Towards Understanding of Complex Communication Networks: Performance, Phase Transitions & Control

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Abstract

The paper discusses a possibility of phase transitions and metastability in various types of complex communication networks as well as implication of these phenomena for network performance evaluation and control. Specific cases include connection-oriented networks with dynamic routing, TCP/IP networks under random flow arrivals/departures, and multiservice wireless cellular networks. Despite reasons for phase transitions and metastability in different types of networks may differ, a number of similarities suggests a possibility of unified theory of complex networks. The microscopic description (statistical physics) of the complex networks is given by a Markov process with a large number of locally interacting components. The relation between microscopic and macroscopic descriptions of complex networks is analogous to the relation between statistical physics and thermodynamics of physical systems.

I. Introduction

As the size and complexity of the existing and emerging communication networks grow, understanding, and especially controlling, network behavior is becoming more and more of a challenge. The microscopic behavior of communication network often can be described by a Markov process $n(t)=(n_1(t),..,n_K(t))$ with a large number of locally interacting components $n_k(t), k=1,..,K$. Even assuming that the number of possible states N_k of each component $n_k(t), k=1,..,K$ is finite, process $n(t)=(n_1(t),..,n_K(t))$ is homogeneous in time and irreducible, and thus ergodic, very large number of process $x(t)=(x_1(t),..,x_K(t))$ states $N_\Sigma = N_1 \times .. \times N_K$ creates a possibility of metastable, i.e., persistent, states on the time scale of practical interest. This possibility is not just of theoretic interest, since metastability has been observed in real large-scale networks. The possibility of metastability has a number of important practical implications for analytical and simulation-based performance evaluation as well as for system control.

Implications for analytical performance evaluation include limitations of power series expansion techniques due loss singularities, which manifest themselves as abrupt and catastrophic changes in the network behavior with small change in the exogenous parameters. Since only very limited classes of networks can be evaluated analytically or numerically, it is often argued that simulation is a general approach to performance evaluation of complex networks. A possibility of phase transitions and metastability, however, may make interpretation of the simulation results a tricky proposition. Implications of metastability for simulation-based performance evaluation include necessity of exploring space of the initial conditions, since one simulation run may reveal only metastable state(s), corresponding to the simulation initial conditions. Due to inherent infeasibility of exploring of the entire space of possible initial conditions in a high-dimensional phase space of a complex system, there is no guarantee that simulation would reveal all desirable and undesirable metastable states. Implications for complex systems control include necessity of keeping the system in a close neighborhood of the most desirable metastable state, since typically different metastable states have different desirability.

The paper is organized as follows. Section II introduces and discusses a model of complex networks: a Markov process with a large number of interacting components, while Sections III, III and IV discusses specific cases of connection-oriented networks with dynamic routing, TCP/IP networks under random flow arrivals/departures, and multi-service mobile cellular networks respectively,.

II. Multi-Component Markov Processes (Ref. [1])

Consider homogeneous in continuous time $t \ge 0$, K-component Markov process $n^K(t)=(n_1^K(t),..,n_K^K(t))$ with finite number N_K of states. Probability distribution $P(t,n)=\Pr ob\{n(t)=n\}$ is uniquely determined by the corresponding Kolmogorov equations and normalization conditions, given initial distribution $P(0,n)$. Under our assumptions process $n(t)$ is ergodic, i.e., unique steady-state distribution exists: $P(n)=\lim_{t\to\infty}P(t,n)$ for any initial distribution $P(0,n)$, and the steady-state distribution $P(n)$ is the unique solution to the corresponding steady-state Kolmogorov linear algebraic equations supplemented with the normalization condition. Moreover, convergence to the steady-state distribution is exponential in time: $\left|P(t,n)-P(n)\right|\le A(n)e^{-\chi(n)t}$, where constants $A(n)$ and $\chi(n)$ are independent of time t, but may depend on the initial state $n(0)=n$ of the process $n(t)$.

Since $\widehat{A}=\max_n A(n)<\infty$ and $\breve{\chi}=\min_n \chi(n)>0$, it is possible to obtain the following uniform with respect to the initial state $n(0)=n$ exponential convergence: $\left|P(t,n)-P(n)\right|\le \widehat{A}e^{-\breve{\chi}t}$. The problem, however, is that constant $\breve{\chi}$ generally depends on the number of process $n(t)$ states N_Σ, and as the number of states increases: $N_\Sigma\to\infty$, it is possible that rate of convergence to the steady-state distributions approaches zero: $\breve{\chi}\to 0$. In a case of a Markov process $n^K(t)=(n_1^K(t),..,n_K^K(t))$ describing a system with K interacting components $x_k^K(t),\ k=1,..,K$, the number of process $n^K(t)=(n_1^K(t),..,n_K^K(t))$ states $N_\Sigma=N_1\times..\times N_K$ is astronomically high even for moderate number of interacting components K. This reconciles ergodicity of Markov "micro process" $n^K(t)=(n_1^K(t),..,n_K^K(t))$ with possibility of metastability.

