QoS Routing under Adversarial Binary Uncertainty

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***Abstract*-A cost based admission control and routing scheme admits an arriving request on the minimum cost route if this cost does not exceed the cost of the request, and rejects the request otherwise. Cost based strategies naturally arise as a result of optimization of the network performance or incorporating Quality of Service (QoS) requirements into the admission and routing processes. In the former case the implied cost of the resources represents expected future revenue losses due to insufficient resources to service future requests. In the latter case the cost of a route represents the expected level of QoS, e.g., bandwidth, delay, packet loss, etc., provided to the request carried on this route. In both cases due to aggregation, statistical nature of the resource costs, propagation and queueing delays in disseminating signaling information, non-steady or adversarial operational environment the cost of the resources may not be known exactly. Usually, this uncertainty is modeled by assuming that resource costs are random variables with fixed probability distributions, which may or may not be known to the network. This paper explores different approach intended to guard against adversarial uncertainty, i.e., worst case scenario, with respect to the resource costs lying within known "confidence" intervals. We assume that the network minimizes and the adversarial environment maximizes the loss or risk resulted from non-optimal admission and routing decisions due to the uncertainty. In a symmetric case we explicitly identify the optimal network strategy by solving the corresponding game of the network against environment.**

I. Introduction

A. Cost Based Admission Control and Routing

A cost based admission control and routing strategy for an arriving request is defined by the set $R=\{r_1,r_2,..,r_K\}$ of feasible routes $r\in R$, the "cost" c_r of a feasible route $r\in R$, and the "cost" of the request w. The strategy admits the arriving request on the minimum cost route

$$r_* = \arg\min_{r\in R} c_r \tag{1}$$

if this minimum cost does not exceed the cost of the request

$$c_* = \min_{r\in R} c_r \le w, \tag{2}$$

and rejects the request otherwise. This cost based admission control and routing strategy can be expressed as follows:

$$r_{opt} = \begin{cases} r_* & if\ c_* \le w \\ \varnothing & if\ c_* > w \end{cases} \tag{3}$$

where $r=\varnothing$ means that the request is rejected.

Cost based strategies naturally arise as a result of optimization of the network performance [1] or incorporating Quality of Service (QoS) requirements into admission and routing process [2]. Admission of a request brings certain revenue w to the network, but also ties up the occupied resources until the service is completed and, consequently, may cause future revenue losses due to insufficient resources for servicing some future requests. The implied cost c_r of the resources on a route r reflects these potential revenue losses, and the surplus value

$$u(c,r|w) = \begin{cases} w-c_r & if\ r\ne\varnothing \\ 0 & if\ r=\varnothing \end{cases} \tag{4}$$

is the difference between the revenue brought by the admitted request and the implied costs of the occupied resources.

In a case of QoS routing cost of a route r reflects expected level of the QoS provided to a request carried on this route. For example, c_r may be the expected delay on route r, may represent the bandwidth b_r available on route r, or may represent the packet loss probability on route r. In a case of QoS routing the request cost w characterizes the minimum acceptable level of QoS for this request: $c_r \le w$.

B. Uncertainty in the Resource Costs

Usually, cost-based admission and routing strategies assume the average, steady-state network behavior implying some, typically simple, stationary or quasi-stationary probabilistic model for the external parameters, e.g., connectivity, capacities, traffic arrival patterns, etc. Since the implied costs and surplus values are determined by future events, e.g., arrival of requests, availability of resources, or network topology, the performance of this strategy depends critically on the accuracy of this probabilistic model. The sources of uncertainty in the resource costs c_r are (for more detailed discussion of a case of QoS routing see [2]): (a) statistical inferences resulted in confidence intervals rather than point estimates, (b) aggregation used to reduce amount of signaling traffic, (c) propagation and queueing delays in disseminating signaling information, (d) non-steady operational environment when costs c_r may change with time, (e) adversarial environment attempting to manipulate available information on costs c_r in order to disrupt the network operations.

Currently, commercial networks, including the Internet, may carry mission-critical applications. Possibility of a disaster or adversary attack necessitates developing management schemes that balance cost efficiency with robustness. In practical situations some limited (incomplete) statistical information about the operating environment is available. Proper utilization of this incomplete information would allow the network to reduce the safety margin and consequently increase the cost efficiency with respect to the resource utilization.

