

# Anomalous relation between time and frequency domain PMD measurements

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## Abstract

We report nearly simultaneous measurements of polarization mode dispersion (PMD) in various samples of highly mode-coupled single-mode fiber using the measurement methods of Jones matrix eigenanalysis (JME) and Fourier-transformed wavelength scanning. The ratio of the PMD values resulting from these two methods differs by approximately 10% from current theoretical predictions. The measurements are verified by demonstrating the theoretical agreement between the JME and wavelength scanning extremum counting results.

## Introduction

In the absence of chromatic dispersion, the data rate of optical communications systems is limited by polarization mode dispersion (PMD) where the group velocity of the traveling light is a function of its polarization state. Several methods for measuring PMD are currently in use and can generally be described as either frequency or time domain techniques. Among the frequency domain methods are Jones matrix eigenanalysis (JME) and wavelength scanning with extremum counting evaluation (WSEC). Two time domain techniques are low coherence interferometry and wavelength scanning Fourier transform evaluation (WSFT). Each of these techniques has its own merits and gives a relative measurement of PMD. Theoretical and experimental work has been done to relate the results of these techniques to each other. The literature has identified the relationship between WSEC and JME measurements [1], between low coherence interferometry and WSFT measurements [2,3], and between frequency domain and time domain measurements [2]. The work described here was designed to experimentally measure the scale factor between frequency and time domain measurements. We find ~10% disagreement with the theoretical prediction of reference 2.

The JME method measures the differential group delay (DGD)  $\Delta\tau$  of the device under test as a function of optical wavelength. PMD is reported from the JME method as either the average DGD  $\langle\Delta\tau\rangle$  or the root-mean-square DGD  $\langle\Delta\tau^2\rangle^{1/2}$ , where the brackets indicate an average over the wavelength range of the test. In the wavelength scanning extremum counting (WSEC) method, the transmitted output optical intensity  $I(\lambda)$  of light through a polarizer, the device under test, and an analyzer is measured over a range of wavelengths. PMD can be determined from this  $I(\lambda)$  by counting the number of extrema  $N_e$  in the response curve. This value is related to the JME result as

$$\langle\Delta\tau\rangle = \frac{kN_e\lambda_1\lambda_2}{2c(\lambda_2-\lambda_1)}, \quad (1)$$

where  $c$  is the speed of light,  $\lambda_1$  and  $\lambda_2$  are the first and last wavelengths of the scan, and  $k$  is the mode coupling factor, determined through simulation to be 0.82 for specimens with high polarization mode coupling [1]. PMD can also be found from the same wavelength scan by taking the Fourier transform of the intensity-vs.-optical frequency curve (WSFT method). For a highly mode coupled fiber, this time domain response should have a Gaussian envelope where  $\sigma$  the square root of the second moment is related to the fiber's PMD.

The current theory [2] relating the time domain measurement of PMD (interferometric or equivalently [2,3] WSFT) to that of the JME method states that  $\sigma$  is exactly equal to the rms differential group delay

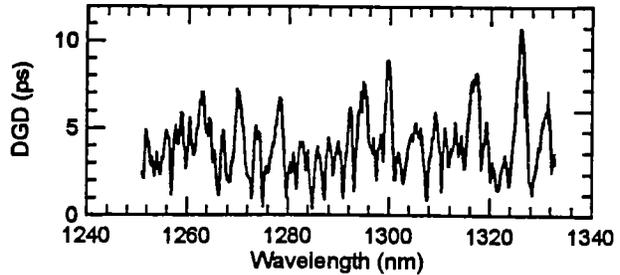
$$\sigma = \sqrt{\langle\Delta\tau^2\rangle}. \quad (2)$$

It is this relationship which is brought into question by our results.

