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J. D. Splett[1] *and C. M. Wang*[1]

Uncertainty in Reference Values for the Charpy V-notch Verification Program

ABSTRACT: We present a method for computing the combined standard uncertainty for reference values used in the Charpy machine verification program administered by the National Institute of Standards and Technology. The technique is compliant with the ISO Guide to the Expression of Uncertainty in Measurement and models the between-machine bias using a Type B distribution. We demonstrate the method using actual data from the Charpy machine verification program.

KEYWORDS: Charpy V-notch, impact certification program, impact testing, ISO GUM, notched-bar testing, reference specimens, uncertainty

Introduction

For the past 13 years, the National Institute of Standards and Technology (NIST) has administered a program to ensure the measurement integrity of Charpy V-notch machines across the nation [1]. A brief description of the program follows. NIST obtains a verification set of 75 impact specimens from a manufacturer and measures the impact toughness of each specimen on one of three "master" Charpy machines. Impact toughness is measured as energy in joules absorbed by the specimen during the test. If the verification set meets certain criteria, then the remaining specimens in the production lot will be machined. A sample of 15 specimens from the production lot is then tested on a single master machine to determine if the production lot is in agreement with the verification set. Once the production lot has been accepted, NIST assigns a reference value to the lot and sells sets of five specimens to companies who wish to certify their own Charpy machine. The program is administered within the guidelines of ASTM E 23-02 [2].

Several other Charpy machine verification programs exist throughout the world; however, they differ widely from the NIST program [3]. Since there are no international standard practices for verifying Charpy machines, it is important to develop some common ground for comparison. There is some interest in conducting a long-term interlaboratory comparison of Charpy machines using a master batch of specimens. To facilitate this comparison, a measure of the uncertainty in the computed reference value is needed. While other Charpy programs already utilize the uncertainty of the reference value, ASTM E 23 does not provide guidelines for computing this quantity.

We propose a method for estimating the combined standard uncertainty in the computed reference value for the NIST Charpy machine verification program and demonstrate the method using actual data from the verification program. The method provides an uncertainty estimate that is compliant with NIST [4] and ISO Guide to the Expression of Uncertainty in Measurement (GUM) [5] guidelines.

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[1]Statistical Engineering Division, National Institute of Standards and Technology, Boulder, CO 80305.

Reference Value and Uncertainty

The reference value is based on verification set data from three master machines and is defined as

$$\bar{X}_{\text{Ref}} = \frac{1}{3}\sum_{i=1}^{3} \bar{x}_i \tag{1}$$

where $\bar{x}_i$ represents the average absorbed energy observed for the 25 specimens tested on the ith master machine.

The combined standard uncertainty of the reference value can be determined by combining three components of standard uncertainty: within-machine standard uncertainty ($u(w)$), standard uncertainty due to machine bias ($u(b)$), and the standard uncertainty of specimen homogeneity ($u(h)$). The combined standard uncertainty (u_c) is

$$u_c = \sqrt{u^2(w) + u^2(b) + u^2(h)} \tag{2}$$

The degrees of freedom associated with each of the three components of uncertainty ($\upsilon_w, \upsilon_b, \upsilon_h$) can be combined to obtain the effective degrees of freedom using the Welch-Satterthwaite formula [5]

$$\upsilon_{\text{eff}} = \frac{u_c^4}{\dfrac{u^4(w)}{\upsilon_w} + \dfrac{u^4(b)}{\upsilon_b} + \dfrac{u^4(h)}{\upsilon_h}} \tag{3}$$

The effective degrees of freedom υ_{eff} associated with the total uncertainty u_c are used to determine the appropriate coverage factor for confidence intervals.

