New York, 1965).
[20]T. Morita, J. Math. Phys. **12**, 2062 (1971).
[21]C. G. Windsor, in *Neutron Inelastic Scattering* (International Atomic Energy Agency, Vienna, 1968), Vol. II, p. 83.
[22]F. Carboni and P. M. Richards, Phys. Rev. **177**, 889 (1969).
[23]T. Morita, J. Math. Phys. **13**, 714 (1972).

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Field-Emission Tails and Tunneling Lifetimes

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Recent observations of high- and low-energy tails in field-emission energy distributions can be interpreted in terms of quasi-stationary-state single-particle tunneling. This imposes a restriction on the observable range of energies in such studies. The tails result from the predicted breakdown of the transfer Hamiltonian theory of tunneling when fourth-order terms in the perturbation expansion of the tunneling matrix element become large. The tunneling lifetimes $\sim 10^{-12}$ sec required to fit the experimental data are consistent with both the RC time constant discussed by Thornber, McGill, and Mead and also a simple intuitive picture. Alternate theories of tunneling lifetimes are critically evaluated.

I. INTRODUCTION

Some interesting speculation has resulted from the experimental discovery of both high- and low-energy tails in the total energy distribution (TED) of electrons field emitted from tungsten.[1-6] For a standard free-electron model, the TED or current per unit energy is[7,8]

$$\frac{dj}{d\epsilon} \equiv j_0'(\epsilon) = \frac{J_0}{d} f(\epsilon) e^{\epsilon/d} \tag{1a}$$

and the total current density is

$$J_0 = \left(\frac{4\pi m e d^2}{h^3}\right) e^{-c} \equiv \gamma_j e^{-c} = \frac{1.537\times 10^{10} F^2 e^{-c}}{\varphi t^2}\ \frac{\text{A}}{\text{cm}^2}, \tag{1b}$$

where $\epsilon = E - E_F$ is the electron energy relative to the Fermi level and $f(\epsilon)$ is the Fermi function. The quantities $c = 0.68\varphi^{3/2} v(F,\varphi)/F$ and $1/d = 1.025\varphi^{1/2} t(F,\varphi)/F$ eV^{-1} are obtained from the exponential tunneling probability where F is the applied field in V/Å, φ is the electron work function in eV, and t and v are tabulated elliptic functions.[8,9] According to Eq. (1a), at zero temperature no electrons with $\epsilon > 0$ will appear in the TED due to the sharpness of the Fermi surface. The experimental facts indicate that both a high- and low-energy tail exist which are several orders of magnitude down from $j_0'(\epsilon = 0) = J_0/d$ and which decrease as inverse powers of ϵ.[1,3,4] The initial suggestion due to Fischer[2] was that the high-energy tail could result from Auger filling of holes arising when electrons with $\epsilon < 0$ are field emitted. From their measurements, Lea and Gomer[3] proved that the total current in the high-energy tail was proportional to J_0^2. Since J_0^2 is also proportional to the square of the tunneling probability at the Fermi level, the findings of Lea and Gomer appeared to support the idea of correlated two-electron tunneling. Gadzuk and Plummer[4] offered a quantitative theory in which the high-energy electrons were considered to be products of a cascade process initiated by the production of a hole beneath the Fermi level.[10] Herring has given a related interpretation.[5] Lastly, Ngai and Bari[6] have attempted to interpret the tails in terms of simultaneous two-particle tunneling, not unrelated to the other cited theories. In all cases, multiparticle tunneling is considered to be the mechanism responsible for $j'(\epsilon) \propto J_0^2$. It is one of our purposes here to point out that the characteristic $j'(\epsilon) \propto J_0^2$ is not sufficient proof that two-particle tunneling has in fact occurred and an alternative is offered which does not require multiparticle tunneling.

