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Mean free paths of very-low-energy electrons: The effects of exchange and correlation

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The mean free paths of low-energy electrons in a free-electron-like material are calculated using a screened electron-electron interaction which is antisymmetrized for the case of parallel-spin electrons. Calculations are carried out for several different approximations of the screening function: Fermi-Thomas, Lindhard, Singwi, and Kukkonen-Overhauser. The first three yield mean free paths that agree to within 10%, while the Kukkonen-Overhauser screening yields mean free paths that are roughly half those given by the other approximations. The effect of the Pauli principal in all cases is that the scattering between antiparallel-spin electrons is roughly three to ten times stronger than between parallel-spin electrons.

I. INTRODUCTION

The inelastic mean free paths of hot electrons in free-electron materials has been the subject of much study, and agreement between theory and experiment is good.[1] However, the mean free paths of electrons at very low energies has received relatively little attention due in large part to the difficulties in obtaining experimental data. Many years ago Ritchie and Ashley (RA)[2] pointed out that when the energy of the "hot" electrons is comparable to those of the solid it is necessary to take exchange into account. They carried out calculations in which the screening was given by Fermi-Thomas theory and the interaction between the hot electron and an electron of the solid was antisymmetrized when both had the same spin. This greatly reduced the interaction of the hot electron with those of parallel spin and the mean free paths they calculated where almost a factor of 2 larger than if exchange had been neglected. On the other hand, Kanter[3] later measured the mean free paths of 5-eV electrons in A1 and found them to be 50 Å compared to 62 Å predicted by the Quinn and Ferrell theory[4] which neglects exchange. Kleinman[5] introduced a different approximation for the exchange-corrected electron-electron interaction and found it gave little change from the Quinn theory. However, Kleinman's effective electron-electron interaction between parallel-spin electrons is not antisymmetric.

Since these early papers much work has been done on the effective electron-electron interaction, i.e., on the effects of exchange and correlation, and our purpose is to incorporate some of those results into a calculation of the electron mean free path for low-energy electrons, $1 \leq \epsilon/\epsilon_F < 1.2$, where ϵ_F is the Fermi energy and ϵ is the energy of the hot electron.

In Sec. II an expression for the mean free path in terms of the effective electron-electron interaction is given and in Sec. III various forms for the electron-electron interaction are discussed. These include the Fermi-Thomas, Lindhard, Singwi, and Kukkonen-Overhauser approximations of the interactions. In Sec. IV numerical results are presented for the electron mean free path and for the spin dependence of the scattering.

II. FORMULA FOR THE MEAN FREE PATH

Let the hot electron have momentum $\vec{p}_0$, energy ϵ_{p_0} and spin σ. After an interaction with an electron from the solid it is scattered into the state $\vec{p}_F$, ϵ_{p_F}, σ and the solid electron is scattered from $\vec{l}_0$, ϵ_{l_0}, σ' to $\vec{l}_F$, ϵ_{l_F}, σ'. The probability of this event per unit time is given by the Born approximation as

$$P_{p_0,\sigma} = \frac{2\pi}{\hbar} \sum_{l_0 l_F, p_F \sigma'} (1 - f_{p_F \sigma})(1 - f_{l_F \sigma'}) f_{l_0 \sigma'} \left| U^{\sigma\sigma'}(|\vec{p}_0 - \vec{p}_F|, \epsilon_{p_0} - \epsilon_{p_F}) - \delta_{\sigma\sigma'} U^{\sigma\sigma'}(|\vec{p}_0 - \vec{l}_F|, \epsilon_{p_0} - \epsilon_{l_F}) \right|^2 \times \delta(\vec{p}_0 + \vec{l}_0 - \vec{p}_F - \vec{l}_F)\, \delta(\epsilon_{p_0} + \epsilon_{l_0} - \epsilon_{p_F} - \epsilon_{l_F}) , \tag{1}$$

where f is a Fermi function and $U^{\sigma\sigma'}(q, \omega)$ is the screened electron-electron interaction between electrons of spin σ and spin σ'. If the electrons have the same spin then the exchange process must be taken into account and this results in the term proportional to $\delta_{\sigma\sigma'}$ in the matrix element of (1). The first delta function insures momentum conservation and the second gives energy conservation. Equation (1) is identical to that used by RA and it assumes that exchange is properly taken into account by antisymmetrizing an appropriately screened interaction, U. This approximation has recently been discussed in detail by Kukkonen and Wilkins.[6]

