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Temperature dependence of the Verdet constant in several diamagnetic glasses

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Measured temperature dependences of the Verdet constants of SiO₂, SF-57, and BK-7 are ~10⁻⁴/K within 3-20% of Becquerel formula estimates. Key words: Verdet constant, Faraday effect, temperature dependence, current sensor, magnetic field sensor.

In recent years, use of the Faraday effect in the measurement of magnetic fields and electric current has expanded substantially.¹ Some of these applications require sensors that can operate over broad temperature ranges with minimal change in calibration. Diamagnetic materials, especially diamagnetic glasses in bulk and fiber form, are commonly chosen as the sensing element in high stability Faraday effect sensors¹ because their temperature dependence is much smaller than that of paramagnetic or ferromagnetic materials.²

The Faraday effect is a magnetic field-induced circular birefringence, most commonly described as a rotation of the plane of polarization of linearly polarized light. As a rotation it is characterized by the following equation:

$$\theta = V \int \mathbf{B} \cdot d\mathbf{h}, \quad (1)$$

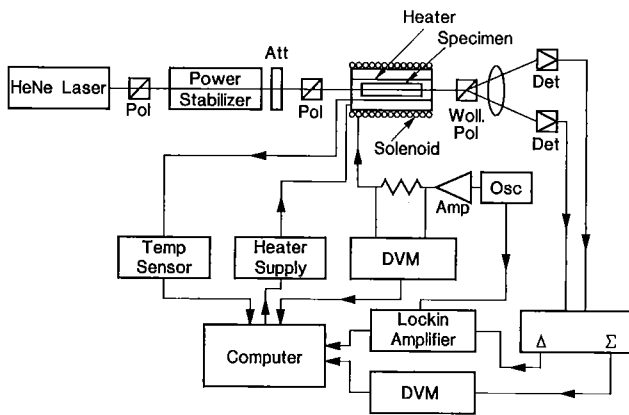


Fig. 1. Measurement system used for $d(VL)/dT$ measurements.

where θ is the induced rotation, \mathbf{B} is the magnetic field vector, $d\mathbf{h}$ is a vector along the direction of propagation of light, and V is a material parameter known as the Verdet constant. The temperature dependence of the Faraday effect arises primarily through the change in the Verdet constant with temperature, although in some applications the thermal expansion of the material may also be significant. In a linear magnetic field sensor, changes in V and L (the length of the sensing element) are indistinguishable. The useful quantity is then $[d(VL)/dT]/(VL)_0$. However, in a current sensing application, the sensor material (optical fiber, for example) makes a closed path around the conductor.¹ By Ampere's law, the line integral of the magnetic field along the optical path is constant and, therefore, independent of changes in the path length. Therefore, for current sensors $(dV/dT)/V_0$ is the relevant parameter. To normalize the temperature dependences, values of dV/dT and $d(VL)/dT$ are reported in a normalized form as $(dV/dT)/V_0$ and $[d(VL)/dT]/(VL)_0$, where V_0 and $(VL)_0$ are the values at 20°C.

Until recently, very little information on the temperature dependence of the Verdet constant in diamagnetic materials has been available other than the general knowledge that it is small.² For cubic diamagnetic crystals, an extensive set of measurements at a single wavelength (633 nm) has recently appeared,³ showing results for $[d(VL)/dT]/(VL)_0$ that generally range between -10^{-3} and $+10^{-3}/K$. The effect is most likely associated with shifts in the band gap and will depend on the wavelength of measurement. One report⁴ of measurements on SF-57 has indicated a value of $[d(VL)/dT]/(VL)_0$ of $\sim 10^{-4}/K$. Data on optical fibers have also been reported.⁵ These results are puzzling in that, even though the fibers were similar, the measured temperature coefficients ranged from -2×10^{-3} to $+8 \times 10^{-5}/K$. We suspect that this set of data can be explained by the temperature dependence of the linear birefringence of the fiber rather than the temperature dependence of the Verdet constant of the glass.

In this Technical Note, we report measurements of the temperature dependence of the Verdet constant of three common glasses: SiO_2 , SF-57, and BK-7. Measurements on SiO_2 are important as an indicator of the performance of sensors that use optical fiber as the sensing element. For sensors that use bulk glass sensing elements, SF-57 is an attractive choice¹ because its Verdet constant is ~ 5.5 times greater than that of silica, and, perhaps more important, its stress optic coefficient is 2 orders of magnitude smaller than that of silica. Measurements on BK-7 are included for comparison.