II. THEORY

It is well known in the theory of α decay, for instance,[11] that owing to the finite lifetime of the quasistationary state of the α particle within the nucleus, the observed energy ϵ of the decaying state is complex with $\epsilon = \epsilon' - i\Delta(\epsilon')$, ϵ' the real part of the energy in the nucleus and $\Delta(\epsilon')$ the lifetime broadening. This fact has a profound effect on the implications of energy conservation in a tunneling or field-emission experiment. This point will be discussed further at the end of this paper. Adapting Eq. (2.23) of the tunneling theory of Appelbaum and Brinkman[12] to the field-emission energy distribution for a noninteracting electron gas, we have that

$$j'(\epsilon)=\int_{-\infty}^{+\infty}\delta(\epsilon-\epsilon')j_0'(\epsilon')\,d\epsilon' \quad , \tag{2}$$

which says that the energy distribution of electrons as a function of energy ϵ' in the metal can be expressed equivalently as a function of observed energy ϵ outside the metal due to the assumed conservation of energy in the tunneling process. In the case of a quasistationary state with lifetime τ, the energy is well defined only within a band of energies $\Delta\simeq\hbar/\tau$ and this uncertainty must be reflected in the apparent energy conservation law.[13]

The uncertainty is handled in Eq. (2) by replacing

$$-\frac{1}{\pi}\,\mathrm{Im}\left(\frac{1}{\epsilon-\epsilon'}\right)=\delta(\epsilon-\epsilon')$$

by

$$-\frac{1}{\pi}\,\mathrm{Im}\left(\frac{1}{\epsilon-\epsilon'+i\Delta(\epsilon')}\right)=\frac{1}{\pi}\;\frac{\Delta(\epsilon')}{(\epsilon-\epsilon')^2+\Delta(\epsilon')^2}\equiv G(\epsilon,\epsilon')$$

and then

$$j'(\epsilon)=\int_{-\infty}^{+\infty}G(\epsilon,\epsilon')j_0'(\epsilon')\,d\epsilon' \tag{3}$$

The key to the problem lies in the level-width function $\Delta(\epsilon')$. Since tunneling rather than quasiparticle decay within the solid is the lifetime limiting factor, then the width must be proportional to the tunneling probability[14] and thus $\Delta(\epsilon')=\Delta_0 e^{(-c+\epsilon'/d)}$ with Δ_0 constant or slowly varying with ϵ'. This point will be discussed at length later. Letting $\epsilon-\epsilon'=x$ and combining Eqs. (1) and (3), the zero-temperature TED becomes

$$j'(\epsilon)=\frac{J_0}{d}\;\frac{\Delta_0 e^{-c}}{\pi}\;e^{2\epsilon/d}\int_{\epsilon}^{+\infty}\frac{e^{-2x/d}}{x^2+\Delta(\epsilon-x)^2}\,dx \quad . \tag{4}$$

Furthermore, expressing the denominator as an infinite series when $|x|>\Delta$,

$$\frac{1}{x^2+\Delta(\epsilon-x)^2}=\frac{1}{x^2}\sum_{n=0}^{\infty}(-1)^n\left(\frac{\Delta(\epsilon-x)}{x}\right)^{2n} \quad ,$$

and realizing that the tabulated nth-order esponential integral is[15]

$$E_n(z)\equiv\int_1^{\infty}\frac{e^{-zt}}{t^n}\,dt$$

allows us to exactly integrate Eq. (4) in terms of the very rapidly converging series

$$j'(\epsilon)=\frac{J_0}{d}\sum_{n=0}^{\infty}\frac{(-1)^n}{\pi}\;\frac{1}{(2n+2)}\;\frac{d}{\epsilon}\left(\frac{\Delta_0 J_0}{\epsilon\gamma_j}\right)^{2n+1}\times e^{-[(2n+2)\epsilon/d]}E_{2n+2}[(2n+2)\epsilon/d] \tag{5}$$

for $\epsilon>0$. Using the asymptotic expansion for the exponential integral,

$$E_m(z)=\frac{e^{-z}}{z}\left\{1-\frac{m}{z}+\frac{m(m+1)}{z^2}-\cdots\right\}$$

and retaining only the $n=0$ term in Eq. (5) which is larger by a factor of $(\epsilon/\Delta)^2\gg1$ than the higher-order terms give the simple result for $\epsilon>\Delta>0$

$$j'(\epsilon)=\frac{J_0^2}{d\gamma_j}\;\frac{d\Delta_0}{2\pi\epsilon^2}\left\{1-\frac{d}{\epsilon}+\frac{3d^2}{2\epsilon^2}-\cdots\right\} \quad . \tag{6}$$

Equation (6) possesses the characteristics observed in the experimental TED tails; j' is proportional to the square of the total current and falls off as various inverse powers of ϵ for $\epsilon>0$. Thus it cannot be concluded that such behavior is necessarily indicative of many-body effects and multiparticle tunneling since Eq. (6) does not contain those ingredients. The crucial parameter determining the importance of uncertainty principle broadening is the scale factor Δ_0. Using the parameters of the Plummer experiments,[4] the observed result that $j'(\epsilon=1\text{ eV})/J_0/d\approx10^{-5}-10^{-6}$ requires from Eqs. (1b) and (6) an electron decay time $\tau=\hbar/\Delta(\epsilon)\approx10^{-11}-10^{-12}$ sec.

