Towards Power and QoS Aware Wireless Networks

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ABSTRACT

The paper studies the optimal use of energy in wireless networking, the feasibility region of tasks that share a multi-access channel, and efficient algorithms for determining if a given set of tasks and resources falls within the feasibility region.

Categories and Subject Descriptors

C.4 [**Performance of Systems**]: Feasibility, Optimality

General Terms

Algorithms, Performance, Economics, Reliability, Theory.

Keywords

Network Information Theory, Pareto optimality

1. INTRODUCTION

The emergence of a multitude of applications relying in wireless networking has resulted in renewed interest in issues of energy efficiency. In many instances, e.g. remote sensor networks, the available energy per device must be considered, in the short run, as non-renewable. A large number of papers address issues of network capacity and optimal energy use [1,2,3,4,5]. Often the problem has been cast as making the optimal decision on committing a single resource to satisfy current demands in the face of uncertainty about future demands under constraints of non-preemptive commitment. A situation of wireless communication presents major complications to this formulation. Among these complications are cooperative nature of the problem due to possibility of multi-user detection, multiple resources, e.g., wireless bandwidth and transmission power, to be allocated, and, possibility that some resources, such as battery power, may not be renewable within the time horizon in question.

In this paper we discuss a model which assumes that at any moment each source has limited battery energy and specific deadlines for transmitting data associated with different applications. The resource allocation problem includes decisions on wireless bandwidth sharing and battery energy expenditure rates. The goal of resource allocation is to maximize the weighted sum of the residual battery energies for different sources, given delay deadlines and current battery energy levels. Since the corresponding weights reflect the expectations on future request arrivals, which are not completely known, in this brief paper we concentrate on approaching the Pareto optimal frontier with respect to the residual battery energies subject to the constraints on delays and numbers of transmitted bits.

The next two sections describe the resource allocation model, and the main results, including the structural properties of optimal allocations. Sections 4 and 5 depict typical feasible areas and summarize our conclusions.and ideas for further research.

2. MODEL

There are $|S|$ sources $s \in S$ attempting to transmit information to the same destination. Initially, at the moment $t = 0$, each source s has battery energy E_s and B_{sk} bits to transmit with deadline $t = T_k$, $k = 1,..,K$, where without loss of generality we assume that $0 = T_0 < T_1 < T_2 < .. < T_K$. We characterize the system performance by the vector of the residual battery powers for all sources $W = (W_1,..,W_K)$. The residual battery power for a source

$$W_s = E_s - \int_0^{T_K} p_s(t)dt \tag{1}$$

where the source $s \in S$ transmission powers $p_s(t)$ and transmission rates $r_s(t)$ at a moment $t > 0$ satisfy the following constraints [1]:

$$\int_0^{T_K} p_s(t)dt \le E_s. \tag{2}$$

$$\int_0^{T_k} r_s(t)dt \ge \sum_{m=1}^{k} B_{sm}, \quad k = 1,..,K \tag{3}$$

$$\sum_{s \in S'} r_s(t) \le \log\left(1 + \sum_{s \in S'} p_s(t)\right), \quad \forall S' \subseteq S \tag{4}$$

Our goal is to find the Pareto optimal frontier with respect to the vector of the residual battery powers $W = (W_1,..,W_S)$ over all $r_s(t) \ge 0,\ p_s(t) \ge 0,\ s = 1,..,S,\ t \in [0, T_k]$ subject to constraints (1)-(4). It is known [6] that this Pareto optimization problem is equivalent to maximization of the weighted sum of the residual battery powers $W_1,..,W_S$, $W_{ave} = \sum_s \alpha_s W_s$ with all possible weights α_s : $\sum_s \alpha_s = 1,\ \alpha_s \ge 0$.

Apparently no reasonable schedule can satisfy all of the constraints (4) above as strict inequalities and each active, i.e., energy expending, source at all times is bound by at least one equality. Sets for which the corresponding (4) constraint holds as an equality are important since they determine the impact of small changes in power on the information transfer rates. We prove that at any time the sets for which (4) holds as equality are linearly ordered by inclusion and that the set of all active sources satisfies (4) as equality. A somewhat inexact but nevertheless informative analog in the discrete world would be slot-renting in such a way that slot k, k=1,2,.. is rented for 2^{k-1} currency units. Whenever a set of sources has rented M slots, the joint expense must be no less than 2^M-1 currency units with equality iff the sources in question rented the first M slots.