For $\epsilon_{p_0}/\epsilon_F \simeq 1$ one can make the approximation $|p_0| \simeq |p_F| \simeq |l_0| \simeq |l_F| \simeq k_F$, where k_F is the Fermi momentum. Consequently, the effective interaction becomes

$$U(q,\omega) \simeq U(q,0) \equiv U(q) \; . \tag{2a}$$

Also,

$$|\vec{p}_0 - \vec{p}_F| \simeq \sqrt{2}k_F\sqrt{1-\mu_F} \;, \quad |\vec{p}_0 - \vec{l}_F| \simeq \sqrt{2}k_F\sqrt{1-\mu_e} \;, \tag{2b}$$

where $\mu_F = \cos\theta_F$, and $\mu_e = \cos\theta_e$ and θ_F, θ_e are the angles between $\vec{p}_F$ and $\vec{p}_0$ and between $\vec{l}_F$ and $\vec{p}_0$, respectively. $\hat{p}_0$ is chosen as the direction of the z axis. Summing over $\vec{l}_0$ in Eq. (1) yields

$$\begin{aligned} p_{p_0,\sigma} = \frac{2\pi}{\hbar} \sum_{l_F p_F \sigma'} &(1-f_{p_F\sigma})(1-f_{l_F\sigma'}) \\ &\times f_{p_F+l_F-p_0\,\sigma'} I^{\sigma\sigma'}(\mu_F,\mu_e) \\ &\times \delta(\epsilon_{p_0} + \epsilon_{\vec{p}_F-\vec{p}_0} - \epsilon_{p_F} - \epsilon_{l_F}) \;, \end{aligned} \tag{3a}$$

where

$$I^{\sigma\sigma'}(\mu_F,\mu_e) = |U^{\sigma\sigma'}(\sqrt{2}k_F\sqrt{1-\mu_F}) - \delta_{\sigma\sigma'}U^{\sigma\sigma'}(\sqrt{2}k_F\sqrt{1-\mu_e})|^2 \; . \tag{3b}$$

Use of the relation

$$\vec{p}_F \cdot \vec{l}_F \simeq k_F^2[\mu_e\mu_F + (1-\mu_e^2)^{1/2}(1-\mu_F^2)^{1/2}\cos(\phi_e-\phi_F)]$$

in the delta function of Eq. (3b) followed by integration over l_F, p_F yields

$$p_{p_0\sigma} = \frac{4\pi}{\hbar}\frac{\pi}{\sqrt{2}}\frac{1}{\epsilon_f}\left(\frac{\Omega}{2\pi^3}\right)^3 \sum_{\sigma'} \int\int \frac{l_F^2 dl_F^2 p_F^2 dp_F \mu_e d\mu_F}{\sqrt{1-\mu_e}\sqrt{1-\mu_F}\sqrt{\mu_e+\mu_F}} I^{\sigma\sigma'}(\mu_F,\mu_e) \;, \tag{4}$$

where the integrations over l_f and p_f are subject to the conditions

$$\epsilon_{l_F}, \epsilon_{p_F} > \epsilon_F > \epsilon_{l_F} + \epsilon_{p_F} - \epsilon_{p_0} \; . \tag{5}$$

In the case $\sigma = \sigma'$ there is also the condition $\epsilon_{p_F} > \epsilon_{l_F}$ so as to avoid double counting. Integrating over l_F and p_F yields

$$p_{p_0}^{\sigma} = p_{p_0}^{\sigma\sigma} + p_{p_0}^{\sigma\bar{\sigma}} \;, \tag{6a}$$

where

$$\begin{aligned} p_{p_0}^{\sigma\sigma'} = A\left(\frac{\epsilon_{p_0}}{\epsilon_f} - 1\right)^2 \int_{-1}^{1}\int_{-1}^{1} &\frac{d\mu_e\, d\mu_F}{\sqrt{1-\mu_e}\sqrt{1-\mu_F}} \frac{1}{\sqrt{\mu_e+\mu_F}} \\ &\times [|U^{\sigma\sigma'}(\sqrt{2}\,k_F\sqrt{1-\mu_e})|^2 - \delta_{\sigma\sigma'}U^{\sigma\sigma'}(\sqrt{2}\,k_F\sqrt{1-\mu_e})U^{\sigma\sigma'}(\sqrt{2}\,k_F\sqrt{1-\mu_F})] \;, \end{aligned} \tag{6b}$$