From the perspective of linear algebraic Kolmogorov equations, metastability and phase transitions correspond to a situation when as the number of states increases: $N_\Sigma\to\infty$, the minimum eigenvalue associated with the corresponding steady-state Kolmogorov system approaches zero, causing singularity of the Kolmogorov system. This singularity reconciles ergodicity of Markov "micro process" $n^K(t)=(n_1^K(t),..,n_K^K(t))$ with possibility of phase transitions, which are the abrupt and catastrophic changes in the steady-state distribution with a small variation in the exogenous parameters. There are two possible frameworks for addressing the issues of phase transitions and metastability in complex systems. One framework is based on asymptotic investigation of Markov process $n^K(t)=(n_1^K(t),..,n_K^K(t))$ as the number of components increases $K\to\infty$. Another framework defines and investigates the limiting process $n^\infty(t)=(n_1^\infty(t),..)$ directly [1].

III. Connection-Oriented Networks with Dynamic Routing (Ref. [2]-[6])

Consider a network with I nodes $i\in\{1,..,I\}$ and link $l\in L$ with capacities C_l. Due to limited space consider a single-service network, where all flows require the same bandwidth b, so that link l of capacity C_l can carry at most $M_l=\max\{m: m\le C_l/b, m=0,1,..\}$ flows. For each origin-destination pair $s=(i,j)$ flows arrive according to a Poisson process with rate Λ_s. The duration of each flow is distributed exponentially with average $1/\mu$. We will characterize the network state by vector $n=(n_r, r\in R)$, where n_r is the number of flows in progress on route r, and R is the set of all feasible routes in the network. A flow with origin-destination $s=(i,j)$ arriving when the network state is n, is admitted on a feasible route $r\in R_s$ with origin-destination $s=(i,j)$ with probability $q_r(s,n)$ and rejected with probability $q_0(s,n)=1-\sum_{r\in R_s}q_r(s,n)$. Thus, set of conditional probabilities $\{q_r(s,n)\}$, which should satisfy obvious self-consistency conditions, determines dynamic admission and routing strategies. Given probabilities $\{q_r(s,n)\}$, vector $n(t)=(n_r(t), r\in R)$ is a homogeneous in time, ergodic Markov process with finite number of states. The network performance is characterized by unconditional blocking probabilities for a flow with origin-destination s: $\pi(s)=E_n[q_0(s,n)]$. A multi-component nature of process $n(t)=(n_r(t), r\in R)$ creates a possibility of phase transitions and metastability. Macro description of process $n(t)=(n_r(t), r\in R)$ can be developed based on approximation of chaos propagation:

$$P(t,m)\approx\prod_l P(t,m_l) \qquad (1)$$

where the number of flows carried on a link l is $m_l(t) = \sum_{r:l\in r} n_r(t)$. Approximation (1) leads to non-linear mean-field equations, and bifurcations of these equations are associated with phase transitions and metastability.

IV. TCP/IP Networks under Random Flow Arrivals/Departures (Ref. [7]-[10])

Flow level Markov model of fair bandwidth sharing under fluctuating demand has been proposed in [7] for a case of file transfer flows and then in [8] for a case of mixture of file transfer and streaming flows. These Markov models assume separation of time scales: given numbers of flows in progress, bandwidth sharing protocol reaches the equilibrium much faster than the numbers of flows in progress change due to flow arrivals/departures. Stability under condition that each link can accommodate its average load had been established in [8]-[9]. Following [8], assume that the network carries file transfer and streaming flows. Introduce vector (n_1, n_2), where $n_1 = (n_{1r})$ and $n_2 = (n_{2r})$ are the vectors of the numbers of file transfer and streaming flows, respectively, carried on all feasible routes $r \in R$. Consider a network with I nodes $i \in \{1,..,I\}$ and link $l \in L$ with capacities C_l. We assume that file transfer and streaming flows arrive at a route $r \in R$ according to Poisson process of rate Λ_{1r} and Λ_{2r} respectively. The size of a file arriving on a route $r \in R$ is distributed exponentially with average b_r, and the holding time of a streaming flow arriving on a route $r \in R$ is distributed exponentially with average τ_r. All flow arrivals, file sizes and holding times are jointly statistically independent.

We assume separation of time scales: given vector $n = n_1 + n_2$ of numbers of flows in progress $n = (n_r)$ where $n_r = n_{1r} + n_{2r}$, the TCP flow control protocol reaches equilibrium bandwidth sharing much faster than the numbers of flows change due to flow arrivals/departures. Under this assumption numbers of flows in progress can be approximated by a homogeneous in time $t \geq 0$ Markov process $(n_1(t), n_2(t))$. Under fluid regime [8]:

$$\Lambda_{ir} = \varepsilon^{-1}\lambda_{ir};\ C_{lr} = \varepsilon^{-1}c_{lr};\ \lambda_{ir}, c_l = O(1);\ i = 1,2;\ l \in L;\ r \in R;\ \varepsilon \to 0 \tag{2}$$

