Fair Bandwidth Sharing under Flow Arrivals/Departures: Effect of Retransmissions on Stability and Performance

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ABSTRACT
A flow-level Markov model for fair bandwidth sharing with packet retransmissions and random flow arrivals/departures is proposed. The model accounts for retransmissions by assuming that file transfer rates are determined by the end-to-end goodputs rather than the corresponding throughputs as in the conventional model. The model predicts the network instability even under light exogenous load. Despite instability, a desirable metastable network state with finite number of flows in progress may exist. The network can be stabilized in a close neighborhood of the metastable state with admission control at the cost of small flow rejection probability.

Categories and Subject Descriptors
C.4 **[Performance of Systems]**: – *modeling techniques, performance attributes, reliability, availability, and serviceability.*

General Terms
Algorithms, Management, Performance, Design, Theory.

Keywords
Fair bandwidth sharing, arriving/departing flows, retransmissions, performance, stability, admission control.

1. INTRODUCTION
Flow level Markov model of fair bandwidth sharing under flow arrivals/departures has been proposed in [1]-[2] for file transfer flows and then in [3] for a mixture of file transfer and streaming flows. These models assume separation of time scales: bandwidth sharing protocol reaches the equilibrium much faster than the numbers of flows in progress change due to flow arrivals/departures. Stability under condition that each link can accommodate its average load has been established in [2]-[3]. However, these Markov models and stability results do not account for retransmissions of "dead" file transferring packets which will be dropped downstream and then retransmitted.

This paper proposes to account for the effect of wasted bandwidth by assuming that file transfer rates are determined by the end-to-end goodputs rather than the corresponding throughputs as in [1]-[3]. The paper shows that the corresponding "goodput-based" Markov model is unstable even under light exogenous load, when the corresponding throughput-based models [1]-[3] are stable. The instability is a result of demand fluctuations: increase in the number of flows in progress causes increase in the packet loss, reducing goodput and further increasing number of flows in progress. This behavior has been demonstrated by simulations in [4]. We show that despite instability, a desirable metastable network state with finite number of flows in progress may still exist. The network can be stabilized in a close neighborhood of this metastable state with flow admission control at the price of small flow rejection probability. Network over provisioning without flow admission control only reduces but not eliminates the instability region.

2. GOODPUT-BASED MODEL
Given a mixture of flows in progress $N=(N_r)$ carried on all feasible routes $r \in R$, the fixed point model [5] determines vector of transmission rates $x(N)=(x_r(N))$ on all feasible routes $r \in R$ and vector of link packet losses $p(N)=(p_r(N))$ on all links $j \in J$. Under approximation of "link independence", the end-to-end packet loss probability on a route r is

$$p_r = 1 - \prod_{j \in r}(1-p_j) \qquad (1)$$

and the corresponding good-put is

$$g_r = (1-p_r)x_r \qquad (2)$$

Following [3], assume that the network carries file transfer and streaming flows. Introduce vector (N_1, N_2), where $N_1=(N_{1r})$ and $N_2=(N_{2r})$ are the vectors of the numbers of file transfer and streaming flows, respectively, carried on all feasible routes $r \in R$. We assume that file transfer and streaming flows arrive at a route $r \in R$ according to Poisson process of rate Λ_{1r} and Λ_{2r} respectively. The size of a file arriving on a route $r \in R$ is distributed exponentially with average b_r, and the holding time of a streaming flow arriving on a route $r \in R$ is distributed exponentially with average τ_{1r}. All flow arrivals, file sizes and holding times are jointly statistically independent.

We assume separation of time scales: given vector $N=N_1+N_2$ of the numbers of flows in progress $N=(N_r)$ where $N_r = N_{1r}+N_{2r}$, the flow control protocol achieves equilibrium bandwidth sharing much faster than numbers of flows in progress change due to flow arrivals/departures. Under this assumption numbers of flows in progress evolve according to a homogeneous in time $t \geq 0$ Markov process $(N_1(t), N_2(t))$. We account for retransmission of file transferring packets by assuming that the file transfer rate on a route $r \in R$ is g_r rather than x_r as in [1]-[3]. This seemingly minor change drastically alters stability properties of the Markov process $(N_1(t), N_2(t))$ due to deterioration of the aggregate file transfer rate $G_{1r} = N_{1r} g_r$ as N_{1r} increases (see Fig. 1).

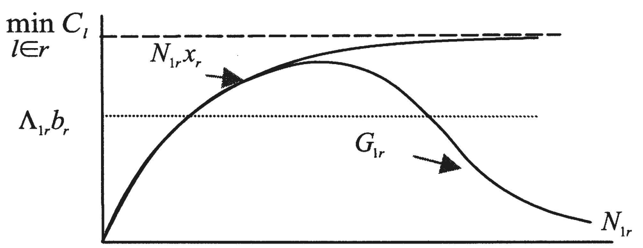

Figure 1. Goodput deterioration.

