Focusing and dispersing properties of a stigmatic crossed-field energy analyzer

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(Received 9 January 1978)

The electron-optical properties of a stigmatic crossed-field energy analyzer (double-focusing Wien filter) have been obtained from exact trajectory calculations. The results are given in the form of focusing and dispersing coefficients to the second order. These coefficients enable the device designer or potential user to calculate the total beam transfer and evaluate the resulting beam quality without additional ray tracing. The specific device for which calculations are made employs a uniform electric field and a toroidal magnetic field. This analyzer is of special interest in our laboratory because it can be constructed with a very small stray magnetic field, and in addition to its dispersive properties it also rotates the spin of a polarized electron beam.

PACS numbers: 41.80.Dd

INTRODUCTION

The principle of using crossed uniform electric and magnetic fields as a velocity analyzer (Wien filter) has been known for many years.[1] It was only much later that, in addition to its velocity dispersive properties, the first-order focusing properties were analyzed,[2–4] but only in the plane of dispersion. More recent analyses[5–7] have concentrated on the first-order properties of short devices, and some of the second-order terms[8,9] in the plane of dispersion. Such crossed-field devices have been used, for example, as mass analyzers,[1,4,5,10,11] energy analyzers,[8,12] and monochromators.[8]

For uniform fields which are mutually perpendicular to each other and the direction of beam propagation, focusing occurs only in the plane of energy dispersion[2,3] which is the direction of the electric field. Additional astigmatic lenses are required for those applications where preservation of rotational beam symmetry is essential. Legler[13] was the first to show that by curving the electric field one can obtain stigmatic focusing while keeping the magnetic field uniform. Legler calculated the second-order angular aberrations for the case of an electric field produced by nonconcentric cylinders and demonstrated a successful electron monochromator. Wahlin[5] showed that, by introducing a gradient into the electric field, the astigmatic focusing could be canceled. This principle has been used in mass analyzers.[5,14] Anderson[9] was the first to discuss the possibility of obtaining stigmatic focusing by using a non-uniform magnetic field. He also showed that a second-order inhomogeneity in the magnetic field will eliminate the second-order angular aberration [the term in α_1^2 in Eq. (6)] in the dispersion plane. This principle has been used in a high transmission electron monochromator and energy analyzer.[15] Seliger[7] showed that astigmatic focusing could be obtained by using tilted pole pieces to produce the magnetic field while keeping the electric field uniform. He calculated the first-order properties both in the plane of dispersion and in the perpendicular plane.

Finally, Collins[16] has calculated the first-order properties for devices in which cylindrical electric fields and toroidal magnetic fields are superimposed, and showed that it is always possible to obtain stigmatic focusing by adjusting the length of the fields and their relative strengths. He has designed and constructed two stigmatic devices, one with uniform electric field and one with uniform magnetic field. Of particular interest to us is the fact, shown by Collins, that the stigmatic devices can be used to transform a spin-polarized electron beam from transverse to longitudinal polarization and vice versa while performing energy analysis. The design of such a device requires knowledge of the second-order aberration coefficients which are not known. We have therefore calculated the complete second-order properties for the case of uniform electric field and toroidal magnetic field. Collins has shown that this device has advantages both in ease of construction and in lack of stray magnetic fields outside of the toroidal winding used to produce the magnetic field. This latter property is especially advantageous for experiments with low energy electrons.

FIELD GEOMETRY AND FIRST-ORDER PROPERTIES

We will discuss the motion of an electron initially moving at the origin with velocity v_0 (energy eV_0) in the z direction under the action of a uniform transverse electric field and a toroidal magnetic field[17] (See Fig. 1):

$$\mathbf{E} = [E_x, E_y, E_z] = [E_0, 0, 0],$$

$$\mathbf{B} = [B_x, B_y, B_z] = \left[B_0 \frac{R_0 y}{(R_0 - x)^2 + y^2},\; B_0 \frac{R_0(R_0 - x)}{(R_0 - x)^2 + y^2},\; 0 \right]. \quad (1)$$

Here R_0 is the radius of the toroidal field at the z axis. In this treatment, as in all previous work,[1–16] we neglect the effects of fringing fields. Because of the small extent of the fringing fields compared to the total length, these effects are expected to be small.