where

$$A = \frac{1}{\hbar}\frac{e^2}{a_0}\left(\frac{1}{2^{7/2}\pi^2}\right)\left(\frac{\Omega k_F^2}{4\pi e^2}\right)^2 \;, \tag{6c}$$

where a_0 is the Bohr radius. Antisymmetrization of the screened interaction gives the term proportional to $\delta_{\sigma\sigma'}$ in (6b).

RA used

$$U^{\sigma\sigma'}(q) = (4\pi e^2/\Omega)[1/(q^2+q_{\mathrm{FT}}^2)] \;,$$

where q_{FT} is the Fermi-Thomas momentum. The mean free path is related to $p_{p_0}^{\sigma}$ by

$$\lambda_{p_0\sigma} = (\hbar k_F/m)(1/p_{p_0}^{\sigma}) \; . \tag{7}$$

For the purpose of numerical computation it is helpful to remove the singular denominator in the integral of Eq. (6b). This can be done by the change of variables, $y_e = \sqrt{1-\mu_e}$, $y_F = \sqrt{1-\mu_F}$ followed by $y_F = (2-y_e^2)^{1/2}\sin\theta$ to obtain the result

$$\begin{aligned} \int\int &\frac{d\mu_e}{\sqrt{1-\mu_F}}\frac{d\mu_F}{\sqrt{1-\mu_e}}\frac{1}{\sqrt{\mu_e+\mu_F}} \times g(\sqrt{1-\mu_e})h(\sqrt{1-\mu_F}) \\ &= 4\int_0^{\sqrt{2}} dy_e\, g(y_e)\int_0^{\pi/2} d\theta\, h((2-y_e^2)^{1/2}\sin\theta) \; . \end{aligned} \tag{8}$$

III. THE ELECTRON-ELECTRON INTERACTION

The interaction between the hot electron and an electron of the solid is screened by the other metal electrons. The effective interaction is shown in Fig. 1(a) in the usual "bubble" approximation. The first diagram represents the direct Coulomb interaction and the second represents the Coulomb interaction mediated by the excitation of an electron-hole pair. The diagrams can be summed to obtain

$$U = \nu + \nu(-\pi^0)\nu + \nu(-\pi^0)\nu(-\pi^0)\nu + \cdots$$
$$= \nu/(1+\nu\pi^0) \ , \tag{9a}$$

where

$$\nu(q) = 4\pi e^2/q^2\Omega \tag{9b}$$

and

$$\pi^0(q) = (0.166 r_s/x^2)L(x) \ , \tag{9c}$$

where

$$L(x) = \tfrac{1}{2} + \{[(1-x^2)/4x]\ln|(1+x)/(1-x)|\} \tag{9d}$$

and

$$x = q/2k_F \ , \tag{9e}$$

where $(4\pi/3)r_s^3 a_0^3 = \Omega N$; a_0 is the Bohr radius and N is the number of electrons. This approximation gives the Lindhard dielectric function; i.e.,

$$U_{\rm L}(q) = \nu(q)/\epsilon_{\rm L}(q) \ , \tag{10a}$$

where the Lindhard dielectric function is

$$\epsilon_{\rm L}(q) = 1 + \nu\pi^0 \ . \tag{10b}$$

$\epsilon_{\rm L}$ can be approximated by

$$\epsilon_{\rm FT}(q) = 1 + (q_{\rm FT}/q)^2 \tag{11a}$$

where

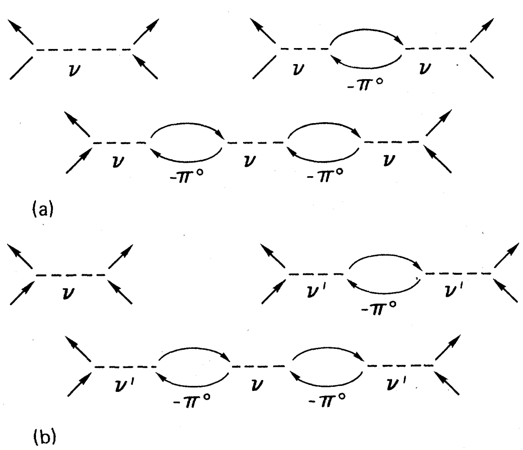

FIG. 1. (a) Diagrams for the effective electron-electron interaction resulting in screening by the Lindhard dielectric function. (b) Diagrams for the case that the two interacting electrons experience an interaction ν different from that of the screening electrons, ν'.