We use the measurement system depicted in Fig. 1. The samples are placed in a temperature-controllable solenoid

~ 14 cm long. An alternating current is applied to the solenoid, giving an rms field of ~ 0.01 T. The solenoid current is monitored through a precision four-terminal resistor. Linearly polarized light from a He-Ne laser (633 nm), externally amplitude stabilized, passes through the sample and is analyzed at the output using a Wollaston prism oriented to separate the light into orthogonal components at $\pm 45^\circ$ to the input polarization direction. The difference and sum of these two signals are determined, and the ratio of the difference to the sum is computed numerically. Using the method of Jones calculus⁶ to analyze the system described above, an expression for the ratio of the difference to the sum (Δ/Σ) can be found. Under the assumptions that there is no linear birefringence in the sample and the field is uniform, the result is

$$\Delta/\Sigma = \sin(2VBL), \quad (2)$$

or for small rotations

$$\Delta/\Sigma \approx 2VBL. \quad (3)$$

The ratio (the difference over the sum) is independent of laser intensity and is subsequently divided by the solenoid current to compensate for drifts in the magnetic field. This normalized signal is proportional to the product of the Verdet constant and the sample length. It is recorded as a function of temperature as the solenoid is heated under computer control from room temperature to $\sim 150^\circ\text{C}$ over a period of ~ 1.5 h. The result is fitted by linear regression, further normalized to its extrapolated 20°C value, and plotted.

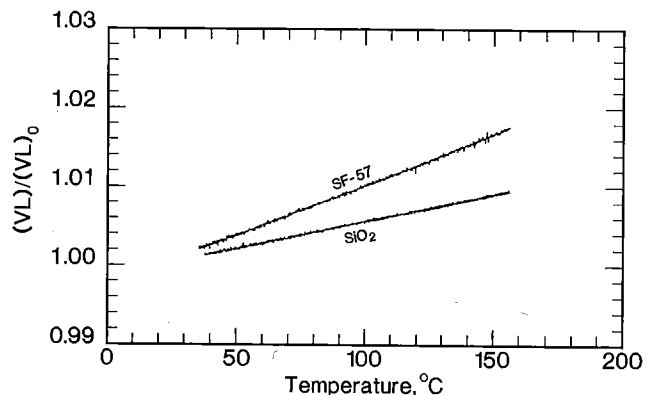


Fig. 2. Typical data for SF-57 and SiO_2 ; data for BK-7 are similar to that for SiO_2 .

We tested three samples of SF-57, three samples of SiO_2 (Dynasil), and four samples of BK-7. The SF-57 samples had a square cross section with dimensions of $10 \times 1 \times 1$ cm. The SiO_2 and BK-7 samples were cylindrical (8 cm long \times 1.4 cm in diameter and 10 cm long \times 1 cm in diameter, respectively). Figure 2 shows typical data for SiO_2 and SF-57. The data for BK-7 are very nearly identical to those for SiO_2 .

The slope of the data in Fig. 2 is $[d(VL)/dT]/(VL)_0$. Again, assuming a uniform magnetic field and no birefringence, we have

$$\frac{1}{(VL)_0} \frac{d(VL)}{dT} = \frac{1}{V_0} \frac{dV}{dT} + \alpha, \quad (4)$$

where $\alpha = (dL/dT)/L$ is the thermal expansion coefficient of the glass for the range of temperatures tested.

In our measurement system, the magnetic field varied somewhat over the sample length. To compensate, we mea-

Table I. Results for SF-57, SiO₂, and BK-7 Glass^a

Material	V	$\frac{1}{V_0} \frac{dV}{dT}$	$\frac{1}{V_0} \frac{dV}{dT}$	$\frac{1}{(VL)_0} \frac{d(VL)}{dT}$	α
		Theory	Experiment	Experiment	
SF-57	11.5[8]	1.29	1.26±0.08	1.35±0.08	9.2[8]
SiO ₂	2.1[10]	0.81	0.69±0.03	0.69±0.03	0.55[9]
BK-7	2.3[8]	0.56	0.63±0.06	0.71±0.06	8.3[8]

^a Units of V are deg/cm · T at 633 nm, units of $(dV/dT)/V_0$ and $[d(VL)/dT]/(VL)_0$ are 10⁻⁴/K, and units of α are 10⁻⁶/K. Numbers in brackets are references.

sured the magnetic field profile and integrated numerically. This gave a small (~1%) correction to $[d(VL)/dT]/(VL)_0$. Values of $(dV/dT)/V_0$ were found by subtracting α from $[d(VL)/dT]/(VL)_0$.