The beam is transmitted undeflected when

$$E_0 = v_0 B_0. \tag{2}$$

First-order stigmatic focusing requires that

$$R_0 = 4V_0/E_0 \tag{3}$$

and the corresponding focusing length L is given by

$$L/R_0 = \pi/\sqrt{2}. \tag{4}$$

GENERAL FOCUSING AND DISPERSIVE PROPERTIES

In the absence of axial symmetry, we use Cartesian coordinates. The parameters of a ray in the input plane $z = 0$ are $(x_1, \alpha_1, y_1, \beta_1)$ where α_1 and β_1 are the ray slopes dx/dz and dy/dz, respectively, z being the direction of beam propagation. At the output plane $z = L$ the corresponding parameters are $(x_2, \alpha_2, y_2, \beta_2)$. We express these output parameters as functions of the input parameters and the energy deviation δ given by $V = V_0(1 + \delta)$ where eV_0 is the energy of the central ray $(x_1 = \alpha_1 = y_1 = \beta_1 = 0)$:

$$\begin{aligned}
\frac{x_2}{R_0} &= f_1\left(\frac{x_1}{R_0}, \alpha_1, \frac{y_1}{R_0}, \beta_1, \delta\right)\\
\alpha_2 &= f_2(x_1/R_0, \alpha_1, y_1/R_0, \beta_1, \delta)\\
\frac{y_2}{R_0} &= f_3\left(\frac{x_1}{R_0}, \alpha_1, \frac{y_1}{R_0}, \beta_1, \delta\right)\\
\beta_2 &= f_4\left(\frac{x_1}{R_0}, \alpha_1, \frac{y_1}{R_0}, \beta_1, \delta\right).
\end{aligned} \tag{5}$$

Note that the position coordinates are expressed in dimensionless form by dividing by the radius of curvature of the toroidal magnetic field at the central ray. In this way the focusing and dispersing properties are independent of the physical size of the crossed-field analyzer.

Assuming that all of the input parameters in Eq. (5) are much smaller than unity, it is possible to express the output parameters as power series in the input parameters[18]:

$$\begin{aligned}
\frac{x_2}{R_0} = {} & A_x\frac{x_1}{R_0} + A_\alpha\alpha_1 + A_\delta\delta + A_y\frac{y_1}{R_0} + A_\beta\beta_1\\
& + A_{xx}\left(\frac{x_1}{R_0}\right)^2 + A_{yy}\left(\frac{y_1}{R_0}\right)^2 + A_{\alpha\alpha}\alpha_1^2 + A_{\beta\beta}\beta_1^2 + A_{\delta\delta}\delta^2\\
& + A_{xy}\frac{x_1}{R_0}\frac{y_1}{R_0} + A_{\alpha\beta}\alpha_1\beta_1 + A_{x\alpha}\frac{x_1}{R_0}\alpha_1 + A_{x\beta}\frac{x_1}{R_0}\beta_1\\
& + A_{x\delta}\frac{x_1}{R_0}\delta + A_{y\alpha}\frac{y_1}{R_0}\alpha_1 + A_{y\beta}\frac{y_1}{R_0}\beta_1 + A_{y\delta}\frac{y_1}{R_0}\delta\\
& + A_{\alpha\delta}\alpha_1\delta + A_{\beta\delta}\beta_1\delta + \ldots
\end{aligned} \tag{6}$$

The expansions for α_2, y_2/R_0, and β_2 are of the same form, with coefficients B, C, and D, respectively. While some of the coefficients were expected to be zero, all of the second-order terms were retained since we did not know *a priori* which ones would vanish. It should be possible, however, to determine which coefficients are zero from a general treatment such as that of Hawkes.[19]