C. *Utility Function*

If the route costs are not known exactly, the network may make erroneous decisions: to accept a request on non-optimal route $r \neq r_{opt}$, or reject the request even if $r_{opt} \neq \varnothing$. In a case of optimization of the network performance the utility of the admission and routing decisions is quantified by the surplus value (4). In a case of QoS routing we propose to quantify the utility of the admission and routing decisions as follows:

$$u(c,r|w) = \begin{cases} \varphi(w - c_r) & if\ r \neq \varnothing \\ 0 & if\ r = \varnothing \end{cases} \tag{5}$$

where function $\varphi(\xi)$ is monotonously increasing, concave for $\xi \in (-\infty, \infty)$, and $\varphi(0) = 0$. Surplus value (4) is a particular case of the utility function (5) when $\varphi(\xi) \equiv \xi$. Note that despite formula (5) can be used in both cases: optimization of the network performance and QoS routing, the meanings of the utility functions φ are different. In the former case, linear function $\varphi(w - c_r) = w - c_r$ represents the *network* utility of allocating resources of total cost c_r to a request that generates revenue w. In the latter case typically nonlinear function $\varphi(\xi)$ represents the *user* utility of receiving QoS c_r, allowing for describing the user "soft" QoS requirements [3]-4]. A particular case of user "hard" QoS requirements corresponds to the following specific selection of the function $\varphi(\xi)$:

$$\varphi(\xi|\omega) = \begin{cases} \omega & if\ \xi > 0 \\ -\infty & if\ \xi < 0 \end{cases} \tag{6}$$

with some positive constant $\omega > 0$.

The following parameterized family of functions provides convenient approximation for the utility function $\varphi(\xi)$:

$$\psi(\xi|\omega,\gamma) = \omega\left(1 - e^{-\gamma\xi}\right) \tag{7}$$

where $\omega > 0$ and $\gamma > 0$ are some parameters. Function (7) is monotonously increasing, concave in ξ for any $(\omega,\gamma) \in (0,\infty)^2$. When $\omega \to \infty$, $\gamma \to 0$, $\omega\gamma = \beta = const$, family (7) yields a linear utility function $\psi(\xi|\omega,\gamma) = \beta\xi$. When $\omega = const$, $\gamma \to \infty$, family (7) yields utility function (6).

D. *Main Results and organization of the paper*

Usually, uncertainty in the resource costs is modeled by assuming that the resource costs are random variables with fixed probability distributions, which may or may not be known to the network [2]. From the decision theoretic perspective this approach lies within Bayesian framework [5]. This paper explores a different approach, which lies within the game theoretic framework, and can be justified as guarding against adversarial environment or as providing bounds for the Bayesian solution by identifying the worst case scenario distributions [5]. The paper follows a general approach to network management under uncertainty proposed in [6] and then discussed in [7] in relation to cost based admission control and routing when route costs are selected by an adversary. Paper [7] used this game-theoretic framework to analyze a case of a single feasible route when the risk results only from the admission decision under uncertainty.

This paper extends game-theoretic framework [7] in two directions: first, into domain of QoS routing by assuming generalized utility function (5), and, second, in terms of practical applicability, to a case of multiple feasible routes, by analyzing risks resulted from admission/rejection as well as routing decisions. In this paper we concentrate on a case of binary adversarial uncertainty, when route costs $c_r = (1-\xi)\breve{c}_r + \xi\hat{c}_r$ where bounds $\breve{c}_r$ and $\hat{c}_r$ are known to the network, and the binary variable $\xi \in \{0,1\}$ is selected by the adversarial environment. We demonstrate that allowing mixed, i.e., random network strategies improves the network performance. This result is in sharp contrast with the Bayesian approach, which suggests the deterministic admission and routing strategies based on the *average* utilities. In a particular case of linear utility function and surplus value (4), Bayesian approach with mutually independent random route costs c_r leads to the cost based admission and routing strategies based on the *average* route costs.