Within-Machine Standard Uncertainty

After establishing that the individual machine variances are equal using Bartlett's test [6], we compute the within-machine standard uncertainty $u(w)$ using the "pooled" standard deviation S_P

$$S_P = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2+(n_3-1)s_3^2}{n_1+n_2+n_3-3}} \tag{4}$$

where n_1, n_2, n_3 are the number of verification specimens tested on each of the three master machines and s_1, s_2, s_3 are the associated standard deviations. (The case in which individual machine variances are unequal is rare and will not be considered in this paper.) Typically 25 verification specimens are tested on each machine, so the within-machine standard uncertainty is

$$u(w) = \frac{S_P}{\sqrt{75}} \tag{5}$$

The divisor in Eq 5 is determined by the number of observations used to compute the pooled variance estimator; usually 75 observations. The degrees of freedom υ_w associated with $u(w)$ are $(25+25+25-3)=72$. Since Charpy testing is destructive, within-machine uncertainty and specimen inhomogeneity cannot be separated, so $u(w)$ contains both within-machine and specimen inhomogeneity.

Standard Uncertainty Due to Machine Bias

The standard uncertainty due to machine bias accounts for possible bias in the observed averages associated with each master machine. The value of $u(b)$ can be quantified using a technique that models the unknown biases with a Type B uncertainty distribution. (See Levenson et al. [7] for details regarding the technique.) Using observed data for 75 verification specimens (25 specimens tested on each of the three master machines), a rectangular distribution bounded by the extremes of the averages of the three master machines is used to model the machine biases. (In the absence of distributional information, a rectangular distribution is commonly used because it provides a more conservative uncertainty estimate than a normal distribution. See Ref. [6] for more information.) Thus, $u(b)$ is

$$u(b) = \frac{|\bar{x}_{\min} - \bar{x}_{\max}|}{2\sqrt{3}} \tag{6}$$

with degrees of freedom

$$\upsilon_b = \left(\frac{1}{2}\right)\frac{(\bar{x}_{\min} - \bar{x}_{\max})^2}{u^2(\bar{x}_{\min}) + u^2(\bar{x}_{\max})} \tag{7}$$

The quantity $\bar{x}_{\min}$ corresponds to the smallest average among the three master machines and $u(\bar{x}_{\min})$ is the associated uncertainty of $\bar{x}_{\min}$. The largest average among the three master machines and the associated uncertainty are denoted by $\bar{x}_{\max}$ and $u(\bar{x}_{\max})$, respectively. (Note that $u(\bar{x}_{\min})$ and $u(\bar{x}_{\max})$ do not correspond to the smallest and largest uncertainties among the three machines. Rather, they are the uncertainties that correspond to the minimum and maximum averages.)

Standard Uncertainty Due to Specimen Inhomogeneity

The final component of standard uncertainty $u(h)$ can be thought of as a correction for specimen inhomogeneity and is based on test results for 25 verification specimens broken on a single master machine and the results for 15 production lot specimens tested on the same master machine. Let μ_0, σ_0 be the unknown true mean and standard deviation of absorbed energy (by a master machine) for the specimens in the verification lot, and μ_1, σ_1 the corresponding parameters in the production lot. Let $\bar{X}_0$, S_0, $\bar{X}_1$, and S_1 be the sample estimates for μ_0, σ_0, μ_1, and σ_1, respectively. We want to make inferences about μ_1 of the production lot based on the sample estimates $\bar{X}_0, S_0$ of the verification lot. That is, we want to find a standard uncertainty S such that

$$P[\bar{X}_0 - 2S \leqslant \mu_1 \leqslant \bar{X}_0 + 2S] \approx 0.95$$

or

$$P[\mu_1 - 2S \leqslant \bar{X}_0 \leqslant \mu_1 + 2S] \approx 0.95$$

or

$$P\left[\frac{\mu_1-\mu_0}{\sigma_0/\sqrt{25}} - \frac{2S}{\sigma_0/\sqrt{25}} \leqslant \frac{\bar{X}_0-\mu_0}{\sigma_0/\sqrt{25}} \leqslant \frac{\mu_1-\mu_0}{\sigma_0/\sqrt{25}} + \frac{2S}{\sigma_0/\sqrt{25}}\right] \approx 0.95$$