$$q_{\rm FT}^2 = (12/\pi)^{2/3}(a_0^2 r_s)^{-1} \ . \tag{11b}$$

This yields

$$U_{\rm FT}(q) = \nu/\epsilon_{\rm FT} \ . \tag{11c}$$

For small q, $\epsilon_{\rm FT} \simeq \epsilon_{\rm L}$, but for large q, $\epsilon_{\rm FT} > \epsilon_{\rm L}$. RA used $U_{\rm FT}$ in calculating the mean free path. This approximation for U is particularly convenient as it allows the integrations in Eq. (6a) to be done analytically.

A more sophisticated approach is to take account of exchange and correlation among the screening electrons as was done by Singwi and coworkers.[7] They derived the screened interaction

$$U_{\rm S}(q) = \nu/\epsilon_{\rm S} \ , \tag{12a}$$

where

$$\epsilon_{\rm S}(q) = 1 + [\nu\pi^0/(1 - G_{\rm S}\pi^0)] \ , \tag{12b}$$

where $G_s(q)$ can be expressed as

$$G_{\rm S}(q) = A(1 - e^{-4Bx^2}) \ , \tag{12c}$$

and A, B are functions of r_s given in Ref. 7. Equation (12) for $U_{\rm S}$ can be understood as follows. Let the interaction between two *screening* electrons be ν' rather than ν due to the effects of exchange and correlation. Then the interaction between the hot electron and an electron in the solid is from Fig. 1(b):

$$U_{\rm S} = \nu - \nu\pi^0\nu + \nu\pi^0\nu'\pi^0\nu - \nu\pi^0\nu'\pi^0\nu'\pi^0\nu + \cdots$$
$$= \nu[1 + (\nu' - \nu)\pi^0]/(1 + \nu'\pi^0) \ . \tag{13}$$

If $-G_{\rm S}\nu$ is the exchange and correlation potential then

$$\nu' = \nu(1 - G_{\rm S}) \ , \tag{14}$$

and use of Eq. (14) in Eq. (13) yields (12). Consequently, $\nu G_{\rm S}$ is the reduction in the electron-electron interaction of the screening electrons due to correlation and exchange.

As emphasized by RA, the hot electron and the electron with which it interacts must also experience an exchange and correlation correction so that a better approximation for U is obtained by replacing ν in Fig. 1(a) and Eq. (9) by $\nu' = \nu(1 - G_{\rm S})$ with the result

$$U_{\rm S'} = \nu' - \nu'\pi^0\nu' + \nu'\pi^0\nu'\pi^0\nu' - \cdots$$
$$= \nu'/(1 + \nu'\pi^0) \ . \tag{15}$$

I will use this form for the "Singwi" screened electron-electron interaction.

A still more realistic way to take exchange and correlation into account is allow for an interaction $\nu^{\sigma\sigma}$ between parallel-spin electrons and a different interaction $\nu^{\sigma\bar{\sigma}}$ between antiparallel-spin electrons. The interaction between the hot electron with spin σ and a metal electron with spin σ' is from Fig. 2,

$$U^{\sigma\sigma'} = \nu^{\sigma\sigma'} - \nu^{\sigma\sigma}\pi^0\nu^{\sigma\sigma'} - \nu^{\sigma\bar{\sigma}}\pi^0\nu^{\bar{\sigma}\sigma'} + \cdots$$
$$= \nu^{\sigma\sigma'} - \nu^{\sigma\sigma}\pi^0 U^{\sigma\sigma'} - \nu^{\sigma\bar{\sigma}}\pi^0 U^{\bar{\sigma}\sigma'} . \tag{16}$$