The paper is organized as follows. Section II quantifies risks associated with rejection, admission and routing decisions under uncertainty, and, also, formulates the game theoretic framework yielding the optimal admission and routing strategy under adversarial uncertainty. Section III derives the best pure strategies for the network and environment in a case of binary adversarial uncertainty. Section IV describes approach to solving of the corresponding game and presents explicit solution in a symmetric case.

II. Risks of Admission and Routing under Uncertainty

A. Game Theoretic Framework

The losses due to non-optimal admission and routing decisions can be quantified by the following loss or risk function [5]-[7]:

$$L(c,r|w) = u_{opt}(c|w) - u(c,r|w) \tag{8}$$

where the utility of the optimal admission and routing decisions is

$$u_{opt}(c|w) = u(c, r_{opt}|w) \tag{9}$$

and r_{opt} is determined by (3). Combining (3), (5), (8) and (9) we obtain:

$$L(c,r|w) = \begin{cases} \max\{0, \varphi(w-c_*)\} - \varphi(w-c_r) & if\ r \neq \varnothing \\ \max\{0, \varphi(w-c_*)\} & if\ r = \varnothing \end{cases} \tag{10}$$

where c_* is determined by (2). Given w, function $L(c,r|w)$ possesses the following properties:

$$L(c,r|w) = 0 \text{ for } r = r_{opt}\text{, and } \forall c \in C$$

$$L(c,r|w) \geq 0 \text{ for } \forall (c,r) \in C \otimes \{\varnothing, R\}$$

and thus

$$L_{\max}^{\min} = \max_{c\in C} \min_{r\in(\varnothing,R)} L(c,r|w) \equiv 0 \tag{11}$$

$$L_{\min}^{\max} = \min_{r\in(\varnothing,R)} \max_{c\in C} L(c,r|w) \geq 0 \tag{12}$$

The best pure strategies for the network and environment $r = \tilde{r}_{opt}$ and $c = \tilde{c}^{\,opt}$ are determined by the solution to the optimization problem (12). It is easy to verify that $\tilde{r}_{opt} = r_{opt}$ if set C consists of a single element: $C \equiv c$. This paper investigates admission and routing strategies guarding against the worst case scenario with respect to the route cost vector $c = (c_r)$ when the available information on the vector c can be quantified in terms of the "confidence" interval C: $c \in C$. The following game theoretic framework with pay-off function (10) provides a natural formalization for this problem.

Consider a zero-sum game with two players, where player (r) represents the network, and player (c) represents the adversarial environment. The set of feasible strategies for the network is $r \in \{\varnothing, R\}$ and the set of feasible strategies for the environment is $c \in C$. The matrix of payoffs made by the network to the environment $L(c,r|w)$ is given by (10). According to this game theoretic framework, the optimal network strategy $r \in \{\varnothing, R\}$ represents the admission and routing strategy guarding against the worst case scenario with respect to the route costs $c \in C$. The value of the game $\upsilon = \upsilon_{C,R}(w)$ represents expected performance loss due to the admission and routing decisions $r \in \{\varnothing, R\}$ for a single request under incomplete information on the implied costs of the resources $c \in C$ selected by adversarial environment. In a particular case when the payoff function $L(c,r|w)$ has a saddle point, i.e., $L_{\min}^{\max} = 0$, the environment and the network have pure optimal strategies $c = \tilde{c}^{\,opt}$ and $r = \tilde{r}_{opt}$ respectively, which are the solution to the optimization problem (12). In a case when the payoff function $L(c,r|w)$ does not have a saddle point, i.e., $L_{\min}^{\max} > 0$, the environment and the network have mixed optimal strategies which are probability distributions on $c \in C$ and $r \in \{\varnothing, R\}$ respectively. The value of the game is

$$\upsilon = \begin{cases} E_{c,r}[L(c,r|w)] & if \quad L_{\min}^{\max} > 0 \\ 0 & if \quad L_{\min}^{\max} = 0 \end{cases} \tag{13}$$

where the expectation is taken with respect to the optimal mixed strategies for the network and environment.