The unfortunate consequences of lifetime broadening occur in the low-energy tail. For $\epsilon<0$ in Eq. (4), the integration picks up the usual TED of Eq. (1) as a pole term at $x=0$. Additional significant contributions for $\epsilon\le x<0$ must also be included. The integration cannot be performed exactly in the way of Eq. (5), but a lower limit on the low-energy tail can be obtained by replacing the denominator by $1/\epsilon^2$. Then for $|\epsilon|>\Delta(\epsilon)$, the lower limit for Eq. (4) is

$$j'(\epsilon)=Z(\epsilon)j_0'(\epsilon)+\frac{J_0^2}{d\gamma_j}\;\frac{d\Delta_0}{2\pi\epsilon^2}\{1+\cdots\} \quad ,$$

with $Z(\epsilon)\lesssim1$ the "residue" at the pole. The leading term for the low-energy lifetime tail is the same as that of Eq. (6) for the high-energy tail. The magnitude of the lifetime tail is down by a factor $\sim10^{-5}$ from $j_0'(0)$. However, for low energies such that $j_0'(\epsilon)\ll10^{-5}j_0'(0)$, it appears that the tails will dominate the observed TED and this unfortunately could place a fundamental limitation upon the range of energies observable in field-emission studies in spite of the sophistication of the energy analyzer.[16]

Also note that the derivation of Eq. (6) in which d/ϵ is the expansion parameter is meaningful only for $|\epsilon|>d$ and thus the behavior of the TED for $|\epsilon|<0.1$ eV is not necessarily described by Eq. (6). Similar restraints apply to the many-body treatments of the high-energy tails.[4] Relevant to this point is the suggestion by Ngai and Bari[6] that the high-energy tail displays a Mahan x-ray-edge singularity[17,18] as $\epsilon\to0^+$. Due to the finite mass of the conduction-band hole, recoil effects smear out the singularity.[19] Although image potential localization[4] tends to suppress recoil smearing, these points need further study before they can be resolved.

III. TUNNELING LIFETIMES

The problem and definition of tunneling lifetimes has a rather diverse history, as pointed out by Mead and co-workers.[14,20,21] Consistent with the varying concepts of tunneling times, a wide spread in their numerical values has been suggested falling in the range $10^{-16} \lesssim \tau \lesssim 1$ sec. Clearly there is room for sharpening up the concept of tunneling times. Briefly stated, the four most prominent tunneling times, spanning the full range of τ above, are the following:

(i) If I is the total tunneling current, then the time interval between electron emission is, from $I = edN/dt$, $\tau = e/I$. Now the time interval is related to the "single-electron lifetime" τ_e by $\tau \simeq \tau_e/N_v$ with N_v the total number of electrons in the conduction band. (This is only a rough outline of the theory.) Consequently, $\tau_e \simeq N_v e/I$. Since $N_v \sim L^3$ whereas $I \sim L^2$ with L a characteristic dimension of the emitter, $\tau_e \sim L$ and the so-called "single electron lifetime" becomes arbitrarily large as the dimensions of the emitter become large. Typically for an emitter with $L^3 \sim 1$ cm^3, $N_v \sim 10^{23}$, and $I = 10^3$ A (as would be obtained with a field emitter,) $\tau_e \sim 1$ sec. This result is unsatisfactory for several reasons. First, we know that any theoretical quantity whose value is dependent upon the size of the crystal must be suspect. Such a quantity has no fundamental significance (other than phenomena such as size quantization). Second, this theory makes no allowance for the fact that a given electron is interacting with the barrier only a small fraction of the time, when it is in the surface region. By taking $N_v \sim 10^{23}$, it is implied that all the electrons are interacting with the barrier all of the time. We feel that this is not a correct picture.