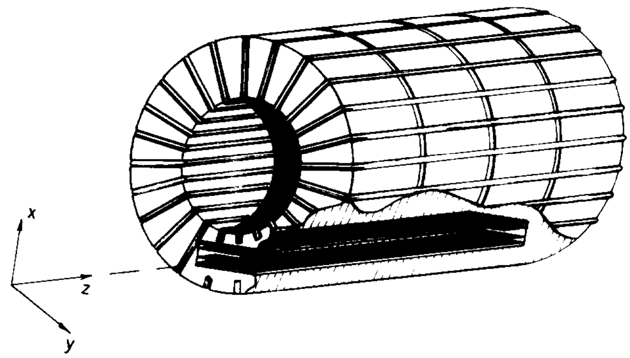

FIG. 1. Stigmatic Wien filter composed of toroidal magnetic and uniform electric field.

We have limited this investigation to second-order effects although it became clear in the course of the calculations that the third-order coefficients can make significant contributions in some cases when the input parameters exceed about 0.05. However, this range of validity is adequate for many applications. Furthermore, in view of the labor which would be needed to evaluate 35 third-order coefficients for each of the four parameters by the present methods, it appears desirable to seek an alternate approach to calculating coefficients of higher orders.

The first five coefficients of the expansion are the first-order matrix elements, which in the plane of dispersion (xz plane) can be written as

$$\begin{bmatrix}\frac{x_2}{R_0}\\ \alpha_2\end{bmatrix} = \begin{bmatrix}A_x & A_\alpha & A_\delta\\ B_x & B_\alpha & B_\delta\end{bmatrix}\begin{bmatrix}\frac{x_1}{R_0}\\ \alpha_1\\ \delta\end{bmatrix} \tag{7}$$

with a similar matrix for y_2/R_0 and β_2. From the work of Seliger,[7] these first-order matrices are given by:

$$\begin{bmatrix}\frac{x_2}{R_0}\\ \alpha_2\end{bmatrix} = \begin{bmatrix}-1 & 0 & 1\\ 0 & -1 & 0\end{bmatrix}\begin{bmatrix}\frac{x_1}{R_0}\\ \alpha_1\\ \delta\end{bmatrix} \tag{8}$$

$$\begin{bmatrix}\frac{y_2}{R_0}\\ \beta_2\end{bmatrix} = \begin{bmatrix}-1 & 0 & 0\\ 0 & -1 & 0\end{bmatrix}\begin{bmatrix}\frac{y_1}{R_0}\\ \beta_1\\ \delta\end{bmatrix} \tag{9}$$

CALCULATIONS OF TRAJECTORIES

The equations of motion are obtained from the Lorentz force $\mathbf{F}$ on an electron of velocity $\mathbf{v}$ in a combined electric and magnetic field:

$$\mathbf{F} = -e[\mathbf{E} + \mathbf{v} \times \mathbf{B}] \tag{10}$$

where e is the absolute value of the electronic charge. With the fields from Eq. (1), the differential equations are

$$\begin{aligned}
m\ddot{x} &= eE_x + e\dot{z}B_y\\
m\ddot{y} &= -e\dot{z}B_x\\
m\ddot{z} &= e\dot{y}B_x - e\dot{x}B_y
\end{aligned} \tag{11}$$

where the dots represent differentiation with respect to time. These three differential equations are integrated simultaneously by a standard Runge–Kutta technique with a time step of 10^{-10} s for 10 eV electrons. The actual calculations were performed on a UNIVAC 1108 computer, using double-precision arithmetic. From the integrations one obtains (x,y,z) and $(\dot{x},\dot{y},\dot{z})$ as a function of time. The ray slopes are obtained from $dx/dz = \dot{x}/\dot{z}$ and $dy/dz = \dot{y}/\dot{z}$. This method was used to calculate two specific cases quoted by Seliger.[7] Excellent agreement was obtained.