Usually, in practical situations, optimization problem (12) is quite tractable while finding optimal mixed strategies is computationally challenging. The performance gain resulted from these computational trouble can be quantified by the following criterion:

$$\delta = \begin{cases} L_{\min}^{\max}/\upsilon - 1 & if \quad \upsilon > 0 \\ 0 & if \quad \upsilon = 0 \end{cases} \tag{14}$$

B. Admission, Rejection and Routing Risks

There is a natural way to separate losses (10) into losses resulted from the admission/rejection decision $L^{a/r}(c|w)$ and losses resulted from the routing decision $L^{rtn}(c,r)$ under uncertainty:

$$L(c,r|w) = L^{a/r}(c|w) + L^{rtn}(c,r) \tag{15}$$

Since the optimal routing decision (1) does not carry any risk, i.e., $L^{rtn}(c, r_*) \equiv 0$, we have from (15): $L^{a/r}(c|w) = L(c, r_*|w)$, and thus:

$$L^{a/r}(c|w) = \begin{cases} L^{adm}(c|w) & if\ r \neq \varnothing \\ L^{rej}(c|w) & if\ r = \varnothing \end{cases} \tag{16}$$

where the admission risk is

$$L^{adm}(c|w) = -\min\{0, \varphi(w-c_*)\} \tag{17}$$

and the rejection risk is

$$L^{rej}(c|w) = \max\{0, \varphi(w-c_*)\} \tag{18}$$

Combining (15)-(18) we obtain the following expression for the routing risk:

$$L^{rtn}(c,r|w) = \begin{cases} \varphi(w-c_*) - \varphi(w-c_r) & if\ r \neq \varnothing \\ 0 & if\ r = \varnothing \end{cases} \tag{19}$$

The value of the game (13) can be also represented as the following sum:

$$\upsilon = \upsilon^{a/r} + \upsilon^{rtn} \quad (20)$$

where the expected performance loss due to the admission/rejection decision is

$$\upsilon^{a/r} = \upsilon^{adm} + \upsilon^{rej}, \quad (21)$$

the expected performance loss due to the admission is

$$\upsilon^{adm} = -\alpha E\left[\min\{0, \varphi(w - c_*)\} \middle| r \neq \varnothing\right], \quad (22)$$

the expected performance loss due to the rejection is

$$\upsilon^{rej} = (1-\alpha)E\left[\max\{0, \varphi(w - c_*)\} \middle| r = \varnothing\right], \quad (23)$$

the expected performance loss due to the routing decision is

$$\upsilon^{rtn} = \sum_{r\in R} \alpha_r E\left[\varphi(w - c_*) - \varphi(w - c_r)\right], \quad (24)$$

the probability of selecting route r is α_r, and the admission probability is

$$\alpha = \sum_{r\in R} \alpha_r \quad (25)$$

Expectations in (22)-(24) are to be calculated with respect to the optimal, in general mixed, strategies for the network and environment.

III. Best Pure Strategies under Binary Uncertainty

A. Binary Uncertainty

Computational tractability of the game with pay-off function (10) depends on the set of feasible strategies for the environment $c = (c_r : r \in R) \in C$. Further in this paper we consider a case of binary strategies for the environment:

$$C = \{\breve{c}, \hat{c}\} \quad (26)$$

or, equivalently,

$$c = (1-\xi)\breve{c} + \xi\hat{c} \quad (27)$$

where the low and upper bounds for the feasible route costs $\breve{c} = (\breve{c}_r : r \in R) \in C$ and $\hat{c} = (\hat{c}_r : r \in R) \in C$ are known to the network, while the binary variable $\xi \in \{0,1\}$ is selected by the adversarial environment. In this extreme case malicious environment can only attack all feasible routes simultaneously. Another extreme case would be a separable set of feasible strategies for the environment C:

$$C = \bigotimes_{r\in R}[\breve{c}_r, \hat{c}_r] \quad (28)$$

Under separable scenario (28) the malicious environment can select route costs independently from each other within their "confidence" intervals $c_r \in [\breve{c}_r, \hat{c}_r]$.

