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Frequency Dependence of a Cryogenic Capacitor Measured Using Single Electron Tunneling Devices

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A new type of capacitance standard based on counting electrons has been built. The operation of the standard has already given very promising results for the determination of the value of a cryogenic vacuum-gap capacitor. The new capacitance standard operates at an effective frequency close to dc, whereas capacitance metrology usually implies the use of bridges operating at a fixed frequency around 1 kHz. Therefore, the frequency dependence of the cryogenic capacitor is critical to the practical application of this standard. We present measurements of this frequency dependence using a technique that involves the same single electron tunneling devices used in the capacitance standard.

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1. INTRODUCTION

Today, capacitance metrology is based on the Thomson-Lampard calculable capacitor.[1] The relative uncertainty of this realization of the SI farad can be as small as 2 parts in 10^8. For several reasons, this experiment is difficult, and only a few calculable capacitors are currently operational throughout the world. As an alternative, we consider a capacitance standard based directly on the definition of capacitance combined with the ability to manipulate single electrons offered by recent advances in single electron tunneling (SET) devices. We charge a capacitor C with a known number N of electrons and measure the resulting voltage change ΔV. The value of the

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capacitance is then given simply by

$$C = \frac{Ne}{\Delta V}, \tag{1}$$

where e is the electron charge. With $C \approx 1$ pF, pumping 10^8 electrons yields a voltage of $\Delta V \approx 15$ V across the capacitor.

The principle of operation is illustrated in Fig.1 and requires three key components: a seven-junction electron pump, a two-junction SET transistor/electrometer and a nearly ideal capacitor. We can reliably operate the electron pump at 40 mK with an error per pumped electron of 0.01×10^{-6}.[2] The capacitor, known as a cryogenic vacuum-gap capacitor (C_{cryo}), is a three-terminal design that is insensitive to stray capacitance and has only vacuum for its dielectric.[3]

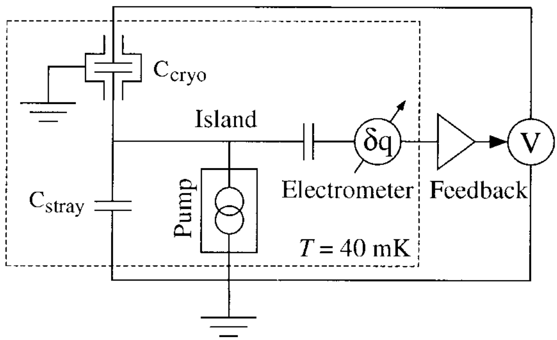

Fig. 1. Schematic view of the SET capacitance standard. As electrons flow through the pump, the feedback loop keeps the island at a virtual ground and avoids any voltage across the pump. After N electrons have been pumped, $C_{cryo} = Ne/\Delta V$.

As we pump electrons onto the capacitor, the island in Fig.1 must be kept at a virtual ground to avoid any voltage-induced errors in N. Besides allowing the pump to work properly, this also ensures that the pumped electrons go onto the cryogenic capacitor and not onto C_{stray}. This is accomplished by using the electrometer as a null detector in a feedback loop which raises the voltage on the *external* side of the capacitor. We pump N electrons in one direction, pause for about 20 s to measure the voltage, and then pump N electrons in the other direction, causing V to move between two levels. This cycle is repeated about 20 times, and C_{cryo} is given by the charge Ne divided by the average voltage change ΔV between the levels.

The value of C_{cryo} can be compared to conventional room-temperature capacitors using standard ac bridge techniques.

The performance of the SET capacitance standard can be summarized as follows:[4]

(a) Repeated measurements of C_{cryo} by counting electrons have a relative standard deviation $\sigma = 0.3 \times 10^{-6}$ over a period of 24 hours.

(b) The value of C_{cryo} agrees with the value measured using a capacitance bridge operating at 1 kHz and calibrated against an artifact traceable to NIST's calculable capacitor. Agreement is well within the $\pm 2.4 \times 10^{-6}$ relative uncertainty of the calibration.

(c) The new standard does not depend on the voltage ΔV (which we can change by choosing different N).[5]

These results indicate that the SET capacitance standard is likely to play an important role in capacitance metrology in the near future. However, the frequency dependence is an important point to check since the SET measurement is done effectively at dc (it takes about 100 s to pump 100 million electrons on and off the capacitor) while comparisons with other standards require the use of bridges operating around 1 kHz. In fact, the comparison with the commercial bridge (point (b) above) is already a first indication that the capacitance is nearly independent of frequency between dc and 1 kHz. However, the unlikely possibility of cancelling errors cannot be completely ruled out unless the frequency dependence can be measured directly.

2. EXPERIMENT

2.1. Principle

The general concept consists of applying a voltage step to the external side of the capacitor and recording the time response of the electrometer. Any deviation from a flat signal in this time response can be converted to a frequency dependence if a specific model of the capacitor and its imperfections is assumed. Our implementation of this concept is based on the fact that the pump in the *hold mode* (when not pumping) can be regarded in a simple way as a switch (see Fig.2a). Indeed, according to its current-voltage characteristic (shown in Fig.2c), the pump behaves like an open switch if the voltage V_p across it does not exceed the Coulomb gap V_{gap}. If $V_p \gg V_{gap}$, the pump acts like a resistor of a few megaohms.