Solving Eq. (16) for $U^{\sigma\sigma}$, $U^{\sigma\bar{\sigma}}$ gives

$$U^{\sigma\sigma} = [\tfrac{1}{2}(V^+ + V^-) + \pi^0 V^+ V^-]/D \tag{17a}$$

and

$$U^{\sigma\bar{\sigma}} = \tfrac{1}{2}(V^+ - V^-)/D , \tag{17b}$$

where

$$D = (1 + V^+\pi^0)(1 + V^-\pi^0) \tag{17c}$$

and

$$V^+ = \nu^{\sigma\sigma} + \nu^{\sigma\bar{\sigma}} , \tag{17d}$$

$$V^- = \nu^{\sigma\sigma} - \nu^{\sigma\bar{\sigma}} . \tag{17e}$$

These results are identical to those of the Kukkonen and Overhauser[8] (KO) equations (32) and (33) for the interactions they denote by $\bar{V}$:

$$\bar{V}_{\uparrow\uparrow} = U^{\sigma\sigma} , \tag{18a}$$

$$\bar{V}_{\uparrow\downarrow} = U^{\sigma\bar{\sigma}} \tag{18b}$$

and

$$\nu(1 - 2G_x) = \nu^{\sigma\sigma} , \tag{18c}$$

$$\nu(1 - 2G_c) = \nu^{\sigma\bar{\sigma}} , \tag{18d}$$

where $-2\nu G_x$, $-2\nu G_c$ are the exchange and correlation potentials introduced by KO. They point out that the direct exchange and correlation potentials should be subtracted from $U^{\sigma\sigma}$ and $U^{\sigma\bar{\sigma}}$ to obtain the screened potentials

$$U_{\mathrm{KO}}^{\sigma\sigma} = U^{\sigma\sigma} - (\nu^{\sigma\sigma} - \nu) , \tag{19a}$$

$$U_{\mathrm{KO}}^{\sigma\bar{\sigma}} = U^{\sigma\bar{\sigma}} - (\nu^{\sigma\bar{\sigma}} - \nu) . \tag{19b}$$

In the notation of KO, $V_{\uparrow\uparrow} = U_{\mathrm{KO}}^{\sigma\sigma}$ and $V_{\uparrow\downarrow} = U_{\mathrm{KO}}^{\sigma\bar{\sigma}}$ with the result [KO, Eqs. (34) and (35)]

$$U_{\mathrm{KO}}^{\sigma\sigma} \equiv V_{\uparrow\uparrow} = \{\nu[1 + (1 - G_+)G_+Q]/D_1\} - (\nu G_-^2 Q/D_2) , \tag{20a}$$

$$U_{\mathrm{KO}}^{\sigma\bar{\sigma}} \equiv V_{\uparrow\downarrow} = \{\nu[1 + (1 - G_+)G_+Q]/D_1\} + (\nu G_-^2 Q/D_2) , \tag{20b}$$

$$G_+ = G_x + G_c , \tag{20c}$$

$$G_- = G_x - G_c , \tag{20d}$$

$$D_1 = 1 + (1 - G_+)Q , \tag{20e}$$

$$D_2 = 1 - G_- Q , \tag{20f}$$

$$Q = \nu\pi^0 . \tag{20g}$$

For the purposes of numerical calculations, values for G_+ and G_- were taken from Ref. 7.

IV. NUMERICAL RESULTS

From Eqs. (6) and (7) the ratio of the mean free path corresponding to the screened interaction $U^{\sigma\sigma'}$ to that of Quinn and Ferrell (QF), (which ignores exchange and uses $U_{\mathrm{L}} = \nu/\epsilon_{\mathrm{L}}$) is

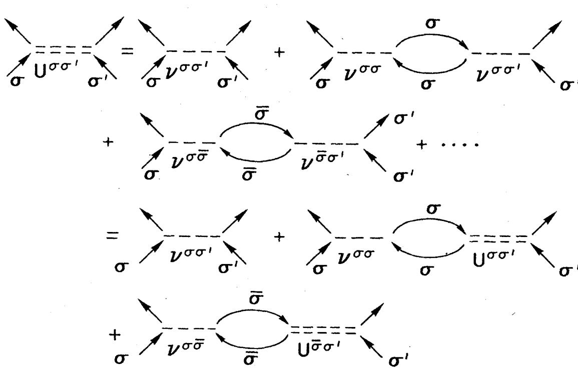

FIG. 2. Diagrams for the effective interaction $U^{\sigma\sigma'}$ between electrons of spin σ and σ' when their direct interaction is $\nu^{\sigma\sigma'}$.

