Representation of Focal Properties of the Equal-Diameter Two-Tube Electrostatic Lens for Computer Calculations

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Previous calculations have given accurate first-order focal properties for the two-tube electrostatic lens at discrete voltage ratios. For computer optimization, calculations involving systems of two-tube lenses, one must be able to calculate the focal properties continuously over some arbitrary range of voltage ratios. Hence the data must be displayed in a continuous manner, and a method of interpolation is needed which yields functions having a high degree of smoothness. Special care must be taken to describe the lens behavior correctly near zero strength or for the voltage ratio approaching unity. A satisfactory solution to this problem has been achieved using cubic splines. The resulting functions of the focal properties are continuous and have continuous first and second derivatives. The total beam behavior, and hence the system design, is determined by the transfer matrix which is obtained from the focal properties. To achieve sufficient accuracy in the lens calculations over the entire range of required focal properties, the region near zero lens strength had to be treated separately.

INTRODUCTION

Accurate paraxial focal properties for equal-diameter two-tube bipotential lenses have been previously obtained[1–4] from results of exact trajectory calculations in precisely mapped electrostatic fields. These focal properties are tabulated for discrete voltage ratios V_2/V_1, where V_1 is the incident beam energy in equivalent volts, and V_2 the energy of the beam emerging from the lens. While analytical expressions have been derived, for example, by Grivet[5] in order to calculate the same focal properties, these imply by definition that a lens must be idealized in order to be amenable to an analytical solution. In situations where high accuracy is needed, such as in designing high-precision instrumentation for experiments involving low-energy electrons, it becomes highly desirable to be able to use results from accurate calculations of realistic lens structures.

While the focal properties have been calculated with a very high degree of accuracy for realistic lenses, their use in subsequent computer applications for designing and optimizing lens systems is limited, because values are available only at discrete points. In an automatic optimization process it is necessary that the lens properties be obtainable continuously over some arbitrary range of voltage ratios with a high degree of smoothness and without discontinuities.

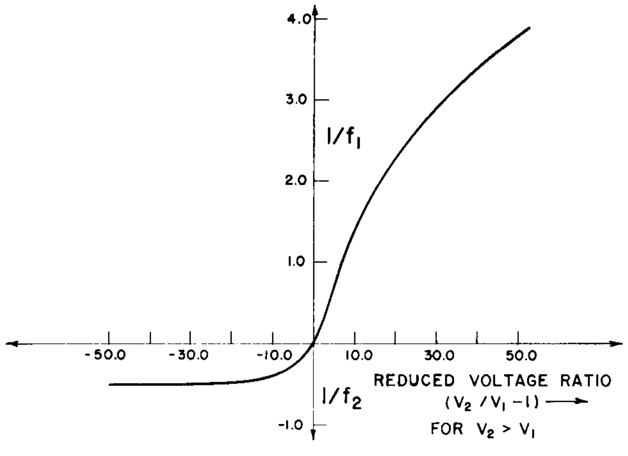

FIG. 1. Reciprocal focal length $1/f$ as function of the reduced voltage ratio.

USE OF INVERSE FOCAL PROPERTIES

As it is well-known in electron optics, for very weak lenses the focal lengths become very large, tending towards infinity as the voltage ratio approaches the unity value for the no-lens case. Since both focal lengths, f_1 and f_2 approach infinity for increasingly weaker lenses, their reciprocal values approach zero. By introducing a new independent variable, a reduced voltage ratio (V_2/V_1-1) for $V_2>V_1$ and $(V_1/V_2-1$ for) $V_1>V_2$, it is possible to represent the two sets of focal lengths as a smooth continuous set of data points through the origin, as shown in Fig. 1. In order to insure smoothness at the no-lens transition point the image focal lengths f_2 were assumed to be negative although the tabulated values[1–4] were shown as positive. At the time these quantities are used in lens equations, the signs are changed as needed to insure a consistent sign convention. At present, all focal properties used in the optimization calculations at NBS are defined to be positive quantities. Since two additional properties are necessary to characterize a thick lens, we used the positions of the object and image principal planes, H_1 and H_2, respectively. The reason for using the principal plane rather than the focal-point positions is that the principal planes give a more slowly-varying function. As can be seen from Fig. 2, they are displayed in a manner similar to the focal lengths. Definition of the various focal properties in the notation used here is given in Fig. 3. These formal focal properties are obtained from exact trajectory calculations by projecting the incoming and outgoing asymptotic rays in accordance with the definitions of geometrical optics.