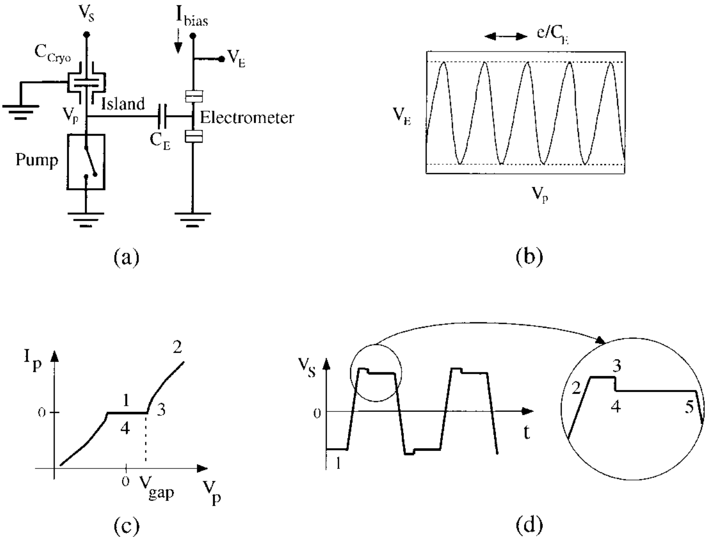

Fig. 2. The experimental configuration is shown in (a) where the pump is represented as a switch. The electrometer response V_E is periodic (of period e/C_E) as a function of V_p, as shown in (b). The typical current-voltage characteristic (I-V curve) of a pump is drawn in (c), while the applied voltage step V_S is sketched not to scale in (d). Points 1 through 5 refer to different phases of the measurement (see text).

During the measurement, we can distinguish four different phases:

(i) We start at point 1 (see Figs.2c+d) with $V_p = 0$, and ramp up the voltage V_S on the external side of the capacitor. The ramp rate is chosen such that only a limited portion of V_S (typically <50 mV) appears across the pump (during 1→2). This voltage still exceeds V_{gap}, and the pump acts like a resistor, allowing the island to discharge.

(ii) At the top of the ramp (2→3), we wait for a short time to finish discharging the island. On the I-V curve of the pump, discharging corresponds to going down the resistive branch to the edge of the gap.

(iii) At point 3, we apply a small reverse step (3→4, typically 1mV) to reset V_p to the middle of the gap, thus putting the pump in its "open" state.

(iv) The island is now floating with $V_p \approx 0$, and the electrometer is monitoring any change in the island charge. If C_{cryo} depends on frequency (or leaks[6]), this charge will change with time, and by recording the electrometer signal between points 4 and 5, we access this information.[7] The time it takes to go from point 1 to point 4 determines the upper limit of the frequency range we can reach,[8] whereas the time between points 4 and 5 sets the lower limit.

We should mention here that this measurement is performed in exactly the same configuration as the one used for the electron counting experiment and therefore ensures that the capacitor sees the same environment as when the standard is operating.

2.2. Results

A typical set of data is presented in Fig.3, where the regions of interest are the wide plateaux in V_E. The horizontal dashed lines represent the upper and lower limits of the electrometer modulation curve (see Fig.2b) and the position of a plateau relative to these lines indicates the gain of the measurement. If a plateau is located close to one of the lines, the gain is close to zero (and therefore the data do not contain much information about C_{cryo}), while a plateau measured near the middle, where the gain is maximum, contains useful information.

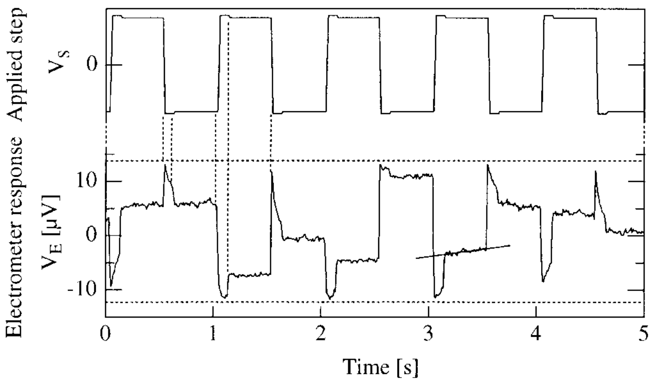

Fig. 3. The upper trace is a schematic view of the voltage V_S applied to the external side of the capacitor. The size of the "reset step" (typically 1 mV) is exaggerated in order to be visible on the 2 V full-scale signal. The lower trace is the corresponding electrometer response V_E.