TABLE I. Ratio of the mean free path to that calculated by the Quinn-Ferrel theory, λ_{QF}, using antisymmetrized electron-electron interactions given by (a) U_{FT}, the Fermi-Thomas interaction of Eq. (11); (b) U_{L}, the Lindhard interaction of Eq. (10); (c) $U_{\mathrm{S'}}$, the modified Singwi interaction of Eq. (15); (d) U_{KO}, the Kukkonen-Overhauser interaction given by Eq. (21). The ratios are independent of the hot-electron energy for low energies but depend on the electron density through r_s.

r_s	$\lambda_{\mathrm{FT}}/\lambda_{\mathrm{QF}}$	$\lambda_{\mathrm{L}}/\lambda_{\mathrm{QF}}$	$\lambda_{\mathrm{S'}}/\lambda_{\mathrm{QF}}$	$\lambda_{\mathrm{KO}}/\lambda_{\mathrm{QF}}$
1	1.53	1.51	1.51	1.01
2	1.75	1.70	1.73	0.90
3	1.90	1.81	1.88	0.80
4	2.01	1.89	1.99	0.71
5	2.10	1.93	2.09	0.64

$$\frac{\lambda}{\lambda_{\mathrm{QF}}}=2\int_{-1}^{1}\int_{-1}^{1}d\mu_e\,d\mu_F\,M(\mu_e,\mu_F)|\,U_{\mathrm{L}}(\sqrt{2k_F}\,\sqrt{1-\mu_E})\,|^2\left(\int_{-1}^{1}\int_{-1}^{1}d\mu_e\,d\mu_F\,M(\mu_e,\mu_F)I_0(\mu_e,\mu_F)\right)^{-1}, \tag{21a}$$

where

$$M(\mu_e,\mu_F)=(\sqrt{1-\mu_e}\,\sqrt{1-\mu_F}\,\sqrt{\mu_e+\mu_F}\,)^{-1} \tag{21b}$$

and

$$I_0(\mu_e,\mu_F)=U^{\sigma\bar{\sigma}}(\sqrt{2k_F}\,\sqrt{1-\mu_e})^2+U^{\sigma\sigma}(\sqrt{2k_F}\,\sqrt{1-\mu_e})^2-2U^{\sigma\sigma}(\sqrt{2k_F}\,\sqrt{1-\mu_e})\,U^{\sigma\sigma}(\sqrt{2k_F}\,\sqrt{1-\mu_F})\,. \tag{21c}$$

The ratio of scattering of the hot electron from electrons with spin parallel to it to that of scattering from antiparallel electrons is from Eq. (6)

$$\frac{p^{\sigma\sigma}}{p^{\sigma\bar{\sigma}}}=\int_{-1}^{1}\int_{-1}^{1}d\mu_e\,d\mu_F\,M(\mu_e,\mu_F)\left[\,U^{\sigma\sigma}(\sqrt{2k_F}\,\sqrt{1-\mu_e})^2-U^{\sigma\sigma}(\sqrt{2k_F}\,\sqrt{1-\mu_e})U^{\sigma\sigma}(\sqrt{2k_F}\,\sqrt{1-\mu_F})\right]$$
$$\times\left(\int_{-1}^{1}\int_{-1}^{1}d\mu_e\,d\mu_F\,M(\mu_e,\mu_F)U^{\sigma\bar{\sigma}}(\sqrt{2k_F}\,\sqrt{1-\mu_e})^2\right). \tag{22}$$

Equations (21) and (22) have been evaluated numerically with the help of Eq. (8) for a number of different screened interactions, $U^{\sigma\sigma'}$: (a) U_{FT}, the Fermi-Thomas interaction of Eq. (11); (b) U_{L} the Lindhard interaction of Eq. (10); (c) $U_{\mathrm{S}'}$, the modified "Singwi" interaction of Eq. (15); and (d) U_{KO}, the Kukkonen-Overhauser interaction of Eq. (21). Results for $\lambda/\lambda_{\mathrm{QF}}$ as a function of r_s are presented in Table I and results for $p^{\sigma\sigma}/p^{\sigma\bar{\sigma}}$ as a function of r_s are given in Table II.

