On Shortest Random Walks under Adversarial Uncertainty*

Vladimir Marbukh
National Institute of Standards and Technology
100 Bureau Drive, Stop 8920, Gaithersburg,
MD 20899-8920, USA
E-mail: marbukh@nist.gov

Abstract

Finding shortest feasible paths in a weighted graph has numerous applications including admission and routing in communication networks. This paper discusses a game theoretic framework intended to incorporate a concept of path stability into the process of shortest path selection. Route stability is an important issue in a wire-line and especially in wireless network due to node mobility as well as limited node reliability and power supply. The framework assumes that the link weights are selected within certain "confidence intervals" by an adversary or set of adversaries. The width of the confidence interval for the path weight represents the path stability. One of the immediate benefits of this framework is justification for randomized routing interpreted as a mixed Nash equilibrium strategy in the corresponding game. To demonstrate a wide range of possible applications of the proposed framework the paper briefly discusses possible application to robust traffic engineering.

1 Introduction

Consider a weighted graph $\Gamma^N(w)$ with N nodes $n \in \{1,2,..,N\}$ and a vector of link $l=(n,k)$ weights $w=(w_l)$, where $w_l > 0$, $k \in \{1,2,..,N\} \setminus n$. Link l exists if $w_l < \infty$, and does not exist if $w_l = \infty$. Define the weight of a route r to be the sum of the weights of the links comprising the route:

$$w_r = \sum_{l \in r} w_l \tag{1.1}$$

A shortest path based admission and routing algorithm for a request with origin $i \in \{1,2,..,N\}$ and destination $j \in \{1,2,..,N\} \setminus i$ admits the request on the shortest feasible route

$$r^*_{i \to j} = \arg\min_{r \in F_{ij}} w_r \tag{1.2}$$

if the corresponding minimum weight (length) does not exceed certain threshold c associated with this request:

$$w^*_{i \to j} = \min_{r \in F_{ij}} w_r \le c, \tag{1.3}$$

and rejects the request otherwise. Here F_{ij} is the set of feasible routes for the request. This shortest path based admission and routing can be expressed as follows:

$$r^{opt}_{i \to j} = \begin{cases} r^*_{i \to j} & if\ w^*_{i \to j} \le c \\ \varnothing & if\ w^*_{i \to j} > c \end{cases} \tag{1.4}$$

where $r = \varnothing$ means that the request is rejected. The shortest feasible routes (1.2) and their lengths (1.3) can be obtained by solving equations [1]-[2]

$$w^*_{i \to j} = \min_{m \ne i} \{w_{im} + w^*_{m \to j}\} \tag{1.5}$$

* This work was supported in part by DARPA Network Modeling and Simulation (NMS) program.

In the networking context shortest path based strategies (1.4) naturally arise as a result of optimization of the network performance [3] or incorporating Quality of Service (*QoS*) requirements into admission and routing processes [4]. Admission of a request brings certain revenue c to the network, but also ties up the occupied resources until the service is completed and, consequently, may cause future revenue losses due to insufficient resources for servicing some future requests. The implied cost w_r of the resources on a route r reflects these potential revenue losses, and the surplus value

$$u(w,r|c)=\begin{cases} c-w_r & if\ r\neq\varnothing \\ 0 & if\ r=\varnothing \end{cases} \tag{1.6}$$

is the difference between the revenue brought by the admitted request and the implied costs of the occupied resources. In a case of *QoS* routing the weight of a link represents the level of *QoS* on the link e.g., available bandwidth, delay, packet loss, etc., the request attribute c characterizes the minimum level of *QoS* acceptable for the request. The shortest path based routing may also represent a step in the iterative process of adaptive load balancing [1].