In practical applications the route costs are typically additive, i.e., the implied cost of a route r is the sum of the implied costs of the links $l \in r$ comprising this route. Due to the "global nature" of the route costs [1], and overlapping of different routes, a realistic scenario lies somewhere between binary scenario (26) and separable scenario (28). In this paper we assume a binary scenario (26) mostly because of the computational tractability allowing us to illustrate the proposed approach to the network management under adversarial uncertainty. Also, binary scenario (26) may serve as a model of less sophisticated or capable adversarial environment than separable scenario (28). Note that binary scenario (26) leads to lower expected risk and to less conservative network strategy than scenarios with less than perfect correlation between costs of different routes.

B. The Best Pure Strategies

The optimal binary environment response $\xi^* = \xi^*(r)$ to selection of a route $r \in \{\varnothing, R\}$ by the network is determined by solution to the following optimization problem:

$$L^{\max}(r|w) = \max_{\xi=0,1} L\left[c(\xi), r \middle| w\right]$$

where $c(\xi)$ is given by (27). It is easy to verify that

$$\xi^*(r) = \begin{cases} 0 & if\ r = \varnothing \\ 1 & if\ r \neq \varnothing \end{cases}$$

The corresponding losses are

$$L^{\max} = \begin{cases} \max\{0, \varphi(w - \hat{c}_{\min})\} - \varphi(w - \hat{c}_r) & if \quad r \neq \varnothing \\ \max\{0, \varphi(w - \breve{c}_{\min})\} & if \quad r = \varnothing \end{cases}$$

where $\breve{c}_{\min} = \min\{\breve{c}_r | r \in R\}$, and

$$\hat{c}_{\min} = \min\{\hat{c}_r | r \in R\} \quad (29)$$

The best pure network strategy is determined by solution to the following optimization problem:

$$\tilde{r}_{opt} = \arg\min_{r\in(\varnothing,R)} L^{\max}(r|w)$$

It is easy to verify that

$$\tilde{r}_{opt} = \begin{cases} \hat{r}_* & if \quad L^{\max}(\tilde{r}_*|w) < L^{\max}(\varnothing|w) \\ \varnothing & if \quad L^{\max}(\tilde{r}_*|w) > L^{\max}(\varnothing|w) \end{cases} \quad (30)$$

where $\hat{r}_*$ is determined by solution to (29).

IV. Optimal Strategy

A. Solution to the Game

In a case of binary uncertainty the payoff function (10) takes a form of the following $2\times(K+1)$ matrix $L = (L_{ik})_{i,k=0}^{2,K+1}$:

$$\begin{array}{ccccc} & r=\varnothing & r = r_1 & \ldots & r = r_K \\ \xi = 0 & L_{00} & L_{01} & \ldots & L_{0K} \\ \xi = 1 & L_{10} & L_{11} & \ldots & L_{1K} \end{array} \quad (31)$$

where

$L_{00} = \max\{0, \varphi(w - \breve{c}_{\min})\}$,

$L_{10} = \max\{0, \varphi(w - \hat{c}_{\min})\}$,

$L_{0k} = \max\{0, \varphi(w - \breve{c}_{\min})\} - \varphi(w - \breve{c}_r)$,

$L_{1k} = \max\{0, \varphi(w - \hat{c}_{\min})\} - \varphi(w - \hat{c}_r)$,

$k = 1,..,K$. A $2\times(K+1)$ game can be explicitly solved [8]. The fundamental simplex of mixed strategies of the environment in this case is the closed interval [0,1]. Let the network select the pure strategy $k = 1,..,K$. Then the payoff of the environment will depend on its chosen probability x of selecting the first pure strategy:

$$g_k(x) = xL_{0k} + (1-x)L_{1k} \tag{32}$$

$k = 1,..,K$, The graph of the function

$$z(x) = \min_{k=0,..,K} g_k(x) = \min_{k=0,..,K} \{xL_{0k} + (1-x)L_{1k}\}$$

is the lower envelop off all straight lines (32). Clearly, such a graph is a broken line that is convex upwards. An upper peak of this broken line characterizes the optimal probability $x = x^*$ and the value of the game

$$\upsilon = \max_{x\in[0,1]} z(x) = \min_{x\in[0,1]} \min_{k=0,..,K} \{xL_{0k} + (1-x)L_{1k}\}$$

It is easy to verify that if $w < \breve{c}_{\min}$, then the network has pure optimal strategy $r = \varnothing$, i.e., reject the request, the environment has a pure strategy $\xi = 0$, and the value of the game $\upsilon = 0$. Due to space constraints, in the next subsection we only describe the optimal strategy in a symmetric case.