The results for $\lambda/\lambda_{\mathrm{QF}}$ are remarkably similar for interactions U_{FT}, U_{L} and $U_{\mathrm{S}'}$. The resultant mean free paths are considerably longer than those given by the Quinn-Ferrell theory because exchange greatly weakens the interaction between parallel-spin electrons. This effect increases with increasing r_s due to the decreased momentum dependence of the screened interactions. The relationship between the parallel-spin interaction and the momentum dependence of the interaction is due to the following. In the limit that the exchange interaction is independent of momentum transfer and energy loss the exchange term must exactly cancel the Coulomb term for interactions between parallel-spin electrons and I have assumed that the interaction is energy independent for low-energy electrons. On the other hand the Kukkonen-Overhauser interaction is much stronger than other interactions due to the fact that *direct* exchange and correlation have been removed in forming the interaction [see Eq. (19)]. Thus, despite the fact that the interaction between the hot electron and those of parallel spin is very weak relative to those of antiparallel spin, the mean free paths are actually shorter than those of the Quinn-Ferrell theory in which both parallel- and antiparallel-spin scattering are of equal importance. The ratios $p^{\sigma\sigma}/p^{\sigma\bar{\sigma}}$ are given in Table II and are in large part a measure of the momentum dependence of the screened interactions; the smaller the momentum dependence the smaller is $p^{\sigma\sigma}/p^{\sigma\bar{\sigma}}$ as is evident from Eq. (22).

TABLE II. Ratio of the scattering probability of a hot electron from electrons of parallel spin to the scattering probability from electrons of antiparallel spin for the Fermi-Thomas, Lindhard, modified Singwi, and Kukkonen-Overhauser interactions.

r_s	$(p^{\sigma\sigma}/p^{\sigma\bar{\sigma}})_{\mathrm{FT}}$	$(p^{\sigma\sigma}/p^{\sigma\bar{\sigma}})_{\mathrm{L}}$	$(p^{\sigma\sigma}/p^{\sigma\bar{\sigma}})_{\mathrm{S}'}$	$(p^{\sigma\sigma}/p^{\sigma\bar{\sigma}})_{\mathrm{KO}}$
1	0.34	0.33	0.47	0.35
2	0.20	0.18	0.34	0.21
3	0.14	0.10	0.25	0.14
4	0.10	0.060	0.19	0.095
5	0.077	0.036	0.14	0.069

V. CONCLUSION

As pointed out by RA the inclusion of exchange through antisymmetrization of the effective interaction results in a drastic reduction in the scattering of the hot electron by those of parallel spin relative to scattering from antiparallel-spin electrons. However, inclusion of exchange and correlation corrections via the KO theory results in a much stronger interaction than that suggested by conventional theory. Consequently, mean free paths predicted by the KO theory are smaller than those given by Quinn-Ferrell despite the fact that in the latter theory the interaction of parallel-spin electrons is equal to that of antiparallel spin. The only low-energy mean-free-path measurements on a free-electron material were carried out by Kanter[3] who found a mean free path of 50 Å for 5-eV electrons in Al. The Quinn-Ferrell theory predicts 62 Å, and from Table I ($r_s = 2$), only the KO theory gives reasonable agreement with Kanter's result which is, however, subject to experimental uncertainties.

[1]C. J. Powell, Surf. Sci. 44, 29 (1974).
[2]R. H. Ritchie and J. C. Ashley, J. Phys. Chem. Solids 26, 1689 (1963).
[3]H. Kanter, Phys. Rev. B 1, 522 (1970).
[4]J. J. Quinn, Phys. Rev. 126, 1453 (1962).
[5]L. Kleinman, Phys. Rev. B 3, 2982 (1971).
[6]C. A. Kukkonen and J. W. Wilkins, Phys. Rev. B 19, 6075 (1979).
[7]K. S. Singwi, A. Sjolander, M. P. Tosi, and R. H. Land, Phys. Rev. B 1, 1044 (1970).
[8]C. A. Kukkonen and A. W. Overhauser, Phys. Rev. B 20, 550 (1979).