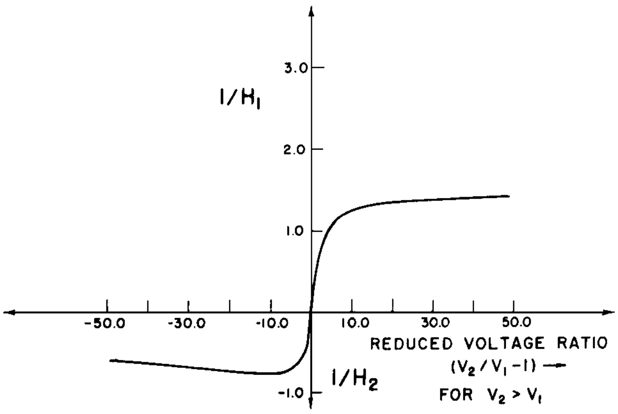

FIG. 2. Reciprocal principal plane position $1/H$ as function of the reduced voltage ratio.

It should be pointed out that the specific values of the focal properties shown in Figs. 1 and 2 are those of an accelerating lens. A decelerating lens is treated simply by exchanging the subscripts 1 and 2 for the object and image spaces, respectively.

TRANSFER MATRIX

Once the focal properties have been determined, they are used for calculating the transfer matrix between points P_1 and P_2 over the distance d_1+d_2. The expression is quite general and the points P_1 and P_2 are not necessarily conjugate. The matrix which applied to the position and angle coordinates of a ray (r_1,r_1') at point

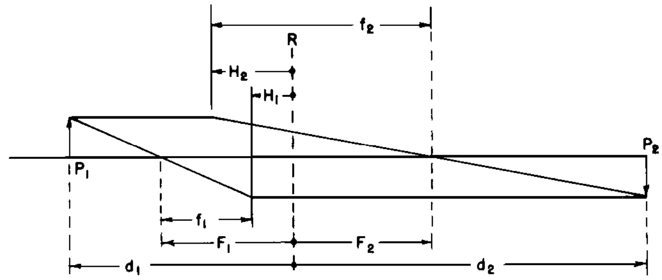

FIG. 3. Definition of the focal properties: R—midplane of lens or reference plane; f_1, f_2—focal lengths; F_1, F_2—distances of focal points from midplane; H_1, H_2—distances of principal planes from midplane.

P_1 gives the new coordinate values (r_2,r_2') at P_2 is given by

$$M_{\text{transfer}}=-\frac{1}{f_2}\begin{bmatrix} d_2-F_2 & (d_1-F_1)(d_2-F_2)-f_1f_2 \\ 1 & d_1-F_1 \end{bmatrix}, \quad (1)$$

where $F_1=H_1+f_1$. This matrix is used in the optimization calculations for lens system design.

INTERPOLATION METHOD

As stated before, the problem to be resolved was to derive some method by which the calculated focal properties could be utilized in a continuous manner. Having achieved the finiteness and smooth continuity

TABLE I. Coefficients $c_{1,k}$ of the cubic spline-fit for the reciprocal focal lengths $1/f$.

$c_{1,k}$	$c_{2,k}$	$c_{3,k}$	$c_{4,k}$	Reduced voltage ratio	Reciprocal focal lengths
0.2067E−9	−0.2391E−9	−0.4623E−3	−0.5458E−3	−999.00	−0.2053
−0.4783E−9	−0.9241E−9	−0.1181E−2	−0.1534E−2	−499.00	−0.3028
−0.1540E−8	−0.2688E−8	−0.2618E−2	−0.3248E−2	−249.00	−0.3979
−0.8065E−8	0.7833E−7	−0.9904E−2	−0.1078E−1	−99.00	−0.4962
0.3916E−6	0.1584E−5	−0.5294E−1	−0.5310E−1	−49.00	−0.5290
0.1584E−5	0.2402E−5	−0.5310E−1	−0.5232E−1	−39.00	−0.5295
0.2402E−5	0.8958E−5	−0.5232E−1	−0.5010E−1	−29.00	−0.5208
0.1792E−4	0.3483E−4	−0.9885E−1	−0.9259E−1	−19.00	−0.4920
0.8708E−4	0.1275E−3	−0.2297E−0	−0.2197E−0	−14.00	−0.4586
0.8497E−4	0.1448E−3	−0.1469E−0	−0.1332E−0	−12.00	−0.4385
0.1448E−3	0.2947E−3	−0.1332E−0	−0.1116E−0	−9.00	−0.3956
0.4421E−3	0.7909E−3	−0.1652E−0	−0.1304E−0	−6.00	−0.3269
0.7909E−3	0.1399E−2	−0.1304E−0	−0.7660E−1	−4.00	−0.2545
0.5597E−2	0.3479E−2	−0.2854E−0	−0.2106E−0	−2.00	−0.1420
0.3479E−2	0.7818E−2	−0.2106E−0	−0.1306E−0	−1.50	−0.1049
0.7818E−2	−0.3598E−1	−0.1306E−0	−0.3889E−1	−1.00	−0.0643
−0.3598E−1	0.4456E−2	−0.3889E−1	−0.1114E−2	−0.50	−0.0239
0.4456E−2	0.6127E−1	−0.1114E−2	0.4334E−1	0.00	0.0000
0.6127E−1	0.8984E−2	0.4334E−1	0.1797E−0	0.50	0.0293
0.8984E−2	0.8234E−2	0.1797E−0	0.3295E−0	1.00	0.0910
0.8234E−2	0.8409E−3	0.3295E−0	0.4917E−0	1.50	0.1658
0.2102E−3	−0.7395E−3	0.1221E−0	0.2875E−0	2.00	0.2460
−0.7395E−3	−0.6571E−3	0.2875E−0	0.4351E−0	4.00	0.5691
−0.4380E−3	−0.3441E−3	0.2922E−0	0.4201E−0	6.00	0.8649
−0.3441E−3	−0.2681E−3	0.4201E−0	0.5294E−0	9.00	1.2511
−0.4022E−3	−0.3144E−3	0.7921E−0	0.8894E−0	12.00	1.5810
−0.1258E−3	−0.9471E−4	0.3584E−0	0.4424E−0	14.00	1.7762
−0.4735E−4	−0.2250E−4	0.2248E−0	0.2875E−0	19.00	2.2002
−0.2250E−4	−0.1829E−4	0.2875E−0	0.3367E−0	29.00	2.8523
−0.1829E−4	−0.9618E−5	0.3367E−0	0.3749E−0	39.00	3.3486
−0.1924E−5	−0.5142E−6	0.7960E−1	0.1005E−0	49.00	3.7397
−0.1714E−6	−0.2225E−7	0.3694E−1	0.4244E−1	99.00	4.9623
−0.1335E−7	−0.6634E−8	0.2600E−1	0.2749E−1	249.00	6.2909
−0.3317E−8	0.3396E−8	0.1437E−1	0.1213E−1	499.00	6.7700
0.0000E−0	0.0000E−0	0.0000E−0	0.0000E−0	999.00	6.4918