It is known [2]-[3] that links ability to sustain their average aggregate file transfer load:

$$\rho_j \overset{def}{=} \frac{1}{C_j} \sum_{r:j \in r} \Lambda_{1r} b_r < 1, \ \forall j \in J \tag{3}$$

is the necessary and sufficient condition for stability of the "throughput-based" Markov model when rates $\Lambda_{ir}, \ \forall i = 1,2; r \in R$ are fixed. One may expect that for the "goodput-based" Markov model condition (3) should be replaced with the following condition, which accounts for the additional load due to retransmissions of file transferring packets:

$$\tilde{\rho}_j(N) \overset{def}{=} \frac{1}{C_j[1 - p_j(N)]} \sum_{r:j \in r} \frac{\Lambda_{1r}(N) b_r}{\prod_{i \in r_j^+}[1 - p_i(N)]} < 1, \tag{4}$$

$\forall j \in J$, where r_j^+ is the part of route r located downstream from link $j \in r$. For diverse network topologies, link packet losses p_i increase, and typically approach 1, as the number of flows carried on the link increases. This "goodput deterioration" causes instability of the goodput-based model even under light exogenous loads when stability conditions for the throughput-based model (3) are satisfied.

Despite instability, the goodput-based model still may have a desirable metastable state with finite number of flows in progress. Under fluid regime

$$\Lambda_{ir} = \varepsilon^{-1}\lambda_{ir}, \ C_j = \varepsilon^{-1}c_j, \ \lambda_{ir}, c_j = O(1), \ \varepsilon \to 0 \tag{5}$$

this metastable state corresponds to a finite locally stable equilibrium (n_{ir}^*), $n_{ir}^* < \infty$ of the following system of ordinary differential equations:

$$\dot{n}_{1r} = \lambda_{1r}(n_1 + n_2) - b_r^{-1} n_{1r} g_r(n_1 + n_2) \tag{6}$$

$$\dot{n}_{2r} = \lambda_{2r}(n_1 + n_2) - n_{2r}\tau_r^{-1} \tag{7}$$

describing evolution of normalized numbers of flows in progress $n_{ir} = \varepsilon N_{ir}$, $i = 1,2$, $\forall r \in R$ [3].

Under fixed exogenous load $\lambda_{ir}, i = 1,2; r \in R$, the numbers of streaming flows in progress stabilize: $\lim_{t \to \infty} n_{2r}(t) = \lambda_{2r}\tau_r$, $r \in R$ [3]. If the initial numbers of flows sufficiently high, typically the numbers of file transfer flows infinitely grow with time for any positive average file transfer load: $\lim_{t \to \infty} n_{1r}(t) = \infty$, $r \in R$, reflecting instability of the corresponding Markov goodput-based model. However, for sufficiently light exogenous load, system (6)-(7) may have a locally stable finite equilibrium $\lim_{t \to \infty} n_{1r} = n_{1r}^* < \infty$, $r \in R$, which realizes for sufficiently small initial number of flows in progress. More detailed analysis shows that this locally stable equilibrium describes a desirable metastable state with finite numbers of flows in progress. The network can be stabilized in a close neighborhood of this desirable metastable state with appropriately designed flow admission strategy at the cost of small flow rejection probability.

3. EXAMPLE: RING NETWORK

Consider a symmetric ring network [4] with K nodes, where each node $k = 1,2,..,K$ is connected to node $(k+1)\,\mathrm{mod}(K)$ by a directed link j_k of capacity C. Let weights $w_r = w$, flow arrival rates $\lambda_{ir} = \lambda_i$, $i = 1,2$, average file sizes $b_r = b$, average streaming flow durations $\theta_r = \theta$ and numbers of carried flows $n_{ir} = n_i, i = 1,2$ be route $r \in R$ independent, where set of feasible l-link routes is $R = \{(j_k, j_{(k+1)\mathrm{mod}(K)}, ..., j_{(k+l)\mathrm{mod}(K)}) : k = 1,..,K\}$. Due to limited space we assume that flows arrive only on feasible routes: $\lambda_{ir} = n_{ir} = 0$, $i = 1,2$, $r \notin R$ and only consider proportionally fair rate assignments $x_r \sim 1/p_r$ [6] under fluid asymptotic regime (5). Introduce utilization by file transfer flows $\rho = l\lambda_1 b/c$, utilization by streaming flows $\beta = l\lambda_2\tau$, normalized numbers of flows carried on a link $\eta_i = n_i l$, $i = 1,2$, and $\eta = \eta_1 + \eta_2$.