(ii) Hartman[22] has developed a theory of tunneling "transmission time" which has been often erroneously equated with a tunneling lifetime, although this was probably not his intention. He considers an electron wave packet incident upon a tunneling barrier and then calculates the time delay before a wave packet of appreciable amplitude appears on the other side of the barrier. For dimensions characteristic of a tunnel junction, this time delay $\tau \simeq 10^{-16}$ sec. This is, however, not the tunneling lifetime as discussed in the present paper since we are concerned with electrons in originally discrete energy eigenstates of the crystal rather than in a wave packet containing arbitrarily-high-energy components. This, in fact, is related to one of the observations of Hartman, that a substantial amount of the transmitted current results from the high-energy components of the wave packet passing over the top of the barrier rather than from the low-energy components tunneling. In addition, there is no *a priori* way to construct the wave packet, dictated by the physics of the solid, (other than invoking something like a Gaussian *Ansatz*), which severely limits the utility of the wave-packet theory in describing real solid-state tunneling events. Consequently, we can reject this approach to lifetimes as being irrelevant for present purposes.

(iii) Kane[23] modeled the tunneling situation within a time-dependent two-state system in which the time-independent basis states were taken to be localized states, Ψ_l and Ψ_r, at the same energy in potential wells on either side of the barrier. In the perturbed situation, the two states couple through the overlap of their exponential tails in the barrier region. The initial boundary condition is that at time $t=0$, the electron is in a state Ψ_l localized on the left. The subsequent time evolution, under the full Hamiltonian including the left-right coupling, is described by the total wave function,

$$\Psi_{\rm Tot}(t) = a_l(t)\Psi_l + a_r(t)\Psi_r \,, \tag{7}$$

with $a_{l(r)}(t)$ determined by the energy states of the coupled system and with the boundary conditions $|a_l(t=0)|^2 = 1$, $|a_r(t=0)|^2 = 0$. The tunneling time τ is defined as the smallest value of t satisfying $|a_r(t=\tau)|^2 = 1$, $|a_l(t=\tau)|^2 = 0$ and is proportional to the square root of the usual WKB tunneling probability. This is equivalently proportional to the coupling matrix element rather than the mod square of the matrix element as obtained from transfer-Hamiltonian or golden-rule perturbation theory. Thus, Kane concludes that even though the barrier transit time is short, the relevant tunneling time, as defined above, can be quite long. The Kane tunneling time is proportional to $e^{-\kappa w}$ rather than $e^{-2\kappa w}$, the WKB result (with κ the decay constant and w the barrier width) because of the peculiarities of a two-state resonance model.[24] In an actual tunneling situation, once an electron passes through the barrier, it is lost forever, and thus cannot resonate between the two wells. This is the major deficiency in applications of the Kane model to real tunneling experiments. One other point of note concerns the possibility of Kane's model describing the lifetime effects discussed in this paper. There is no reason why the unperturbed states Ψ_l and Ψ_r in Eq. (7) must be at the same energy. In fact, if the right-hand side wave function is replaced by the linear superposition of states $\sum_r a_r(t)\Psi_r$ with r representing the quantum numbers describing the unperturbed right-hand states at various energies ϵ_r, then the left-hand state Ψ_l stands a chance of appearing in any of the Ψ_r within a continuous spectrum of energies for the same reasons that the "lifetime tails" appear in the theory presented in this paper.

(iv) The problems of tunneling lifetimes have been considered by Mead and co-workers[14,20,21] with emphasis on tunnel junctions. After considering various alternatives, they have concluded that the effective RC time constant $\tau = RC$ of the tunnel structure is the appropriate lifetime when discussing energy broadening. Here R is the resistance and C the capacitance of the tunnel structure. Parenthetically, we have encouragingly noted that they found lifetimes $\sim 10^{-12}$ sec for Al-AlN-Mg sandwiches with 1.76-eV Fermi level to AlN conduction-band minimum separations and 30-Å insulator thicknesses, roughly yielding the same WKB phase-integral area in the forbidden region as in a field-emission configuration. We also agree with Mead that the RC time constant is the appropriate lifetime. Although at present we are unable to derive this from a first-principles many-body theory, several enlightening arguments can be advanced to illustrate and clarify the use of the RC time constant as the tunneling lifetime.