TABLE II. Coefficients $c_{i,k}$ of the cubic spline-fit for the reciprocal principal plane positions $1/H$.

$c_{1,k}$	$c_{2,k}$	$c_{3,k}$	$c_{4,k}$	Reduced voltage ratio	Reciprocal principal plane positions
$0.3544E-8$	$-0.9383E-9$	$-0.1247E-2$	$-0.3059E-3$	-999.00	-0.1804
$-0.1877E-8$	$-0.6359E-8$	$-0.9637E-3$	$-0.1078E-2$	-499.00	-0.2702
$-0.1060E-7$	$0.2344E-7$	$-0.2221E-2$	$-0.4244E-2$	-249.00	-0.3690
$0.7032E-7$	$-0.3641E-6$	$-0.1132E-1$	$-0.1124E-1$	-99.00	-0.5574
$-0.1821E-5$	$-0.2931E-6$	$-0.6056E-1$	$-0.6375E-1$	-49.00	-0.6074
$-0.2931E-6$	$-0.1649E-5$	$-0.6375E-1$	$-0.6711E-1$	-39.00	-0.6378
$-0.1649E-5$	$0.7894E-6$	$-0.6711E-1$	$-0.7147E-1$	-29.00	-0.6728
$0.1579E-5$	$0.8349E-5$	$-0.1428E-0$	$-0.1468E-0$	-19.00	-0.7139
$0.2087E-4$	$0.5680E-4$	$-0.3666E-0$	$-0.3697E-0$	-14.00	-0.7330
$0.3787E-4$	$0.8385E-4$	$-0.2467E-0$	$-0.2481E-0$	-12.00	-0.7390
$0.8385E-4$	$0.3427E-3$	$-0.2481E-0$	$-0.2450E-0$	-9.00	-0.7420
$0.5141E-3$	$0.1193E-2$	$-0.3649E-0$	$-0.3464E-0$	-6.00	-0.7257
$0.1193E-2$	$0.5298E-2$	$-0.3464E-0$	$-0.2992E-0$	-4.00	-0.6832
$0.2119E-1$	$0.4267E-1$	$-0.1117E+1$	$-0.9908E-0$	-2.00	-0.5560
$0.4267E-1$	$0.4922E-1$	$-0.9908E-0$	$-0.8001E-0$	-1.50	-0.4901
$0.4922E-1$	$0.1729E-0$	$-0.8001E-0$	$-0.5357E-0$	-1.00	-0.3939
$0.1729E-0$	$0.4772E-1$	$-0.5357E-0$	$-0.1193E-1$	-0.50	-0.2462
$0.4772E-1$	$-0.1549E-0$	$-0.1193E-1$	$0.5834E-0$	0.00	0.0000
$-0.1549E-0$	$-0.4901E-1$	$0.5834E-0$	$0.9463E-0$	0.50	0.2723
$-0.4901E-1$	$-0.4776E-1$	$0.9463E-0$	$0.1236E+1$	1.00	0.4670
$-0.4776E-1$	$-0.2767E-1$	$0.1236E+1$	$0.1453E+1$	1.50	0.6119
$-0.6917E-2$	$-0.2176E-2$	$0.3893E-0$	$0.5033E-0$	2.00	0.7233
$-0.2176E-2$	$-0.1031E-2$	$0.5033E-0$	$0.5651E-0$	4.00	0.9892
$-0.6874E-3$	$-0.2426E-3$	$0.3802E-0$	$0.4110E-0$	6.00	1.1220
$-0.2426E-3$	$-0.1416E-3$	$0.4110E-0$	$0.4288E-0$	9.00	1.2265
$-0.2123E-3$	$-0.1067E-3$	$0.6421E-0$	$0.6535E-0$	12.00	1.2825
$-0.4268E-4$	$-0.2134E-4$	$0.2623E-0$	$0.2692E-0$	14.00	1.3062
$-0.1067E-4$	$-0.1640E-5$	$0.1354E-0$	$0.1375E-0$	19.00	1.3434
$-0.1640E-5$	$-0.1277E-5$	$0.1375E-0$	$0.1386E-0$	29.00	1.3734
$-0.1277E-5$	$-0.1607E-6$	$0.1386E-0$	$0.1390E-0$	39.00	1.3850
$-0.3213E-7$	$0.8707E-8$	$0.2787E-1$	$0.2794E-1$	49.00	1.3897
$0.2902E-8$	$0.1180E-8$	$0.9255E-2$	$0.9581E-2$	99.00	1.3980
$0.7079E-9$	$0.8561E-9$	$0.5720E-2$	$0.6259E-2$	249.00	1.4411
$0.4281E-9$	$0.5764E-9$	$0.3049E-2$	$0.4069E-2$	499.00	1.5781
$0.0000E-0$	$0.0000E-0$	$0.0000E-0$	$0.0000E-0$	999.00	2.1067