## Experimental Setup

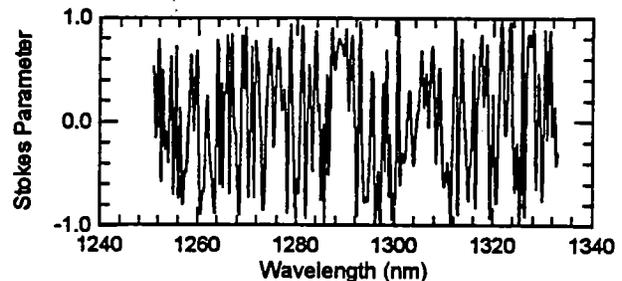
The goal of this experiment was to perform repeated JME and WSFT measurements on several fiber samples to determine if  $\langle \Delta \tau^2 \rangle^{1/2}$  and  $\sigma$  are indeed equivalent. We made measurements on 7 samples of highly mode-coupled single-mode fiber with lengths from 1 to 44 km and whose PMD values ranged from 0.2 to 15 ps (not necessarily respectively).

JME measurements were made using a polarimeter and a tuneable diode laser source with tuning range from 1251 nm to 1333 nm and a linewidth of 100 kHz. The automated polarimeter measures the Jones matrix for the fiber under test by sequentially probing the fiber with light of three linear polarization states and measuring the Stokes parameters of the output light giving the Jones matrix for the fiber under test at the measurement wavelength. The differential group delay of the fiber at the wavelength  $\lambda_0$  is determined from measurements of the Jones matrix at two wavelengths  $\lambda_0 - \Delta\lambda/2$  and  $\lambda_0 + \Delta\lambda/2$  (where  $\Delta\lambda$  is sufficiently small) [4]. In this experiment, the differential group delay was measured for values of  $\lambda_0$  ranging from 1251 nm to 1333 nm. The number of data points was selected to give a  $\Delta\lambda$  which was well below the criterion  $\Delta\lambda \Delta\tau < 2.8 \text{ ps}\cdot\text{nm}$  to prevent aliasing [5]. For the lowest PMD fibers with  $\sim 0.2$  ps of PMD, no less than 64 data points were taken with  $\Delta\lambda \sim 1.3$  nm. The highest PMD measured was  $\sim 15$  ps and no less than 1600 points were taken, giving a maximum  $\Delta\lambda$  of  $\sim 0.05$  nm. Typical JME measurement results are shown for an 8 km length fiber in Fig. 1.



**Figure 1** Typical data from JME measurement. Differential group delay vs. wavelength for an 8 km spool of single mode fiber.

The wavelength-scanning measurements were performed using the same polarimeter and tuneable diode laser. This permitted sequential measurements of JME and wavelength scanning without disturbing the fiber under test. Whereas a simple wavelength-scanning apparatus measures the transmission of light through a polarizer-fiber under test-analyzer combination, the polarimeter used in this experiment measures the three normalized Stokes parameters  $S_1$ ,  $S_2$ , and  $S_3$  of the light exiting the fiber under test. This effectively triples the number of wavelength scanning measurements and thus helps reduce the random uncertainty. For each wavelength scanning measurement, the tuneable source was varied over its full 82 nm range in steps of constant wavelength. In order to perform the Fourier transform, the data steps must be constant in frequency. This was accomplished through software interpolation of the acquired data. For the  $\sim 0.2$  ps fiber, 64 data points were taken with wavelength steps of  $\sim 1.3$  nm. For the largest PMD  $\sim 15$  ps, 2048 points were taken with a wavelength step of 0.04 nm. Figure 2 shows an example wavelength scanning data set for the 8 km fiber.



