacity at T_c is

$$\Delta C = kN(0)\beta_c^2 \left(\frac{\partial \Delta^2(T)}{\partial \beta} \right)_{T_c}, \tag{1}$$

where $\beta = 1/kT$, $\beta_c = 1/kT_c$, and $N(0)$ is the density of states at the Fermi level. For the normal state $C = \frac{2}{3}\pi^2 N(0)k^2 T_c$, and defining $A = 2\Delta(0)/kT_c$ we find

$$\frac{\Delta C}{C} = \frac{-3}{8\pi^2} A^2 T_c \frac{\partial}{\partial T} \left(\frac{\Delta(T)}{\Delta(0)} \right)^2 \Bigg|_{T_c}. \tag{2}$$

For the weak-coupling BCS model $A = 3.52$, and

$$T_c \frac{\partial}{\partial T} \left(\frac{\Delta(T)}{\Delta(0)} \right)^2 \Bigg|_{T_c} = -(1.74)^2. \tag{3}$$

Substituting back into Eq. (2) gives the BCS weak-coupling result.

If we now assume that the result $A \sim 3.3$ obtained by Brunel *et al.*[5] is correct and that it implies weak coupling, a consistent way to obtain values of $\Delta C/C \sim 6$ is to explore the variation with temperature of the superconducting energy gap. For example, if $T_c(\partial/\partial T)[\Delta(T)/$

$\Delta(0)]^2|_{T_c} \sim 9$, then both experiments can be reconciled within weak-coupling theory.

In Fig. 1, the data of Brunel *et al.*[5] is reproduced. The experiment involved a measurement of the reflectivity R at various fixed energies $\hbar\omega < \Delta(0)$ as a function of temperature. As the temperature is increased, the energy gap decreases until absorption at the fixed energy occurs and is observed as a sharp drop in R. At higher temperatures, R again becomes constant. The sharp feature in R is identified with $2\Delta(T)$, and this is given by the left-hand side of the horizontal lines in Fig. 1. The horizontal lines indicate the regions over which the drop in R occurs.

The solid lines in Fig. 1 represent fits to the data given by the arbitrary functional form for values near T_c.

$$\left(\frac{\Delta(T)}{\Delta(0)}\right)^2 = \frac{S(1-T/T_c)}{1+S(1-T/T_c)}, \tag{4}$$

where the slope of the function at $T/T_c = 1$ is determined by the specific-heat jump $\Delta C/C$. The curves labeled 3,5,7,9 correspond to specific-heat discontinuities having these values. In contrast, the BCS fit near T_c is

$$\left(\frac{\Delta(T)}{\Delta(0)}\right)^2 \approx 3.03\left(1-\frac{T}{T_c}\right). \tag{5}$$

In conclusion, it has been shown that recent optical measurements of the gap, $\Delta(T)$, for which $2\Delta(T)/kT_c = 3.3$ are consistent with a specific-heat jump $\Delta C/C > 4$ even though the former result implies weak coupling, and the latter result is normally interpreted as implying strong coupling. The apparent conflict between the observed $\Delta C/C$ values and the results of strong-coupling theory is resolved.

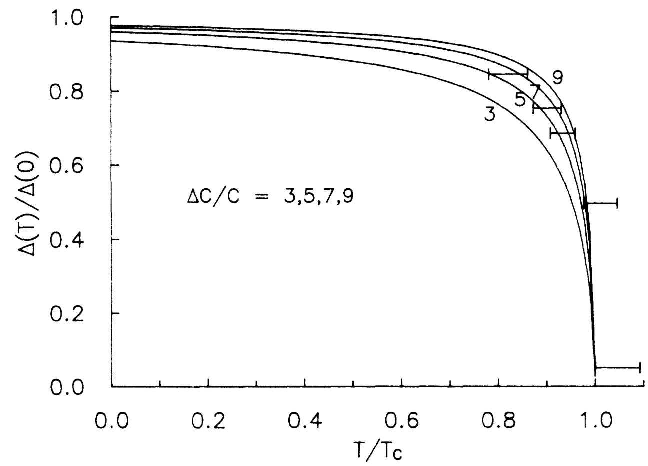

FIG. 1. Plot of the normalized energy gap vs normalized temperature. The horizontal lines represent the data of Brunel *et al.* (Ref. 5) as described in the text. The solid lines are given by Eq. (4) and represent a fit to the experiment for values of the specific-heat jump corresponding to $\Delta C/C = 3,5,7,9$.

We thank S. G. Louie and G. Martinez for communicating their results to us before publication and N. E. Phillips and R. A. Fisher for helpful conversations. One of us (M.L.C) was supported by National Science Foundation Grant No. DMR88-18404 and by the Director, Office of Energy Research, Office of Basic Energy Sciences, Materials Sciences Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

[1] M. L. Cohen, in *Novel Superconductivity,* edited by S. A. Wolf and V. Z. Kresin (Plenum, New York, 1987), p. 1095.

[2] R. Meservey and B. B. Schwartz, in *Superconductivity,* edited by R. D. Parks (Marcel Dekker, New York, 1969), p. 117.

[3] M. F. Crommie, G. Briceno, and A. Zettl, in *Proceedings of the International Conference on Materials and Mechanisms of Superconductivity: High Temperature Superconductors II,* edited by R. N. Shelton, W. A. Harrison, and N. E. Phillips (North-Holland, Amsterdam, 1989), p. 1397.

[4] J. M. Imer, F. Patthey, B. Dardel, W. D. Schneider, Y. Baier, Y. Petroff, and A. Zettl, Phys. Rev. Lett. **63**, 102 (1989).

[5] L. C. Brunel, S. G. Louie, G. Martinez, S. Labdi, and H. Raffy (unpublished).

[6] N. E. Phillips, R. A. Fisher, J. E. Gordon, S. Kim, A. M. Stacy, M.K. Crawford, and E. M. McCarron III, Phys. Rev. Lett. **65**, 357 (1990).

[7] R. A. Fisher, S. Kim, Y. Wu, N. E. Phillips, H. M. Ledbetter, and K. Togano, in *Proceedings of the International Conference on Materials and Mechanisms of Superconductivity: High Temperature Superconductors II* (Ref. 3), p. 502.

[8] N. Okazaki, T. Hasegawa, K. Kishio, K. Kitazawa, A. Kishi, Y. Ikeda, M. Takano, K. Oda, H. Kitaguchi, J. Takada, and Y. Miura, Phys. Rev. B **41**, 42967 (1990).

[9] N. E. Phillips, R. A. Fisher, and J. E. Gordon, in *Progress in Low-Temperature Physics B,* edited by D. Brewer (Elsevier, Amsterdam, in press).

[10] J. Blezius and J. P. Carbotte, Phys. Rev. B **36**, 3622 (1987).