through the origin, various least-square curve-fitting procedures were considered. These were ultimately abandoned for a straightforward interpolation method using cubic splines.[6] This method has the merit that the original data points, or in this case, the precisely calculated values for the focal properties, are retained. In the regions between the calculated points the focal properties are obtained as continuous functions with continuous first and second derivatives.

In a present version of a lens-system optimization program which makes use of the accurately calculated focal properties, the spline-fit method is being used as a two-step procedure. In a separate computer run four coefficients are calculated with the cubic spline-fit interpolation routine for each of the data points on the curves shown in Figs. 1 and 2. These coefficients are then entered as a built-in look-up table in the lens optimization program, so that for any intermediate voltage ratio the focal properties are calculated from third-order polynomials using the stored coefficients.

Tables I and II give the complete list of coefficients $c_{i,k}$ which are used in the computer program for calculating the focal lengths and principal plane positions continuously over the range of voltage ratios up to 1000. The cubic spline-fit interpolation consists of determining between which two points (x_k,y_k) and (x_{k+1},y_{k+1}) the given value of x lies, and then calculating the corresponding y by the formula

$$y=c_{1,k}(x_{k+1}-x)^3+c_{2,k}(x-x_k)^3 +c_{3,k}(x_{k+1}-x)+c_{4,k}(x-x_k). \quad (2)$$

In this case, x is the reduced voltage ratio and y the corresponding value of the focal property. While the spline-fit enables one to obtain the intermediate focal properties to a very good accuracy, the interpolated values do not necessarily satisfy the well-known relationship between the focal lengths

$$f_1/f_2=(V_1/V_2)^{\frac{1}{2}}. \quad (5)$$

Since this relation is used in testing the flow and accuracy of lens system calculations, one can apply the spline-fit procedure to one focal length only and calculate the other by applying the above expression. No such relationship exists for the focal point or principal plane positions.

WEAK LENS PROCEDURE

The spline-fit method provides a convenient and accurate tool for using previously calculated precise values of the focal properties in an automatic lens system design. It does not, however, eliminate those problems arising from numerical uncertainties in the region near zero lens strength. As can be seen from Eq. (1), the quantities which enter into the matrix elements are the focal properties directly rather than their reciprocals used in the spline-fit procedure. The problem is particularly severe where small quantities are derived as differences between very large numbers. With double precision the region of reliability can be extended but it does not ultimately resolve the problem. Matrix element a_{12} of Eq. (1) was found to be particularly sensitive to small inaccuracies in the description of the lens behavior. As a temporary solution, this matrix element was calculated separately from the tabulated focal parameters and its value extrapolated for the near-zero lens region. In the future we expect to bypass this problem by using all of the matrix elements directly, without the intermediary focal properties.

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