Assuming the distribution of $(\bar{X}_0 - \mu_0)/\sigma_0/\sqrt{25}$ is approximately standard normal, then

$$P\left[-2 \leqslant \frac{\bar{X}_0-\mu_0}{\sigma_0/\sqrt{25}} \leqslant 2\right] \approx 0.95$$

If we know the true values of μ_0, μ_1, and σ_0, one can show that an appropriate choice for S is

$$S = \frac{\sigma_0}{5}\left(1 + \frac{|\mu_1 - \mu_0|}{2\sigma_0/5}\right)$$

Thus, the uncertainty $\sigma_0/5$ is inflated by a factor of $1+|\mu_1 - \mu_0|/2\sigma_0/5$.

Substituting the sample estimates $\bar{X}_0$, $\bar{X}_1$, and $u(\bar{X}_0)$ from the verification lot data for the true values, the inflation factor becomes $1+|\bar{X}_1 - \bar{X}_0|/2u(\bar{X}_0)$. Once the inflation factor is estimated, we can use the uncertainty information from all the verification lot data to obtain $u(h)$. That is,

$$\sqrt{u^2(w)+u^2(b)}\left(1 + \frac{|\bar{X}_1 - \bar{X}_0|}{2u(\bar{X}_0)}\right) = \sqrt{u^2(w)+u^2(b)+u^2(h)}$$

Thus, if the production lot is accepted, $u(h)$ is calculated using

$$u(h) = \sqrt{(u^2(w)+u^2(b))\left[\left(1 + \frac{|\bar{X}_1 - \bar{X}_0|}{2u(\bar{X}_0)}\right)^2 - 1\right]} \tag{8}$$

where $\bar{X}_1$ is the average absorbed energy of the 15 production lot specimens tested using a master machine, and $\bar{X}_0$ is the average absorbed energy of the 25 verification specimens tested on the same master machine. A conservative estimate of the degrees of freedom associated with S_h is $\upsilon_h = 15-1=14$. The magnitude of the uncertainty due to specimen inhomogeneity depends on how closely the production lot agrees with the pilot lot. A large discrepancy will result in a larger uncertainty than a small discrepancy.

Example

The quantitative measurement results for an actual verification set are shown in Table 1. Figure 1 displays box plots of the verification

TABLE 1—*Verification set measurement results.*

Machine #1	Machine #2	Machine #3
$n_1=25$	$n_2=25$	$n_3=25$
$\bar{x}_1=219.782$ J	$\bar{x}_2=226.761$ J	$\bar{x}_3=226.408$ J
$s_1=7.488$ J	$s_2=6.077$ J	$s_3=6.669$ J
$u(\bar{x}_1)=1.498$ J	$u(\bar{x}_2)=1.215$ J	$u(\bar{x}_3)=1.334$ J

set measurements for each master machine. Each box plot shows the minimum, 25th percentile, median, 75th percentile, and maximum impact energy observed for each machine.

The reference value (see Eq 1) associated with the verification set shown in Table 1 is 224.317 J. To compute the combined standard uncertainty associated with the reference value, we must first compute $u(w)$. Bartlett's equality of variance hypothesis was not rejected ($p=0.3$), so a pooled standard deviation is appropriate. Compute S_P using Eq 4 and $u(w)$ from Eq 5 (with $\upsilon_w=72$ degrees of freedom).

$$S_P = \sqrt{\frac{s_1^2+s_2^2+s_3^2}{3}} = \sqrt{\frac{(7.488)^2+(6.077)^2+(6.669)^2}{3}} = 6.769 \text{ J}$$

$$u(w) = \frac{S_P}{\sqrt{75}} = \frac{6.769}{\sqrt{75}} = 0.782 \text{ J}$$

Next, Eqs 6 and 7 are used to compute $u(b)$ and υ_b, respectively, where $\bar{x}_{\min}=219.782$ J, $\bar{x}_{\max}=226.761$ J, $u(\bar{x}_{\min})=1.498$ J, and $u(\bar{x}_{\max})=1.215$ J.

