Uncertainty Analysis for Four Terminal-Pair Capacitance and Dissipation Factor Characterization at 1 MHz and 10 MHz[‡]

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Introduction

The Electricity Division at the National Institute of Standards and Technology (NIST, formerly NBS) has implemented a system to characterize capacitance and dissipation factor for four terminal-pair (4TP) air dielectric capacitors at frequencies from 1 kHz to 10 MHz [1]. The method is based on work by Cutkosky [2,3] and Jones [4,5] of NBS and recent developments by Yokoi, Suzuki, and Aoki [6,7] as well as Yonekura and Wakasugi [8] of Hewlett-Packard Japan. This paper describes an extensive uncertainty analysis of the measurement system. The analysis has been divided into three areas: 1 kHz capacitance measurements; network analyzer impedance measurements (covering frequencies from 40 MHz to 200 MHz); and a mathematical extrapolation algorithm that regresses the high-frequency characterization down to frequencies of 10 MHz and below [6,7]. This algorithm is referred to as the capacitor frequency characteristic prediction (CFCP) method. The capacitance and dissipation factor characteristics at 1 MHz and 10 MHz are produced by applying the CFCP algorithm to the 1 kHz capacitance as well as the high-frequency (40 MHz to 200 MHz) impedance measurements.

Capacitors characterized using this technique will be used as impedance reference standards for a general-purpose digital impedance bridge recently developed at NIST to calibrate inductors and ac resistors [9]. The technique is also to be employed in a future NIST Special Test for 4TP capacitance and dissipation factor.

1 kHz Capacitance Measurement Uncertainty

The capacitance components of the 4TP standard capacitors, C_{lh}, C_{lg}, and C_{hg} are repeatedly measured over time to establish repeatability using a 1 kHz capacitance bridge. Figure 1 shows the simple 4TP capacitor circuit model, where C_{lh} is the low-to-high capacitance, and C_{lg} and C_{hg} are low-to-ground and high-to-ground leakage capacitances, respectively. Table 1 presents Type A relative standard uncertainties for 1 kHz bridge measurements of the 10 pF, 100 pF, and 1000 pF standard capacitors. Uncertainties are given as parts in 10^6 and labeled ppm. The Type B relative standard uncertainty for the 1 kHz capacitance bridge is about 10 ppm [10].

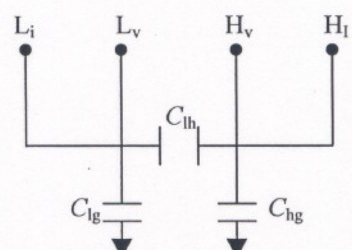

Figure 1. Four Terminal-Pair Capacitor: Simple Model

Since the 1 kHz capacitance measurements are used in the CFCP method, there will be an uncertainty contribution from the 1 kHz measurements to the final capacitance and dissipation factor results. Software

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simulations were performed to determine the uncertainty components of the 4TP capacitance and dissipation factor due to the 1 kHz capacitance measurements.

Table 1. Type A Standard Uncertainties for 1 kHz Measurements

Nominal Capacitance (pF)	Measured Parameter	Measured Capacitance (pF)	Type A Relative Standard Uncertainty(ppm)
10	C_{lh}	10.000745	4.0
	C_{lg}	29.168758	13.5
	C_{hg}	25.024177	29.9
100	C_{lh}	99.99892	6.8
	C_{lg}	32.27685	29.5
	C_{hg}	31.03866	36.8
1000	C_{lh}	1000.0739	5.4
	C_{lg}	34.98904	163.6
	C_{hg}	33.36330	263.4

Data from the Yonekura model [8] were used to solve the 4TP capacitor circuit at high frequencies to simulate network analyzer measurements (40 MHz to 200 MHz) as well as to provide exact solutions for capacitance and dissipation factor at 1 MHz and 10 MHz. The components of the Yonekura model were determined by dissembling a number of capacitors and measuring the impedance of each part. The average values of these components and their uncertainties are given in [8].

A single set of the high-frequency data was fed into the CFCP method using many different sets of 1 kHz capacitance measurement data generated randomly (normal distribution) and centered around the means of the capacitor circuit model components and spanning twice the standard deviations of the measurement data. Predictions of the 4TP capacitance and dissipation factor from the CFCP method were obtained for the multiple simulations at 1 MHz and 10 MHz. Ten thousand such iterations were performed for each standard capacitor. The predictions were compared with the exact solutions of the circuit computed from the Yonekura model. The exact capacitance and dissipation factor values computed from the Yonekura model were used as reference points to estimate the Type A uncertainty contribution in the final results due to the 1 kHz measurements. The Type B uncertainty simulations run just as the Type A simulations except that the 1 kHz capacitance values are offset by 10 ppm instead of randomly scattered prior to running the CFCP algorithm. The uncertainties produced from this simulation are reported in the Uncertainty Results section as 1 kHz Types A and B standard uncertainties.

Note that the Type B uncertainties have been approximated conservatively to be 10 ppm; we have not yet calibrated our 1 kHz capacitance meter in order to incorporate an exact value. Additionally, the 10 ppm Type B standard uncertainty for the 1 kHz capacitance bridge is root-sum-squared with the Type B 1 kHz component of the standard uncertainty of the CFCP method.

Network Analyzer Measurement Uncertainty

A network analyzer is used to measure one-port impedances of a standard capacitor at frequencies from 40 MHz to 200 MHz. The actual measured quantity is the scattering parameter, S11. The network analyzer converts S11 into impedance. The network analyzer contributes both Type A and Type B uncertainties. The Type A component of uncertainty is estimated by using repeat measurements from the network analyzer and the CFCP method to characterize the capacitance and dissipation factor of the standard capacitors.