In all these cases due to aggregation, delays in disseminating signaling information, non-steady external conditions, etc., the link weights may not be known exactly. In a situation of adaptive load balancing, the route stability becomes an issue in a case of equal length multi-path (route flapping instability) [5]. In a mobile network due to node mobility as well as limited node reliability and power supply, the link weights may be a subject to significant uncertainty. Proposed in [4] approach to accounting for uncertainty in the link weights is based on the assumption that the link weights are random variables with some fixed probability distributions. This approach lies within the Bayesian framework for making decisions under uncertainty, and results in finding shortest paths with respect to the corresponding average lengths. In many practical situations, however, the Bayesian approach is not sufficient. The distributions of the link weights usually depend on some parameters, e.g., moments, estimated from the available data. These parameters can be more reliably quantified in terms of the "confidence regions" than point estimates, leaving the problem of uncertainty unresolved. Usually route instability indicates that allowing randomization of the routing decisions may improve the routing algorithm performance. For example, randomization, leading to the optimal traffic split, may significantly improve the network load balancing capabilities [1]. Based on the average link weights Bayesian approach, however, is not able to address these issues.

This paper discusses a game-theoretic framework for shortest path based admission and routing, intended to guard against the worst-case scenario with respect to the uncertain link weights. An immediate benefit of the game theoretic perspective is justification and quantification of the randomized routing interpreted as a mixed Nash equilibrium strategy in the corresponding game. Stabilization and optimization properties of randomized routing naturally follow from the corresponding properties of Nash equilibrium. The paper is organized as follows. Section 2 quantifies losses (regrets) resulted from admission/rejection and routing decisions under uncertain link weights. Averaging these losses over fixed probability distributions yields the corresponding Bayesian risks. Minimization of the Bayesian risk results in the shortest path based strategy under the average case scenario with respect to uncertain link weights. Section 3 describes a game theoretic framework for shortest path based admission and routing intended to guard against the worst-case scenario with respect to uncertain link weights. Assuming that the distribution of the link weights depends on some unknown parameters controlled by the adversarial environment, the framework assumes that the admission and routing protocols attempt to minimize the corresponding risks while the environment attempts to maximize these risks. Section 4 demonstrates the load balancing and routing stabilization capabilities of the "Nash equilibrium" routing on an example of robust traffic engineering under uncertain demands.

2 Admission and Routing under Average Case Scenario

The regrets (losses) due to non-optimal admission and routing decisions can be quantified by the following regret (loss) function [6]:

$$L(w,r|c) = u^{opt}(w|c) - u(w,r|c) \tag{2.1}$$

where the utility of the optimal admission and routing decisions is

$$u^{opt}(w|c) = u(w,r^{opt}|c) \tag{2.2}$$

and r^{opt} is determined by (1.4). Combining (1.4), (1.6), and (2.1)-(2.2) we obtain:

$$L(w,r|c) = \begin{cases} \max\{0, c-w^*\} - (c-w_r) & if\ r \neq \varnothing \\ \max\{0, c-w^*)\} & if\ r = \varnothing \end{cases} \tag{2.3}$$

where w^* is determined by (1.3). Given c,

$$L(w,r|c) \geq L(w,r^{opt}|c) = 0,\ \forall w \in W,\ \forall r \in \{\varnothing, F\}. \tag{2.4}$$

Since a decision to admit a request on some feasible route $r \in F$ exposes the network to potential admission and routing losses $L^{adm}(c)$ and $L^{rtn}(w,r)$, and a decision to reject a request $r = \varnothing$ exposes the network to potential rejection loss $L^{rej}(c)$ it is natural to expressed total loss (2.1) as follows:

$$L(w,r) = \begin{cases} L^{adm}(c) + L^{rtn}(w,r) & if\ r \neq \varnothing \\ L^{rej}(c) & if\ r = \varnothing \end{cases} \tag{2.5}$$