B. *The Optimal Strategy in a Symmetric Case*

Consider a symmetric case with $K \ge 2$ feasible routes $c_r \in [\breve{c}, \hat{c}]$, $r \in R = \{r_1,..,r_K\}$. The network has two pure admission strategies: $r = \varnothing$, i.e., to reject the request, and $r \in R$, i.e., to accept the request. We assume that once accepted a request is carried on a route randomly, with equal probabilities, selected from all feasible routes $r \in R = \{r_1,..,r_k\}$. It is easy to verify that the optimal admission probability is

$$\alpha = \begin{cases} 1 & if \quad \hat{c} \le w \\ \dfrac{\varphi(w-\breve{c})}{\varphi(w-\breve{c}) - \varphi(w-\hat{c})} & if \quad \breve{c} < w < \hat{c} \\ 0 & if \quad w \le \breve{c} \end{cases} \tag{33}$$

and the value of the game is

$$\upsilon = \begin{cases} -\dfrac{\varphi(w-\breve{c})\varphi(w-\hat{c})}{\varphi(w-\breve{c}) - \varphi(w-\hat{c})} & if \quad \breve{c} < w < \hat{c} \\ 0 & otherwise \end{cases} \tag{34}$$

Note that $\upsilon^{adm} = \upsilon$ since $\upsilon^{rtn} = 0$. For utility function (7) we get from (33)-(34): $\alpha = \dfrac{\exp[\gamma(w-\breve{c})]-1}{\exp[\gamma(\hat{c}-\breve{c})]-1}$ and $\upsilon = \omega\alpha\left(e^{\gamma(\hat{c}-w)} - 1\right)$ if $\breve{c} < w < \hat{c}$. In a case of linear utility function $\varphi(\xi) \equiv \xi$, expressions (33)-(34) yield: $\alpha = \dfrac{w-\breve{c}}{\hat{c}-\breve{c}}$ and $\upsilon = \dfrac{(w-\breve{c})(\hat{c}-w)}{\hat{c}-\breve{c}}$. In a case of hard QoS requirements (6), expressions (33)-(34) yield: $\alpha = 0$ and $\upsilon = \omega$.

The following matrix shows the network pure strategies: $r = \varnothing$ and $r \in R$, the best environment response $\xi = \xi^*(r)$, and the corresponding loss $L^{\max}$ in a symmetric case:

r	ξ^*	$L^{\max}$
$r = \varnothing$	0	$\max\{0, \varphi(w-\breve{c})\}$
$r \in R$	1	$-\min\{0, \varphi(w-\hat{c})\}$

The best pure admission strategy (30) is to admit the request if $w \ge \overline{w}$, and reject the request otherwise. The gain (14) is

$$\delta_1 = \begin{cases} -\dfrac{\varphi(w-\hat{c})}{\varphi(w-\breve{c}) - \varphi(w-\hat{c})} - 1 & if \quad \breve{c} < w < \overline{w} \\ \dfrac{\varphi(w-\breve{c})}{\varphi(w-\breve{c}) - \varphi(w-\hat{c})} - 1 & if \quad \overline{w} < w < \hat{c} \end{cases}$$

and $\delta_1 = 0$ otherwise, where $w = \overline{w}$ is the unique solution to the following equation:

$$\varphi(w-\breve{c}) + \varphi(w-\hat{c}) = 0 \tag{35}$$

For utility function (7) equation (35) can be solved explicitly:

$$\overline{w} = \frac{1}{\gamma}\log_e\left(\frac{e^{\gamma\breve{c}} + e^{\gamma\hat{c}}}{2}\right)$$

In a case of linear utility function $\varphi(\xi) \equiv \xi$, equation (35) yields $\overline{w} = (\breve{c} + \hat{c})/2$, and in a case of hard QoS requirements (6), equation (35) yields $\overline{w} = \hat{c}$.

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