Under these assumptions the efficiency of the bandwidth sharing, measured by the fraction of the link bandwidth occupied by packets to be delivered to their destinations as opposed to packets to be dropped downstream, is

$$\gamma(\eta) = \frac{\eta}{(1 + \eta/l)^l - 1} \tag{8}$$

Consider a case of fixed arrival rates λ_i, $i = 1,2$ assuming that the numbers of flows carried on a link have already reached equilibrium $\eta_2 = \beta$ [3]. In this case system (6)-(7) equilibriums are given by the following fixed-point equation:

$$\eta_1 = f(\eta_1) \overset{def}{=} \rho\left[\left(1+\frac{\eta_1+\beta}{l}\right)^l - 1\right] \quad (9)$$

Figure 2 shows solution to "steady-state" equation (9) in two cases: $\rho > \rho_*(\beta)$ and $\rho < \rho_*(\beta)$, where "metastability threshold" $\rho_*(\beta)$ for exogenous file transfer load ρ monotonously decreases from $\rho_*(0) = 1$ to $\rho_*(\infty) = 0$ with increase in the exogenous streaming load β.

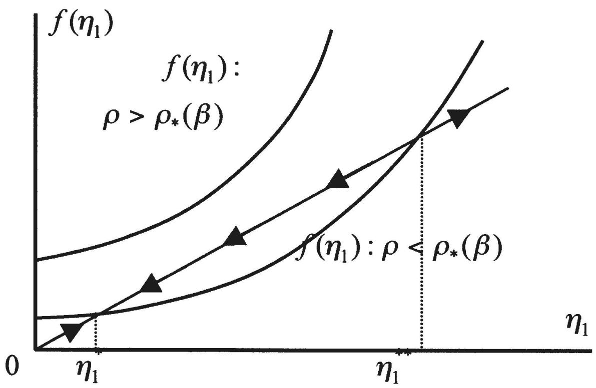

Figure 1. Solution to the equilibrium equation (9).

If exogenous file transfer load exceeds the metastability threshold: $\rho > \rho_*(\beta)$, equilibrium equation (9) has no solution $\eta_1 \geq 0$ and, according to dynamic equation (6)-(7), number of file transfer flows in progress $\eta_1(t)$ infinitely grows with time for any initial $\eta_1(0) \geq 0$. If exogenous file transfer load is below the metastability threshold: $\rho < \rho_*(\beta)$, equilibrium equation (9) has two solutions $\eta_1 = \eta_1^*(\rho,\beta)$ and $\eta_1 = \eta_1^{**}(\rho,\beta)$ describing respectively stable and unstable equilibriums of dynamic equations (6)-(7). Number of file transfer flows in progress $\eta_1(t)$ approaches stable equilibrium: $\lim_{t\to\infty}\eta_1(t) = \eta_1^*(\rho,\beta)$ if initial number of flows is sufficiently small: $\eta_1(0) < \eta_1^{**}(\rho,\beta)$. Otherwise, i.e., if $\eta_1(0) > \eta_1^{**}(\rho,\beta)$ then the number of file transfer flows in progress $\eta_1(t)$ infinitely grows: $\lim_{t\to\infty}\eta_1(t) = \infty$.

Thresholds $\rho_*(\beta)$, $\eta_1^*(\rho,\beta)$ and $\eta_1^{**}(\rho,\beta)$ can be easily evaluated numerically. If $\beta = 0$, i.e., the network carries only file transfer flows, metastability threshold is $\rho_* = 1$. If $\rho < 1$, the metastable equilibrium is $\eta_1^* = 0$, and unstable equilibrium η_1^{**} is the unique positive solution of equation $\eta/\rho = (1+\eta/l)^l - 1$. This equation yields $\eta_1^{**} = 4(\rho^{-1} - 1)$ if $l = 2$, and takes form $\eta/\rho = e^\eta - 1$ if $l \to \infty$. When $\beta \to 0$, $\delta \overset{def}{=} 1 - \rho \to 0$ the following asymptotic formulas can be derived from (9):

$$\delta_* \overset{def}{=} 1 - \rho_* = \sqrt{2\frac{l-1}{l}\beta} \quad (11)$$

$$\eta_1^* = \frac{l}{l-1}\delta - \sqrt{\frac{l}{l-1}\left(\frac{l}{l-1}\delta^2 - 2\beta\right)} \quad (12)$$

$$\eta_1^{**} = \frac{l}{l-1}\delta + \sqrt{\frac{l}{l-1}\left(\frac{l}{l-1}\delta^2 - 2\beta\right)} \quad (13)$$

More detailed analysis shows that equilibrium η_1^* represents the desirable metastable network state with finite number of flows in progress. This metastable state can be transformed into stable state with flow admission control admitting arriving file transfer flows if and only if $\eta_1 < \eta_1^{**}$. The stabilization is achieved at the cost of small flow rejection probability at the fluid asymptotic regime.

The possibility of metastability raises numerous research issues including existence, stability margins, and queuing performance of the metastable state for a general topology network.

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