Assuming that the optimal routing decision (1.2) does not carry any risk, i.e., $L^{rtn}(w,r^*) \equiv 0$, the admission, rejection and routing risks are uniquely identified as follows:

$$L^{adm}(w|c) = -\min\{0, c-w_*\}, \tag{2.6}$$

$$L^{rej}(w|c) = \max\{0, c-w_*\}, \tag{2.7}$$

$$L^{rtn}(w,r) = w_r - w_*. \tag{2.8}$$

Using identity $L^{rtn}(w,r) = L^{rej}(w|c) - L^{adm}(w|c) + w_r - c$, total loss (2.5) can be rewritten as follows:

$$L(w,r|c) = \begin{cases} L^{rej}(w|c) + w_r - c & if\ r \neq \varnothing \\ L^{rej}(w|c) & if\ r = \varnothing \end{cases} \tag{2.9}$$

Note the following properties of the loss functions (2.5)-(2.8). First,

$$\left(L^{adm}(w|c) = 0\right) \wedge \left(L^{rej}(w|c) > 0\right) \text{ if } w^* < c, \tag{2.10}$$

$$\left(L^{adm}(w|c) > 0\right) \wedge \left(L^{rej}(w|c) = 0\right) \text{ if } w^* > c. \tag{2.11}$$

Second, the admission loss $L^{adm}(w|c)$ monotonously decreases and the rejection loss $L^{rej}(w|c)$ monotonously increases with increase of c. Third, the admission loss $L^{adm}(w|c)$ increases, and the rejection loss $L^{rej}(w|c)$ decreases if route weights w_r increase for all $r \in F$. Fourth, the routing loss $L^{rtn}(w,r)$ stays constant if all route weights change by the same amount a: $L^{rtn}(w,r) = L^{rtn}(w+a,r)$.

Bayesian approach to shortest path routing under uncertainty [4] assumes that the vector of link weights $w = (w_r : r \in F)$ is a random variable with some known probability distribution $Q(w)$. Averaging (2.5) over distribution $Q(w)$ we obtain the risks or average regrets (losses) resulted from non-optimal admission and routing decisions due to incomplete information on the link weights:

$$R(r|c) = \begin{cases} R^{adm}(c) + R^{rtn}(r|c) & if\ r \neq \varnothing \\ R^{rej}(c) & if\ r = \varnothing \end{cases} \tag{2.12}$$

where the admission risk is $R^{adm}(c) = Q(w_* > c)E[w_* - c|w_* > c]$, the rejection risk is $R^{rej}(c) = Q(w_* \le c)E[c - w_*|w_* \le c]$, and the routing risk is $R^{rtn}(r) = E[w_r - w_*]$. Properties of the risk functions can be derived from the properties of the corresponding loss functions. In particular, we have from (2.9)

$$R(r|c) = \begin{cases} R^{rej}(c) + \overline{w}_r - c & if\ r \ne \varnothing \\ R^{rej}(c) & if\ r = \varnothing \end{cases} \tag{2.13}$$

where $\overline{w}_r = E_Q[w_r]$. Risk minimization

$$r_{ij}^{opt} = \arg \min_{r \in \{\varnothing, F\}} R(r|c) \tag{2.14}$$

yields

$$r_{ij}^{opt} = \begin{cases} r_{ij}^* & if\ \overline{w}_{i\to j}^* \le c \\ \varnothing & if\ \overline{w}_{i\to j}^* > c \end{cases} \tag{2.15}$$

where

$$r_{ij}^* = \arg \min_{r \in F_{ij}} \overline{w}_r \tag{2.16}$$

$$\overline{w}_{i\to j}^* = \min_{r \in F_{ij}} \overline{w}_r, \tag{2.17}$$

The shortest routes (2.16) and their lengths (2.17) can be obtained by solving equations

$$\overline{w}_{i\to j}^* = \min_{m \ne i} \{\overline{w}_{im} + \overline{w}_{m\to j}^*\} \tag{2.18}$$

3 Admission and Routing under Worst Case Scenario

In applications the distribution of the link weights $Q(w) = Q(w|\theta)$ and thus risks depend on some unknown parameters, e.g., moments, $\theta \in \Theta$, where Θ is the corresponding "confidence region". It is natural to define the admission and routing intended to guard against the worst-case scenario with respect to $\theta \in \Theta$ as follows.