The Type B component of uncertainty due to the network analyzer was estimated using software simulations. Again, the solution of the Yonekura circuit model [8] was used to compute 4TP capacitance and dissipation factor at frequencies of 1 MHz and 10 MHz. S11 parameters were varied within an uncertainty band of the network analyzer and applied to the CFCP algorithm to predict 1 MHz and 10 MHz capacitance and dissipation factor behavior. Repeated computations on the regressed data were produced with simulations using the specified measurement uncertainty to determine variability in the 4TP capacitance and dissipation factor due to network analyzer errors.

The uncertainty values depend on the reflection coefficient of the measurement and, hence, 1 percent magnitude and 1 degree phase are upper bounds on the error, based upon manufacturer-specified uncertainty [11].

The simulation program estimates components of standard uncertainty due to the network analyzer for the 4TP capacitance and dissipation factor for each of the standard capacitors. The uncertainties are estimated by introducing random errors that vary within one standard deviation of the network analyzer S11 magnitude and phase uncertainty specifications. Repeated computations of the 4TP capacitance and dissipation factor are produced with the assumption that all of the network analyzer error is an offset error. One thousand iterations were performed for positive and negative sign combinations of the magnitude and phase error components.

Additionally, S11 measurements of a precision 20 cm air line were used to estimate and compensate for residual directivity, frequency tracking, and source match errors in network analyzer measurements. The air line has been calibrated at NIST to achieve traceability to national standards. A comparison of the air line calibration values with S11 measurements of the air line from the network analyzer must be performed to verify the reported Type B component of the uncertainty. The Types A and B standard uncertainties in the 4TP characterization due to the network analyzer are reported in the Uncertainty Results section. Further analysis of the air line calibration will be performed and reported in the transaction paper.

Regression Algorithm Uncertainty Due to Variations in Capacitor Manufacturing

Still more simulations were performed to determine the uncertainty components of the 4TP capacitance and dissipation factor introduced by the regression algorithm (and due to the variability in manufacturing of the capacitor standards). The regression algorithm extrapolates the network analyzer impedance measurements (made over a range of frequencies chosen somewhere between 40 MHz and 200 MHz) down to 10 MHz and below using the 1 kHz capacitance measurement values, described briefly above, as references. The regression parameters were selected to optimally predict the capacitance frequency characteristic for nominal capacitor values.

The regression algorithm uncertainty simulations use the Yonekura model of the standard capacitor to solve the circuit at 1 MHz and 10 MHz. The circuit solution provides reference values of capacitance and dissipation factor at the frequencies of interest. The Yonekura circuit is also used to obtain high-frequency impedance values (simulations of network analyzer measurement data). The simulated network analyzer measurement data are then used to iteratively extrapolate to the frequencies of 10 MHz and 1 MHz with normally random errors injected into the Yonekura model components according to each component's uncertainty. For each standard capacitor, the extrapolation was performed ten thousand times and a distribution was created by subtracting the 1 MHz and 10 MHz mean model solutions.

Uncertainty Results

Table 2 labels the standard uncertainty components and Table 3 shows the values of the components as well as the expanded standard uncertainties for capacitance and dissipation factor characterization of the standard 4TP capacitors (10 pF, 100 pF, and 1000 pF) at frequencies of 1 MHz and 10 MHz. The uncertainty components are root-sum-squared and then multiplied by two to produce

Table 2. Uncertainty Component Descriptions.

a1	Type A 1 kHz capacitance standard uncertainty
a2	Type A network analyzer standard uncertainty
b1	Type B 1 kHz capacitance standard uncertainty
b2	Type B network analyzer standard uncertainty
b3	Type B CFCP standard uncertainty

Table 3. Relative Uncertainty Components and Expanded Uncertainties ($k = 2$).

	a1	a2	b1	b2	b3	U
10 pF Capacitance						(ppm)
1 MHz	4.0	0.8	10.0	1.4	0.8	22
10 MHz	4.0	76.2	30.6	133.7	75.4	348
10 pF Dissipation Factor						(rad)
1 MHz	4.0	0.1	10.0	10.5	2.2	31
10 MHz	4.0	2.0	10.0	332.7	70.3	681
100 pF Capacitance						(ppm)
1 MHz	6.8	0.1	10.0	0.7	0.1	24
10 MHz	6.8	9.1	10.0	65.5	7.2	135
100 pF Dissipation Factor						(rad)
1 MHz	6.8	0.0	10.0	9.5	0.1	31
10 MHz	6.8	0.2	10.0	301.2	2.1	603
1000 pF Capacitance						(ppm)
1 MHz	5.4	0.1	10.0	0.9	2.4	23
10 MHz	5.4	8.1	36.3	167.6	255.3	615
1000 pF Dissipation Factor						(rad)
1 MHz	5.4	0.0	21.6	26.4	8.3	71
10 MHz	5.4	0.2	610.1	844.4	265.8	2150

the expanded standard uncertainty values. All capacitance uncertainty components are given in parts in 10^6, labeled as ppm, and all dissipation factor uncertainty components are given in microradians, labeled as rad. These values will be refined and reevaluated as the authors gain experience with the measurement system.

Future Work

The uncertainty analysis reported in this paper consists of simulations that determine the statistical variation of the components of the 4TP capacitor characterization technique. Some assumptions are made regarding the errors and the circuit model derived from measurements of a set of standard capacitors. A theoretical analysis of the network analyzer should be performed in order to compare with the simulations performed and reported upon here.

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