**Figure 2** Typical data from a wavelength scanning measurement of an 8 km spool of single mode fiber.

The wavelength-scanning data were analyzed as follows. Each of the three Stokes parameter-vs.-wavelength traces was analyzed in terms of extremum counting and the resulting PMD for the system was calculated as the average of the results of the three Stokes traces. The Fourier transform results were obtained by first interpolating the Stokes parameter-vs.-wavelength data to convert it to Stokes parameter-vs.-optical frequency data with data points spaced by constant frequency. Then, the Fourier transform of each of the three data sets was taken using a fast Fourier transform with no windowing function. The results were analyzed using a method similar to that prescribed by the Telecommunications Industry Association (TIA) [6] with one significant change: a curve fitting method instead of the second-moment method was used to determine the PMD. The TIA suggests

measuring  $\sigma$  directly as the square root of the second moment of the time domain data

$$\sigma = \sqrt{\frac{\sum_{p=1}^N I_p t_p^2}{\sum_{p=1}^N I_p}}, \quad (3)$$

where  $I_p$  and  $t_p$  are the intensity and delay time of the  $p$ th data point and  $N$  is the number of data points. However, we found that this method yields 3-7% systematic uncertainties for noisy data. These errors are attributable mostly to the fact that noise dominates in the tail of the time domain data forcing the use of an incomplete data set in Eq. (3). This incomplete sum yields incorrect values of  $\sigma$  which are smaller than they should be.

To avoid this problem, we measured  $\sigma$  by fitting the Gaussian curve

$$I(t) = A \exp\left(\frac{-t^2}{2\sigma^2}\right) \quad (4)$$

to the Fourier-transformed data with  $A$  and  $\sigma$  as fitting parameters. Unlike the second-moment calculation, this Gaussian curve fitting shows no systematic biases when incomplete data sets are used. Of course, for noiseless data, the curve-fitting method and the second-moment calculation produce the same value of  $\sigma$ . We also found that the effects of noise could be eliminated from the second-moment method by calculating correction factors based on the amount of data which was unusable due to noise. When using these correction factors on the second-moment calculation we found good agreement with the curve-fitting results even for noisy data. Having shown these two methods equivalent, we chose to use the curve-fitting method because of its robustness. Figure 3 shows the Fourier transform of the wavelength scanning data of Fig. 2 with the Gaussian fit drawn over the data.

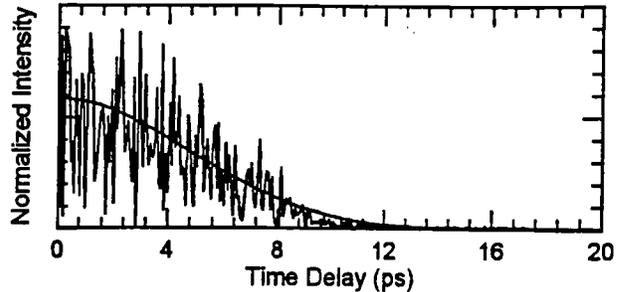


Figure 3 Typical Fourier-transformed wavelength scanning data from 8 km spool of single mode fiber.

During these measurements, the fiber under test was held in a low-vibration, temperature-controlled oven. After a JME measurement and a wavelength scanning measurement had been completed (taking a total of between 30 minutes to 3.5 hours depending on the number of data points sampled), the temperature of the oven was changed and the fiber was allowed between 30 minutes and an hour to equilibrate. Then the JME and wavelength scanning measurements were repeated. Measurements were made at various temperatures from about 20°C to 60°C with a temperature change of at least 5°C between consecutive measurements. The temperature changes allowed statistically independent measurements of each fiber spool. About 10 measurements were made on each spool with the exception of the three highest PMD samples which gave statistically sufficient results with only 3-4 measurements each.

## Results

Figure 4 shows a plot of  $\langle \Delta \tau^2 \rangle^{1/2} / \sigma$  vs.  $\langle \Delta \tau^2 \rangle^{1/2}$ . The current theory predicts that for our measurements, this ratio should be equal to 1 for PMD values which are larger than  $\sim 0.1$  ps (the effective coherence time of the 82 nm scan range of the source). However, the data show a result which is approximately 10% lower than expected. Generally, the inherent randomness of PMD measurements on highly mode-coupled fibers prevents conclusions from being drawn based on experimental data alone. However, that is not the case here. While the inherent randomness of the data yields a large spread in values on the y-axis, the mean is clearly not 1. In fact, not even one measurement yielded a ratio at or above the predicted value of 1.