Principle 1: Admission and Routing. S*election* $r \in \{\varnothing, F\}$ *attempts to minimize the total risk* $R(\theta, r|c)$*, while selection* $\theta \in \Theta$ *attempts to maximize the total risk* $R(\theta, r|c)$.

The deterministic worst-case scenario parameters $\theta^{wst} \in \Theta$ and the optimal admission and routing decisions $r^{opt} \in \{\varnothing, F\}$ guarding against this worst-case scenario are determined by solution to the following optimization problem:

$$R(\theta^{wst}, r^{opt}|c) = \min_{r\in\{\varnothing,F\}} \max_{\theta\in\Theta} R(\theta, r|c) \tag{3.1}$$

A game theoretic interpretation of the Principle 1 provides the methodological and computational apparatus of the game theory for developing robust admission and routing algorithms under uncertainty. In particular, randomized strategies may be interpreted as mixed Nash equilibrium solutions in the corresponding game. A mixed admission and routing strategy is determined by probability distribution $p = (p_r)$ on $r \in \{\varnothing, F\}$. If $p_\varnothing < 1$ and the request is admitted, the feasible route $r \in F$ should selected with probabilities $\pi = (\pi_r)$, where

$$\pi_r = \frac{p_r}{1 - p_\varnothing} \tag{3.2}$$

Note that different game theoretic interpretations leading to different solutions $p = (p_r)$ are possible. In a case of connection oriented network, when the entire route for the flow (session) is selected at the set-up stage, an appropriate game theoretic model is a single stage game with the admission and routing protocols represented by a single player attempting to minimize $R(\theta, r|c)$ by selecting a feasible strategy $r \in \{\varnothing, F\}$. The adversarial environment, however, may be represented by a single player or by multiple players depending on possible coordination in the adversarial selections of the weights for different links [7].

In a connectionless network routing decisions at each node are based on the local routing table. Since the packet travel time is typically much shorter than the period between the routing table updates, a sequence of nodes traversed by a packet in a connectionless network is in effect a Markov process. However, distribution (3.2) does not necessarily describe a Markov process. Also, maintaining routing tables with entries containing a wide range of parameters c is technologically unfeasible. It is natural to define a robust connectionless routing intended to guard against the worst-case scenario with respect to $\theta \in \Theta$ as follows.

Principle 2a: Routing. S*election* $r \in F$ *attempts to minimize the routing risk* $R^{rtn}(\theta, r)$*, while selection* $\theta \in \Theta$ *attempts to maximize the routing risk* $R^{rtn}(\theta, r)$.

The deterministic worst-case scenario parameters $\theta^{wst} \in \Theta$ and the optimal route $r^* \in F$ guarding against this worst-case scenario are determined by solution to the following optimization problem:

$$R^{rtn}(\theta^{wst}, r^*) = \min_{r \in F} \max_{\theta \in \Theta} R^{rtn}(\theta, r) \tag{3.3}$$

In a case of connection-oriented network an appropriate game theoretic interpretation of the Principle 2a is, again, a single stage game with the routing protocols represented by a single player attempting to minimize $R^{rtn}(\theta, r|c)$ by selecting a feasible strategy $r \in F$. However, in a case of connectionless network an appropriate game theoretic interpretation of the Principle 2a is a multi-stage game, where the routing algorithm selects the next hop and the adversarial environment selects link weights based on the current node, but without knowledge of the next hop selection by the routing algorithm. In both, connection-oriented as well as connectionless, cases the adversarial environment may be represented by a single or by multiple players depending on possible coordination in the adversarial selections of the weights for different links [7]. Due to space constraints we only consider a case of a single stage game theoretic interpretation of the Principle 2a. It is easy to verify the following statement for a Nash equilibrium routing strategies in these games.