Consider the actual sequence of events in a field-emission experiment. A field is applied which is screened at the emitter surface, thus creating an excess of electron charge. According to the laws of an RC circuit, this excess total charge number N_0 (not charge density) decays as

$$N(t) = N_0 e^{-t/RC} \quad . \tag{8}$$

Call t_d the time interval for electron to be emitted. Thus $N(t_d) = N_0 - 1 = N_0 e^{-t_d/RC}$ and consequently the total transition probability per unit time is $1/t_d = N_0/RC$. The transition probability per unit time per electron is the total probability divided by the number of excess electrons, so $1/\tau = 1/RC$, independent of N_0. The inverse of the single-electron transition probability is what Mead[14,20,21] and the present writers take to be the tunneling lifetime.

Rather than explicitly calculating an effective R and C for the field emission configuration in order to obtain a theoretical lifetime, τ can be directly related to the applied field and total field emitted current density. From Eq. (8), the total emitted current under steady-state conditions is

$$AJ_0 = e\frac{dN}{dt} = \frac{eN_0'}{RC} = \frac{eN_0'}{\tau} \quad , \tag{9}$$

with A the emitting surface area and J_0 the field-emission current density given by Eq. (1b). The quantity labeled N_0' differs from N_0, the total induced excess charge number, in the following way. The total screening charge N_0 arises from small distortions of all the electron wave functions throughout the conduction band, whose width is the Fermi energy $E_F \sim 10$ eV. On the other hand, owing to the exponential reduction of the tunneling probability only those electrons within ΔE_{obs} ~ 0.1 eV of the Fermi level are candidates for tunneling. Thus N_0 in Eq. (8) should be replaced by $N_0' \simeq (\Delta E_{\text{obs}}/E_F)N_0$ since only a small fraction of the total induced charge is involved in decaying in the tunneling RC system. For a perfect conductor, the surface charge density σ is related to the applied field F by Gauss's law: $F = 4\pi\sigma$. Since $N_0 = \sigma A/e = FA/4\pi e$, combination of this result with Eq. (9) yields the relationship between single-electron tunneling lifetime and the controllable parameters

$$\tau = \left(\frac{\Delta E_{\text{obs}}}{E_F}\right)\frac{F}{4\pi J_0} \quad , \tag{10}$$

which is independent of system dimensions as it must be. Using the experimental values[4] of $F \sim 0.355$ V/Å and $J_0 = 2.5\times10^3$ A/cm^2 and taking $(\Delta E_{\text{obs}}/E_F) \sim 10^{-2}$ yields Fermi-level electron lifetimes in the range $10^{-12} \lesssim \tau \lesssim 10^{-11}$ sec as required for the lifetime broadening mechanism to contribute to the experimentally observed tails.

It is also possible to directly calculate $\tau = RC$ by adopting a plausible model. In field emission, a triangular shaped barrier (appropriately reduced by the image potential) is formed by the surface potential plus applied field. An electron at the Fermi level must tunnel through a distance $s_T = \varphi/eF$. By identifying this distance with the effective capacitor-plate separation, the "capacitance" is $C = A/4\pi s_T$. The analog of resistance is taken to be the inverse conductivity so $R \equiv (Aedj/d\epsilon)^{-1}_{\epsilon=0}$ with $j'(0) = J_0/d$. Consequently, the directly calculated τ_{RC}, within the proposed model is,

$$\tau_{RC} = \left(4\pi s_T e\frac{dj}{d\epsilon}\right)^{-1} = \left(\frac{d}{\varphi}\right)\frac{F}{4\pi J_0} \quad ,$$

which bears a striking resemblance to Eq. (10) when it is realized that $\Delta E_{\text{obs}} \simeq d$. In general $E_F \sim 2\varphi$, so the two expressions, obtained from different points of view, give the same end result within a factor of 2.

It should be emphasized that the tunneling lifetime is not the average time interval between emission of electrons. In effect, the tunneling lifetime determines the width (in the time domain) of an emitted electron wavepacket which is fundamentally different than the time interval between packets. The wave-packet width is independent of the system geometry or size, whereas the time interval is directly proportional to the size.