3. Main Results

Consider class of schedules $\{r_{sk}, p_{sk}\}$ with constant transmission and power rates for each source $s = 1,..,S$ in each of the intervals $[T_{k-1}, T_k]$, $k = 1,..,K$: $r_s(t) = r_{sk} \ge 0$, $p_s(t) = p_{sk} \ge 0$, where it is assumed $T_{k-1} = 0$. For this class of schedules conditions (1)-(4) take respectively the following forms:

$$W_k = E_s - \sum_{k=1}^{K}(T_k - T_{k-1})p_{sk} \qquad (5)$$

$$\sum_{k=0}^{K-1}(T_k - T_{k-1})p_{sk} \le E_s, \; s \in S \qquad (6)$$

$$\sum_{m=1}^{k}(T_m - T_{m-1})r_{sm} \ge \sum_{m=1}^{k} B_{sm}, \; k = 1,..,K, \; s \in S \qquad (7)$$

$$\sum_{s \in S'} r_{sk} \le \log\left(1 + \sum_{s \in S'} p_{sk}\right), \; k = 1,..,K, \; S' \subseteq S \qquad (8)$$

We prove that the Pareto optimal frontier with respect to the vector of the residual battery powers $W = (W_1,..,W_S)$ $W = (W_1,..,W_S)$ over all possible schedules $r_s(t) \ge 0, \; p_s(t) \ge 0, \; s = 1,..,S, \; t \in [0, T_k]$ satisfying (1)-(4) is attainable by using schedules $\{r_{sk}, p_{sk}\}$ satisfying (5)-(8). This result allows us to reduce finding the optimal schedules resulting in the corresponding Pareto frontier to $2|S|K$-dimensional convex (but not necessarily strictly convex) optimization problem. This is a direct consequence of the convexity of the logarithm function [7].

The study of the different schedules that may produce a Pareto-optimal vector $(W_s : s \in S)$ leads us to consider two concepts: (a) The (S',k) pairs for which (8) holds as an equality while $p_{sk}>0$ for all s in S'; in which case we will call S' *exact* for the k-th interval, and (b) Schedules which satisfy as few as possible of the restrictions (7) and (8) as equalities; such schedules will be called *minimally constrained.*

We prove that: (a) For each interval $[T_{k-1}, T_k]$, the exact sets are linearly ordered with respect to inclusion. (b) For every optimal schedule and every interval $[T_{k-1}, T_k]$, the set of all sources expending energy in $[T_k, T_{k+1}]$ is exact. (c) If a schedule is optimal, then the total power is a non-increasing function of time. (d) If the total power in interval $[T_{k-1}, T_k]$ exceeds the total power in $[T_k, T_{k+1}]$, then every source active in the first interval is either satisfies constraint (7) as equality or is a member of an exact set for which the total power does not exceed the total power in $[T_k, T_{k+1}]$. (e) If two minimally constrained schedules lead to the same Pareto optimum vector, then they satisfy the same instances of (7) as equality, have the same exact sets for every interval, and over each exact set S' for $[T_{k-1}, T_k]$ the $[T_{k-1}, T_k]$-aggregate-power-expenditure of the sources in S' is schedule independent.

Indeed (a) and (b) are reformulations of the structural results. Statement (c) implies that if aggregate power in the k-th exceeds the aggregate power in the j-th, j<k, then a better schedule can be devised if any source that is bound through a single exact set in the k-th interval transfers energy to the j-th interval. Condition (d) states that if the schedule is optimal the aggregate power in the j-th interval exceeds the aggregate power in the k-th, j<k, then no source s can profitably transfer power from j-th to the k-th interval either because corresponding inequality of type (7) will be violated or because the drop in information transfer rate in interval j (determined by the aggregate power the smallest exact set containing s) exceeds the gains in interval k. Finally, (e) is based on the simple fact that when two schedules are available, a third feasible schedule can be constructed by averaging the transfer rates and the power consumption rates. Among the constraints of type (7) only those, which hold as equalities for both of the original schedules, will hold as equalities for the third. And among the constraints of type (4) only those which hold as equalities for both of the original schedules and for which the aggregate power is the same for both schedules will hold as equalities for the third.

Some interesting results can be proven if each source has a single meaningful deadline to meet, i.e., for each s, all B_{sk} but one are zero. For this particular case we can prove that each minimally constrained schedule can induce an equivalence relationship $\sim$ and a partial ordering $\prec$ in S such that (a) if sources α and β are both active in $[T_{k-1}, T_k]$ then $\alpha \sim \beta$ if and only if they belong to the same exact sets for $[T_{k-1}, T_k]$; (b) if sources α and β are both active in $[T_{k-1}, T_k]$ then $\alpha \prec \beta$ if and only if the smallest exact set for $[T_{k-1}, T_k]$ that contains α does not contain β. (c) If sources α,β,γ, and δ satisfy $\alpha \sim \beta$, $\beta \prec \gamma$, and $\gamma \sim \delta$, then $\alpha \prec \delta$.