The plateau positions are random because the time between points 2 and 3 in Figs.2c+d is not exactly repeatable. Since V_p is still changing during this time (due to the discharging through the pump), this time variation leads to a variation in V_p at point 4, and this variation corresponds to a large fraction of the period in Fig.2b. The sign of the gain, which corresponds to a positive or negative slope in Fig.2b, cannot be extracted from the data. This prevents us from directly averaging many plateaux to reduce the noise of the measurement. However, an estimate of the absolute value of the gain can be obtained by fitting a sine wave to the electrometer modulation curve.[9]

Any deviation from a flat plateau in V_E could be due to frequency dependence of C_{cryo}. To quantify the deviations, we fit each plateau with a line. Its slope can be converted to a change in the island charge and interpreted as a change in C_{cryo}. This change in C_{cryo} is presented as a function of the normalized gain in Fig.4. We analyze only those plateaux for which the gain is at least half of the maximum gain (which represents more than 80% of the plateaux). Since approximately half of the data have a positive gain and the other half a negative gain, a real systematic behavior should appear as a grouping of the values in Fig.4 on two lines (one for each sign of the gain). Since this is obviously not the case, we can average all of the data and put an upper bound on the relative change in C_{cryo} per second of $(0.6 \pm 2) \times 10^{-6}$ s^{-1}.

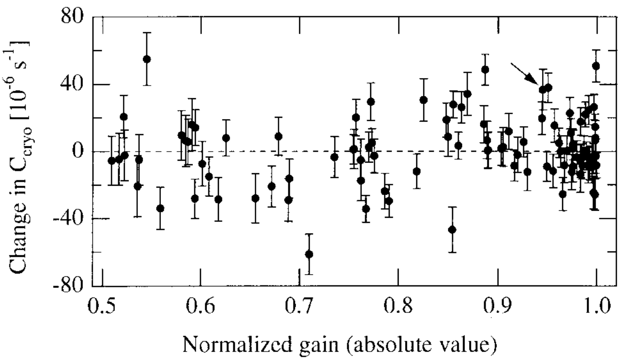

Fig. 4. The slope of each plateau converted to a change in C_{cryo} per second is presented as a function of the normalized gain of the measurement (●). Since no systematic trend is observed, we average this set of data to obtain an upper bound of the change in C_{cryo} of $(0.6 \pm 2) \times 10^{-6}$ s^{-1}. The arrow (↘) indicates the point corresponding to the fitted slope on Fig.3.

It is important to note that the electrometer itself can produce plateaux that are not flat because of its $1/f$ input noise. Even with a fixed island voltage (which we apply using cryogenic needle switches[2, 4]), the electrometer signal sometimes displays drifts due to slow charge relaxation or jumps due to two-level fluctuators. We expect that this noise will be the ultimate limit on the sensitivity of the frequency dependence measurement, as well as on the performance of the capacitance standard.

3. CONCLUSIONS

We have described a technique for measuring the frequency dependence of a cryogenic vacuum-gap capacitor within the same environment in which it is used for an SET capacitance standard. Using this technique, we have placed an upper bound on the relative change in C_{cryo} of $(0.6 \pm 2) \times 10^{-6}$ s^{-1}, measured between 0.1 s and 0.5 s after applying the voltage. Since we did not observe any actual frequency dependence, our result cannot be used to distinguish between different models of the imperfections in the capacitor, and thus we do not feel it is useful to apply a specific model in our analysis. Instead, we simply infer that the relative frequency dependence of C_{cryo} over the corresponding frequency range, 10 Hz to 2 Hz, is no larger than about 1×10^{-6}. This preliminary result indicates that the cryogenic vacuum-gap capacitor is a promising candidate for the purpose of a new capacitance standard. We are currently pursuing two improvements to our measurement technique. First, we will use custom electronics to shorten the time between points 2 and 3 in Fig.2d to less than 1 ms, allowing us to measure frequency dependence up to about 1 kHz. Second, we will measure the actual electrometer gain at each plateau so that we may directly average many measurements together and reduce the uncertainty of the final result.

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5. These measurements ranged from 22 V to 4 V, and were done at a constant pumping rate. The corresponding effective frequency range was 5 mHz to 14 mHz, but this small range is expected to have a negligible effect.

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6. A direct measurement of the leakage of C_{cryo} has given a lower bound for its parallel resistance of $10^{21}\Omega$.
7. In principle, any imperfections in C_{stray} will also be detected by our measurement. However, the voltage across C_{stray} during the time we measure the electrometer signal never exceeds V_{gap} ~1 mV, whereas the voltage across C_{cryo} is of order 1 V. C_{stray} and C_{cryo} have similar values, so our measurement is of order 1000 times less sensitive to C_{stray} than to C_{cryo}. Since the primary dielectric in C_{stray} is vacuum, its frequency dependence is presumably similar to that of C_{cryo}.
8. In principle, taking the pump resistance $R_p \approx 2$ MΩ and the total capacitance $C_{cryo} + C_{stray} \approx 10$ pF gives an RC time $\tau_{RC} \approx 20$ μs or an upper limit in the 10 kHz range.
9. By using a sine wave instead of the actual electrometer modulation curve, we ignore the difference between positive and negative slopes. For our electrometer, this procedure overestimates the gain on the positive slope and underestimates the gain on the negative slope, by about 20% in each case. Since the relative uncertainty in our estimate of an upper bound for the change in C_{cryo} is much larger than ±20% (see discussion of Fig. 4), we neglect this additional uncertainty in the slope.