IV. RELATION TO OTHER THEORIES

The single-particle results presented here are formally similar to a simple multiparticle tunneling theory. Consider an electron in the metal with energy ϵ'. It can tunnel to a non-energy-conserving state at $\epsilon' + \delta$ provided a second electron at ϵ' tunnels to a state at $\epsilon' - \delta$, thus conserving over-all energy. Single electron energy can be nonconserved up to $\Delta \simeq \hbar/\tau$, where τ is the time interval between the two events, which is also of order 10^{-12} sec. Thus an electron observed at ϵ could come from some other ϵ' in the metal. This effect would be felt by folding the $j_0'(\epsilon')$ energy distribution into a smearing function like $G(\epsilon, \epsilon')$. This result could be derived (the case of the hot-hole cascade[4] is one specific model derivation) and in the simplest models, $G(\epsilon, \epsilon')$ would be just the Lorentzian used in the single-particle theory. Thus it appears that the end results of the two different approaches are quite similar, although for different reasons. The distinction in the theories would arise in a proper microscopic theory for the constant Δ_0. In the case of two-particle tunneling, Δ_0 would require information about electron-electron interactions. Coincidence experiments between electrons emitted beneath and above the Fermi level might help in deciding whether one mechanism dominates in the tail current. Another possibility is field emission from a material with an energy gap above the Fermi level, at least in the direction of emission. If the tails are due to many-body effects, then the lack of final states in the gap region might significantly alter the tail structure.

Next, we note that the single-electron energy broadening discussed here is also similar to Fano's[25] configuration interaction between a spatial ly localized discrete state, in our case the metal states, and a continuum, the states outside the metal. Such interactions allow mixing between the quasistationary discrete state and a band of continuum states effectively "diluting" the discrete state into many continuum states of different energies.

Further insight into phenomenon of "lifetime broadening" can be gained by considering the extensive literature on quasistationery or decaying states[26–29] and also the literature of the theory of the transfer Hamiltonian approach to tunneling.[30–32] Take decaying states first. The situation is that there exists a limited geometrical subspace (the nucleus, atom, or solid) within which a set of eigenfunctions φ_k which are solutions to an unperturbed Schrödinger equation $H_0\varphi_k' = \epsilon_{k'}\varphi_{k'}$. Now at time $t=0$, the possibility of decay is turned on by such mechanisms as application of a field or placement of another potential well with unfilled states at the energy of the occupied states of H_0 next to the "bound" system. The new total system of the quasibound region plus the rest of the universe on the other side of the barrier are now described by a total Hamiltonian H which defines a new basis set ψ_k with $H\psi_k = \epsilon_k\psi_k$. Since the ψ_k form a complete set, we can describe the original localized state φ in terms of the ψ's, that is $\varphi_{k'}(t=0) = \sum_{k''} A(k', k'')\psi_{k''}$. The coefficients $A(k', k'')$ describe the projection of $\varphi_{k'}$ on any given $\psi_{k''}$, a stationary state of the total system. The quantity $|A(k', k'')|^2$ yields the probability that an electron originally in the bound state with energy $\epsilon_{k'}$ will appear in the exact state with energy $\epsilon_{k''}$. Physically, for any given energy $\epsilon_{k'}$ there are many system wavefunctions with varying energies around $\epsilon_{k'}$ that have appreciable amplitudes in the bound-state region. If we insist on saying that the original state is both localized and also in an energy eigenstate, then the consequence of this demand (which is implicit in the lowest-order form of the transfer Hamiltonian) is apparent energy-non-conserving tunneling processes.

This point of view is quite consistent with the observations of Prange[32] with regards to the transfer Hamiltonian. He notes that localization of electrons requires a large number of high-energy components of the system wave functions. He then points out that this problem can be of limited consequence in the transfer Hamiltonian theories *provided* one deals with tunneling situations which are adequately described by the square of the tunneling matrix element. The case of field-emission tails is different, since they vary as J_0^2 or as the tunneling matrix element to the fourth power. Thus it should not be surprising, upon reflection, that the standard theories based upon the transfer Hamiltonian breakdown is this regime. The unique features of field emission, namely, that an energy analysis system is basically a triode configuration, make it possible to observe such effects which are not observable in diode tunnel junctions. This observation is not meant to imply that the only fault of the transfer Hamiltonian theory is its failure to treat higher-order terms of the tunneling matrix element. Certainly the transfer Hamiltonian theory must be modified when dealing with many-body effects or impurity-assisted tunneling.[12,33–35] In addition to these problems, we have shown here how the theory breaks down for a purely one-electron effect when fourth-order terms are significant.

V. CONCLUSIONS

We are thus led to the following conclusions (which do not negate the hot-hole cascade effects but are added features).

(i) The level-broadening mechanism could be particularly simple (in contrast to n-electron tunneling) and leads immediately to a J_0^2 dependence.

(ii) The order of magnitude of the effect is right with no adjustable parameters.

(iii) $j' \propto J_0^n$ does not necessarily imply multiparticle tunneling.