The following system of ordinary differential equations describing evolution of the vector (η_1, η_2), where $\eta_i = (\eta_{ir}, r \in R)$ and $\eta_{ir} = \varepsilon n_{ir}$, has been derived in [8]

$$\dot{\eta}_{1r} = \lambda_{1r}(\eta_1, \eta_2) - b_r^{-1} \frac{\eta_{1r}}{\eta_{1r} + \eta_{2r}} x_r(\eta_1 + \eta_2) \tag{3}$$

$$\dot{\eta}_{2r} = \lambda_{2r}(\eta_1, \eta_2) - n_{2r}\tau_r^{-1} \tag{4}$$

In (3)-(4) the bandwidth allocated to a flow carried on a route $r \in R$ is $x_r(\eta_1 + \eta_2)$, and it is assumed that arrivals rates depend on the numbers of flows already in progress: $\lambda_{ir} = \lambda_{ir}(\eta_1, \eta_2)$. System (3)-(4) global stability under condition that each link can accommodate its average load had been established in [8]-[9]. However, these stability results for the Markov model and system (3)-(4) under fluid regime do not account for the bandwidth wasted on transmissions of "dead" file transferring packets which will be dropped downstream and then retransmitted [10].

Probably, the simplest way to account for retransmissions is replacing throughput $x_r(\eta_1 + \eta_2)$ with the corresponding good-put $g_r(\eta_1 + \eta_2) = [1 - \pi_r(\eta_1 + \eta_2)]x_r(\eta_1 + \eta_2)$, where the end-to-end packet loss on a route $r \in R$ is $\pi_r(\eta_1 + \eta_2)$. This seemingly minor change drastically alters stability properties of the Markov process $(n_1(t), n_2(t))$ due to deterioration of the good-put as the numbers of flows in progress increase. The corresponding "good-put based" Markov model is unstable even under light average load, when the corresponding throughput-based models are stable. The instability is a result of demand fluctuations: sufficient increase in the number of flows in progress causes increase in the packet loss, reducing good-put and further increasing number of flows in progress [10]. Despite instability, desirable meta-stable network state may still exist. The network can be stabilized in a close neighborhood of this meta-stable state with appropriately designed flow admission strategy at the price of a small flow rejection probability. Network over provisioning without flow admission control only reduces but not eliminates the instability region.

V. Multi-Service Mobile Cellular Networks (Ref. [11])

Consider a cellular wireless network with set of cells L serving S classes of wireless users. Cell $l \in L$ has capacity C_l while each user of class $s = 1,..,S$ requires capacity b_s and has exponentially distributed "life-span" τ_s with average

$\bar{\tau}_s = 1/\lambda_{s0}$. Numbers of users in different cells are described by vector $n = (n_{sl}, s = 1,.., S; l = 1,.., L)$, where the number of users of class s in cell l is n_{sl}. We assume that the feasible region for vector X is as follows:

$$F = \left\{ n : \sum_{s=1}^{S} b_s n_{sl} \le C_l, l = 1,.., L \right\} \tag{5}$$

This assumption describes Frequency Division Multiple Access (FDMA) network, and under some assumptions can be justified for Code Division Multiple Access (CDMA) networks [11].

We assume that new users of class $s = 1,.., S$ originate in cell $l \in L$ according to a Poisson process of rate Λ_{sl}. A new user originated in cell $l \in L$ is admitted with probability $\alpha_{sl}(n)$ and rejected with probability $\beta_{sl}(n) = 1 - \alpha_{sl}(n)$. Each admitted into the network user of class $s = 1,.., S$ performs a random walk over set of cells $l \in L$ during the user "life-span" $[0, \tau_s)$. The random walk is described by Markov process $\xi_s(t)$ with set of states L, continuous time and transitional rates from cell $i \in J$ to cell $j \in J \setminus i$ equal λ_{sij}. Under these assumptions vector $n(t) = (n_{sl}(t), s = 1,.., S; l = 1,.., L)$ is a homogeneous in time multi-component Markov process with finite number of states. Under some natural assumptions process $n(t)$ is irreducible and thus ergodic.

Similar to a case of connection-oriented networks (1), steady-state macro description of process $n(t) = (n_{sl}(t), s = 1,.., S; l = 1,.., L)$ can be developed based on approximation of chaos propagation:

$$P(n) \approx \prod_{l} P_l(n_l) \tag{6}$$

where vector $n_l = (n_{sl}, s = 1,.., S)$ characterizes numbers of users in cell $l \in L$. In absence of admission control or when admission control is based only on the average (but not instantaneous) values of vector $n(t) = (n_{sl}(t), s = 1,.., S; l = 1,.., L)$, further simplification is possible: $P_l(n_l) \approx \frac{1}{Z_l} \prod_{s=1}^{S} \frac{\rho_{sl}^{x_{sl}}}{x_{sl}!}$, where normalization constant is $Z_l = \sum_{x \in F_l} \prod_{s=1}^{S} \frac{\rho_{sl}^{x_{sl}}}{x_{sl}!}$ and "effective loads" ρ_{si} satisfy the corresponding "mean-field" equations. Bifurcations of these, generally non-linear, mean-field equations can be naturally associated with phase transitions and metastability, predicted in [11].

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