Using this method we can calculate any desired set of trajectories through the device. One might, for example, evaluate beam performance by using the phase-space approach where input and output beams are represented by emittance diagrams. We have chosen to evaluate the coefficients in Eq. (5), because they allow the device designers to evaluate device performance without making detailed trajectory calculations.

CALCULATION OF THE TRANSFER COEFFICIENTS

The transfer coefficients are obtained from selected sets of trajectories using a procedure we developed from a method used by Foster.[20] For coefficients which involve a single parameter, say α_1, trajectories are traced for several values of α_1 with all other input parameters zero. Each output parameter is plotted as a function of α_1 and the slope of the initial straight line portions gives the first-order coefficients. The first-order portions are subtracted, the results divided by α_1, and replotted. The slope of the initial straight line portion now gives the second order coefficient. For coefficients which involve two parameters, say α_1 and β_1, trajectories are traced for several values of $\alpha_1 = \beta_1$ with the other input parameters zero. After subtracting the terms in α_1, β_1, α_1^2 and β_1^2 the term in $\alpha_1\beta_1$ is obtained by dividing by $\alpha_1 = \beta_1$, plotting, and determining the slope of the initial straight-line portion. An advantage of this procedure is that it shows clearly where higher-order terms begin to influence the results.

The results of these calculations are given in Table I. The first-order coefficients were found to be well within 1% of those expected from Seliger's work[7] [Eqs. (8) and (9)]. In extracting the second-order coefficients, the first-order coefficients were equated to the theoretical values. From the scatter of points about the straight-line fits we estimate that the second-order coefficients are accurate to about 5%, except for the smaller coefficients where the accuracy is about 10%. As an independent check of the accuracy of the second-order coefficients we carried out second-order analytical slolutions of Eqs. (11), following Legler's treatment,[13] and obtained values for $A_{\alpha\alpha}$, $A_{\alpha\beta}$, $A_{\beta\beta}$, $C_{\alpha\alpha}$, $C_{\alpha\beta}$, and $C_{\beta\beta}$. We found $A_{\alpha\alpha} = -4.33$, $A_{\beta\beta} = -1.00$, $C_{\alpha\beta} = -2.00$ and $A_{\alpha\beta} = C_{\alpha\alpha} = C_{\beta\beta} = 0$. The largest disagreement is about 7% in $A_{\alpha\alpha}$. Coefficients less than 0.02 are not considered to be significant and are given as zero in Table I.

By extending Legler's method, it should be possible with considerable labor to calculate all of the second-order coefficients analytically. However, it would seem more desirable to use a general treatment, such as that of Hawkes,[19] to obtain the coefficients in the form of aberration integrals. In any case, the method presented here of obtaining the coefficients directly from trajectories is valuable as an independent check of such analytic forms.

TABLE I. Stigmatic Wien filter focusing and dispersing coefficients to the second order. Vertical column designates the coefficient subscripts as in Eq. (6).

	A	B	C	D
x	−1.0	0.0	0.0	0.0
α	0.0	−1.0	0.0	0.0
y	0.0	0.0	−1.0	0.0
β	0.0	0.0	0.0	−1.0
δ	1.0	0.0	0.0	0.0
xx	−3.61	0.0	0.0	0.0
yy	−2.0	0.0	0.0	0.0
$\alpha\alpha$	−4.0	0.0	0.0	0.0
$\beta\beta$	−1.0	0.0	0.0	0.0
$\delta\delta$	−1.2	−2.8	0.0	0.0
xy	0.0	0.0	0.0	0.0
$\alpha\beta$	0.0	0.0	−1.93	0.0
$x\alpha$	0.3	9.95	0.0	0.0
$x\beta$	0.0	0.0	0.0	4.0
$y\alpha$	0.0	0.0	0.0	3.5
$y\beta$	0.0	0.0	0.0	0.0
$x\delta$	3.73	5.35	0.0	0.0
$\alpha\delta$	−3.22	−4.60	0.0	0.0
$y\delta$	0.22	0.3	0.0	−3.18
$\beta\delta$	0.0	0.0	−1.65	−1.81

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