The presence of linear birefringence acts to reduce the apparent magnitude of the Faraday effect. This problem is minimized by aligning the input polarizer along one of the birefringence axes of the sample.¹ In this case, Eq. (2) becomes

$$\frac{\Delta}{\Sigma} = 2VBL \left[\frac{\sin \sqrt{\delta^2 + (2VBL)^2}}{\sqrt{\delta^2 + (2VBL)^2}} \right], \quad (5)$$

where δ is the linear retardance of the sample.⁷ Equation (5) reduces to Eq. (2) if the retardance is small or if it is small compared with the Faraday rotation. The room temperature retardances of the samples used in these experiments were measured to be $\delta \sim 2.2^\circ$ for SF-57, $\delta \sim 3.2^\circ$ for SiO₂, and $\delta \sim 1.8^\circ$ for BK-7. These values are comparable to the peak rotation resulting from the applied fields. More important, typical values of $d\delta/dT$ for our samples were 0.01°/K for BK-7, 0.003°/K for SiO₂, and 0.0005°/K for SF-57. Computations using these numbers and Eq. (5) suggest that the error in $(dV/dT)/V_0$ resulting from the temperature dependence of the birefringence could be as much as 3% for BK-7 and 1% for SiO₂; it is negligible for SF-57.

In Table I, we report the results for each glass. The experimental values for $(dV/dT)/V_0$ are the means of 40–45 measurements distributed among the different samples of each glass. The uncertainties listed for SF-57 are the random uncertainties (two standard deviations) for the full set of measurements. For SiO₂ and BK-7, the uncertainties resulting from the temperature dependence of the linear birefringence are added to the random uncertainty. We believe that other systematic uncertainties lie within the quoted imprecisions. Differences between the samples of each glass were statistically insignificant.

It is possible to estimate the temperature dependence of the Verdet constants for diamagnetic materials using Becquerel's formula for the Verdet constant.² In SI units, the Becquerel formula is

$$V_{\text{diam}} = \gamma \frac{e\lambda}{2mc} \left(\frac{dn}{d\lambda} \right), \quad (6)$$

where λ is the wavelength, e and m are the electronic charge and mass, c is the speed of light, n is the index of refraction of the material, and γ is a correction factor known as the magneto-optic anomaly, which depends on the type of bonding in the material. The relative change in Verdet constant with temperature can then be expressed as

$$\frac{1}{V_0} \frac{dV}{dT} = \frac{\frac{d}{dT} \left(\frac{dn}{d\lambda} \right)}{\left(\frac{dn}{d\lambda} \right)} = \frac{d}{d\lambda} \left(\frac{dn}{dT} \right). \quad (7)$$

This value of $(dV/dT)/V_0$ can be estimated by obtaining values of $dn/d\lambda$ from the empirical dispersion formula of each glass and data on $dn(\lambda)/dT$ at various wavelengths.^{8,9} We used a least-squares fitting routine to calculate $d(dn/dT)/d\lambda$. These results are also shown in Table I. The difference between the measured and estimated results ranges from 3% for SF-57 to 20% for SiO₂.

Trade names are used to specify adequately the material used. No recommendation or endorsement by the National Institute of Standards and Technology is implied.

References

1. See, for example, G. W. Day and A. H. Rose. "Faraday Effect Sensors: The State of the Art," *Proc. Soc. Photo-Opt. Instrum. Eng.* **985**, 138–150 (1988).
2. H. Piller, "Faraday Rotation," in *Semiconductors and Semimetals*, Vol. 8, R. K. Willardson and A. C. Beer, Eds. (Academic, New York, 1972), Chap. 3.
3. S. Haussuhl and W. Effgen, "Faraday Effect in Cubic Crystals. Additivity Rule and Phase Transitions," *Z. Kristallogr.* **183**, 153–174 (1988).
4. M. A. Machado Gama, "Faraday Effect in Optical Glass at Low Temperatures," *Opt. Quantum Electron.* **7**, 335–336 (1975).
5. Z. Ren, Y. Wang, and P.-A. Robert, "Faraday Rotation and its Temperature Dependence Measurements in Low-Birefringence Fibers," *IEEE/OSA J. Lightwave Technol.* **LT-7**, 1275–1278 (1989).
6. R. C. Jones, "A New Calculus for the Treatment of Optical Systems," *J. Opt. Soc. Am.* **31**, 488–493 (1941).
7. Equation (5) can be derived using Jones calculus and the Jones matrix for a medium having both linear and circular birefringence. The method is illustrated, for example, in A. M. Smith, "Polarization and Magneto-optic Properties of Single-Mode Fiber," *Appl. Opt.* **17**, 52–56 (1978).
8. Optical Glass, Schott Optical Glass, Inc., York Ave., Duryea, PA 18642.
9. D. E. Gray, Ed., *American Institute of Physics Handbook* (McGraw-Hill, New York, 1972).
10. W. B. Garn, R. S. Caird, C. M. Fowler, and D. B. Thomson, "Measurement of Faraday Rotation in Megagauss Fields over the Continuous Visible Spectrum," *Rev. Sci. Instrum.* **39**, 1313–1317 (1968).