(iv) The low-energy tails may place a fundamental restriction on the energy range accessible to field emission (and other tunneling) experiments.

(v) The effective RC time constant is a relevant time parameter for discussing tunneling lifetime effects.

(vi) The predicted breakdown of the transfer Hamiltonian theory does occur in circumstances in which fourth-order terms contribute to measurable currents.

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[1]E. W. Plummer, in Thirtieth Annual Physical Electronics Conference, Milwaukee, Wis., March, 1970 (unpublished).

[2]T. E. Fisher, in Ref. 1.

[3]C. Lea and R. Gomer, Phys. Rev. Lett. **25**, 804 (1970).

[4]J. W. Gadzuk and E. W. Plummer, Phys. Rev. Lett. **26**, 92 (1971).

[5]C. Herring (private communication).

[6]K. L. Ngai and R. A. Bari, Bull. Am. Phys. Soc. **16**, 431 (1971).

[7]R. D. Young, Phys. Rev. **113**, 110 (1959).

[8]J. W. Gadzuk and E. W. Plummer, Rev. Mod. Phys. (to be published).

[9]R. H. Good and E. W. Müller, in *Handbuch der Physik*, edited by S. Flugge (Springer, Berlin, 1956), Vol. 21, p. 176.

[10]R. H. Ritchie, J. Appl. Phys. **37**, 2276 (1966).

[11]J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (Wiley, New York, 1952), p. 588.

[12]J. A. Appelbaum and W. F. Brinkman, Phys. Rev. **186**, 464 (1969).

[13]L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Pergamon, London, 1958), p. 150.

[14]K. K. Thornber, T. C. McGill, and C. A. Mead, J. Appl. Phys. **38**, 2384 (1967).

[15]*Handbook of Mathematical Functions,* Natl. Bur. Stds. Appl. Math. Series No. 55 (U.S. GPO, Washington, D.C., 1965), p. 228.

[16]C. E. Kuyatt and E. W. Plummer, Rev. Sci. Instrum. **43**, 108 (1972).

[17]G. D. Mahan, Phys. Rev. **163**, 612 (1967).

[18]S. Doniach, P. M. Platzman, and J. T. Yue, Phys. Rev. B **4**, 3345 (1971).

[19]S. Doniach, Phys. Rev. B **2**, 3898 (1970).

[20]G. Lewicki and C. A. Mead, Phys. Rev. Lett. **16**, 939 (1966).

[21]S. L. Kurtin, T. C. McGill, and C. A. Mead, Phys. Rev. B **3**, 3368 (1971).

[22]T. E. Hartman, J. Appl. Phys. **33**, 3427 (1962).

[23]E. O. Kane, in *Tunneling Phenomena in Solids*, edited by E. Burstein and S. Lundqvist (Plenum, New York, 1969), p. 1.

[24]R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics, Quantum Mechanics* (Addison-Wesley, Reading, Mass., 1965), Vol. III.

[25]U. Fano, Phys. Rev. **124**, 1866 (1961).

[26]C. Lanczos, Z. Phys. **62**, 518 (1930); Z. Phys. **65**, 431 (1930); Z. Phys. **68**, 204 (1931).

[27]M. H. Rice and R. H. Good, Jr., J. Opt. Soc. Am. **52**, 239 (1962).

[28]B. A. Lippman and R. F. O'Malley, Phys. Rev. A **2**, 2115 (1970).

[29]R. M. More, Phys. Rev. A **3**, 1217 (1971); and (unpublished).

[30]J. Bardeen, Phys. Rev. Lett. **6**, 502 (1961).

[31]M. H. Cohen, L. M. Falicov, and J. C. Phillips, Phys. Rev. Lett. **8**, 316 (1962).

[32]R. E. Prange, Phys. Rev. **131**, 1083 (1963).

[33]C. B. Duke, *Tunneling in Solids* (Academic, New York, 1969).

[34]C. Caroli, R. Combescot, P. Nozières, and D. Saint James, J. Phys. C **4**, 916 (1971); C. Caroli, R. Combescot, D. Lederer, P. Nozières, and D. Saint James, J. Phys. C **4**, 2598 (1971); R. Combescot, J. Phys. C **4**, 2611 (1971).

[35]C. B. Duke, G. C. Kleiman, and T. E. Stakelon, Phys. Rev. B **6**, 2389 (1972).