In addition, given a vector $(W_s, T_k, E_s, B_{sk} : s \in S, \; k = 1,..,K)$, we develop an algorithm, which can determine if this vector is feasible, i.e., constraints (5)-(8) can be satisfied with some schedule $\{r_{sk}, p_{sk}\}$, and if so produces such a schedule. The algorithm has low complexity and easy to implement in real-time. Due to limited space we do not describe the algorithm formally. Intuitively, given the power expenditures by sources, the algorithm assumes the "most altruistic" rate assignment by each source. In terms of the slot-analogy, the algorithm would have source #k "buy" the most expensive available slots in the first k intervals which are compatible with its information transfer requirements and energy budget. In our case of course the problem is continuous and the algorithm will be modified as follows.

A "bottom x" water filling algorithm for source #k is a water filling algorithm for the first k intervals that acts as it whatever is available below level x is impermeable, and if feasible schedules exist, one –not necessarily an optimal one- can be found by determining for each k a level x=x[k] so that given the preceding power allocations, a bottom-x water filling algorithm will satisfy the information transfer requirements of source #k while using all available energy for said source.

4 EXAMPLES

For lack of space we only present Figures 1 and 2, respectively the (B_1, B_2) and (T_1, T_2) feasibility regions of a two source, two task system and note that the graphical interpretation of the model constraints provides a better guide to the qualitative aspects.

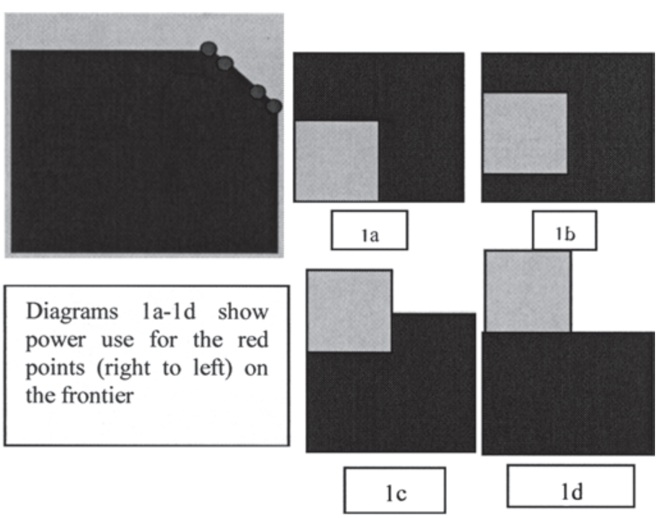

Figure 1. (B_1, B_2) feasibility region

5 CONCLUSION

This paper has proposed a model for on-line transmission scheduling in a wireless multi-access system and presented some initial analysis results for this model. The goal of resource allocation is to maximize the weighted sum of the residual battery energies for different sources, given delay deadlines and current battery energy levels. Current efforts include (a) developing a model for adjusting the corresponding weights to reflect expectations on the future request arrivals, and (b) decentralized solution to the corresponding optimization

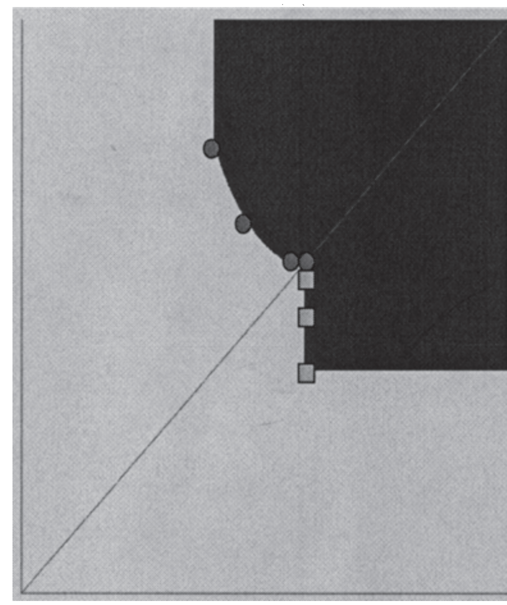

Figure 2. (T_1, T_2) feasibility region

problem, based on Lagrange multiplies associated with the constraints.

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