$$u(b) = \frac{|\bar{x}_{\min}-\bar{x}_{\max}|}{2\sqrt{3}} = \frac{|219.782-226.761|}{2\sqrt{3}} = 2.015 \text{ J}$$

$$\upsilon_b = \left(\frac{1}{2}\right)\frac{(\bar{x}_{\min}-\bar{x}_{\max})^2}{u^2(\bar{x}_{\min})+u^2(\bar{x}_{\max})} = \left(\frac{1}{2}\right)\frac{(219.782-226.761)^2}{(1.498)^2+(1.215)^2} = 6.546$$

and υ_b is rounded down to 6.

If 15 production lot specimens are tested on Machine #3 with $\bar{x}_{\text{Production}}=223.738$ J, then from Eq 8, $u(h)$ (with $\upsilon_h=14$) is

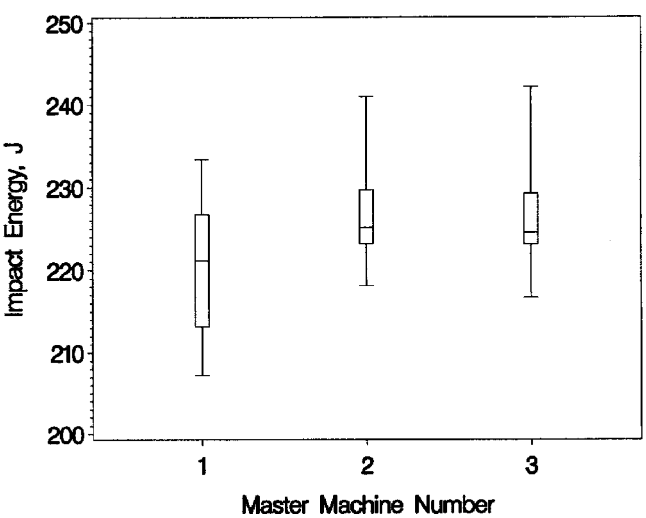

FIG. 1—*Distribution of verification set measurements for each master machine.*

$$u(h) = \sqrt{(u^2(w)+u^2(b))\left[\left(1+\frac{|\bar{x}_{\text{Production}}-\bar{x}_3|}{2u(\bar{x}_3)}\right)^2-1\right]}$$

$$= \sqrt{((0.782)^2+(2.015)^2)\left[\left(1+\frac{|223.738-226.408|}{2(1.334)}\right)^2-1\right]}$$

$$= 3.745 \text{ J}.$$

The combined standard uncertainty (see Eq 2) associated with the reference value is

$$u_c = \sqrt{u^2(w)+u^2(b)+u^2(h)} = \sqrt{(0.782)^2+(2.015)^2+(3.745)^2}$$

$$= 4.324 \text{ J}$$

The degrees of freedom, calculated using Eq 3, are

$$\upsilon_{\text{eff}} = \frac{u_c^4}{\frac{u^4(w)}{\upsilon_w}+\frac{u^4(b)}{\upsilon_b}+\frac{u^4(h)}{\upsilon_h}} = \frac{(4.324)^4}{\frac{(0.782)^4}{72}+\frac{(2.015)^4}{6}+\frac{(3.745)^4}{14}}$$

$$= 20.805$$

which rounds down to 20.

Thus, the expanded uncertainty, corresponding to a 95% confidence interval on the true reference value, is $t_{0.025,20}(4.324) = 2.086(4.324) = 9.020$ J.

Conclusions

We have presented a method for computing the combined standard uncertainty of a reference value for the Charpy machine verification program and have demonstrated the application of the method for actual data. The method is compliant with ISO GUM and NIST uncertainty guidelines.

By developing a procedure for computing the standard uncertainty of a reference value, we hope to provide a means for improving the limits used to certify customer Charpy machines. The certification limits currently in use are somewhat arbitrary and do not account for uncertainty in the reference value. The development of an uncertainty for a reference value has no practical effect on the verification program at this time.

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