As a proof that this discrepancy is not due to a measurement error, we used the same JME data and wavelength scanning data to verify Eq. (1). The left side of Eq. (1) is the average differential group delay and the right side is PMD calculated using the WSEC method. Figure 5 shows a plot of the ratio of these two values for the various fibers measured. The data have average values which are within 2-3% of the theoretical value of 1.

## Conclusions

We have calculated the ratio of PMD values measured using JME and Fourier-transformed wavelength scanning. We find  $\langle \Delta \tau^2 \rangle^{1/2} / \sigma$  gives average values close to 0.9 rather than the theoretically predicted 1. Because of the random nature of PMD in highly mode-coupled fiber, this result has been overlooked so far. However by making numerous measurements on highly mode-coupled fibers with significantly large PMD and large wavelength tuning ranges, we have demonstrated clear disagreement with theory.

A current concern in the optical communications industry is the issue of establishing one PMD measurement technique as a reference standard to which the other techniques can be compared and calibrated. However, the results presented here demonstrate a systematic discrepancy between time and frequency domain PMD measurements. This issue must be resolved before a reference standard technique can be useful.

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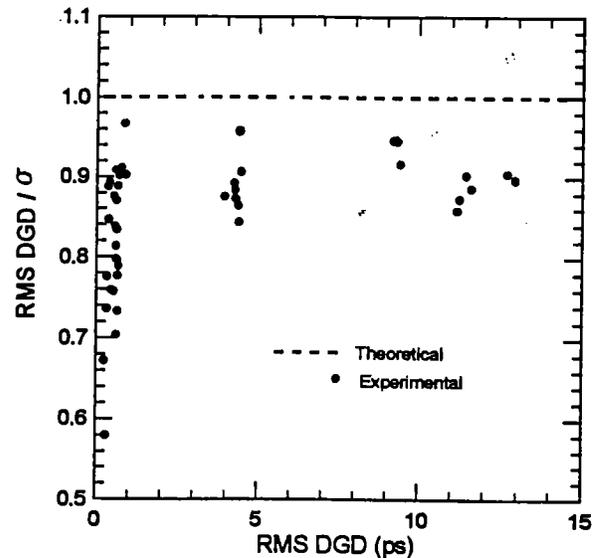


Figure 4 Comparison of PMD measured by JME and WSFT methods. Dashed line indicates theoretical prediction [2].

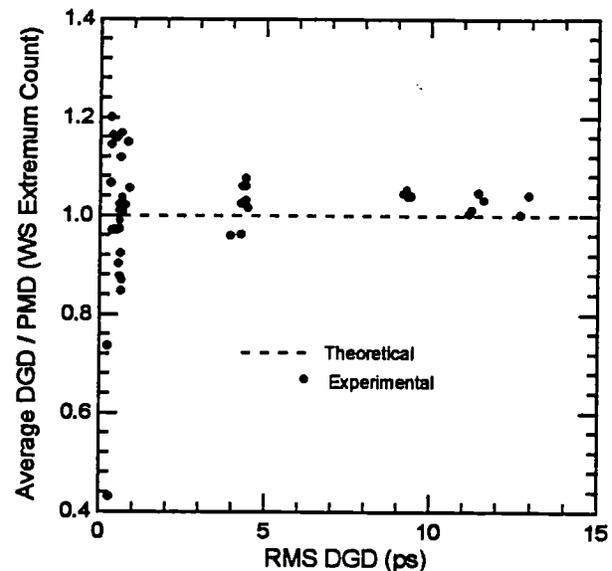


Figure 5 Comparison of PMD measured by JME and WSEC methods. Dashed line indicates theoretical prediction [1].

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