Route Optimality Principle: *Only feasible routes of minimum expected length can be selected with non-zero probabilities.*

Let $\overline{w}^{opt}$ be this minimum expected length, and $\widehat{w}^* = \overline{w}_{r^*}(\theta^{wst})$ be the length of the route r^* determined by (3.3). The following difference represents reduction in the shortest path length resulted from allowing randomized route selection:

$$\Delta = \widehat{w}^* - \overline{w}^{opt} \tag{3.4}$$

Given $\overline{w}^{opt}$, the natural admission strategy follows from (2.13).

Principle 2b: Admission. *Admission probability is*

$$p_{\neg\varnothing} \overset{def}{=} 1 - p_{\varnothing} = \begin{cases} 1 & if\ \overline{w}^{opt} < c \\ 0 & if\ \overline{w}^{opt} > c \end{cases} \tag{3.5}$$

Our conjecture is that under "reasonable" assumptions on the type of uncertainty Principle 1 and Principles 2a-b result in the same optimal admission and routing strategies.

4 Case Study: Connection Oriented Network with Parallel Structure

Consider a particular case of connection-oriented network with a set of K feasible paths $r \in \{r_1,..,r_K\} = F$ without overlapping links. Due to space constraints we only consider a case when the adversary is capable of synchronizing the link lengths selection for all links. In this case the confidence interval for the path $r_k \in F$ is

$$w_k \in [\breve{w}_k, \widehat{w}_k] \tag{4.1}$$

where $\breve{w}_k = \sum \breve{w}_l$, $\widehat{w}_k = \sum \widehat{w}_l$, $l \in r_k$. An optimal adversarial response to routing decision $r_i \in F$ is

$$w_k^{wst}(i) = \begin{cases} \widehat{w}_k & if\ k = i \\ \breve{w}_k & if\ k \neq i \end{cases} \quad (4.2)$$

The optimal deterministic admission and routing strategies, given by (3.1), are

$$r^{opt} = \begin{cases} r^* & if\ \widehat{w}^* \le c \\ \varnothing & if\ \widehat{w}^* > c \end{cases} \quad (4.3)$$

where the optimal deterministic shortest path r^*, given by (3.3), and its length $\widehat{w}^*$ are determined by graph $\Gamma^N(\widehat{w})$:

$$\widehat{w}^* = \widehat{w}_{r^*} = \min_{k=1,..,K} \widehat{w}_k \quad (4.4)$$

The corresponding losses can be easily evaluated. It is easy to see that all adversarial strategies (4.1) are dominated by strategies

$$w_l = (1-\delta_l)\breve{w}_l + \delta_l \widehat{w}_l \quad (4.5)$$

where binary vector $\delta = (\delta_l) \in \{0,1\}^K$. Thus the corresponding game model is equivalent to the following finite, single stage, zero sum, two-player game $B(\breve{w}, \widehat{w}, F)$. In the game $B(\breve{w}, \widehat{w}, F)$ one player (r) represents the admission and routing algorithm, and attempt to minimize the total binary loss

$$S(\delta, r|c) = L(w, r|c)\Big|_{w_k = (1-\delta_k)\breve{w}_k + \delta_k \widehat{w}_k} \quad (4.6)$$

by selecting a feasible strategy $r \in \{\varnothing, F\}$, and another player (δ) represents the adversary, and attempts to maximize the total binary loss (4.6) by selecting a binary vector $\delta = (\delta_1,..,\delta_K) \in \{0,1\}^K$. Note that so far described in this section results after some minor adjustments are also valid for an arbitrary topology network.

In a case of a single feasible route $K = 1$, the optimal rejection probability is [6]: $p_\varnothing = 1$ if $c \le \breve{w}$, $p_\varnothing = (\widehat{w} - c)/(\widehat{w} - \breve{w})$ if $\breve{w} < c \le \widehat{w}$, and $p_\varnothing = 0$ if $c > \widehat{w}$. Further in this section we consider a case when number of feasible routes $K \ge 2$. In a homogeneous case

$$w_k \in [\breve{w}, \widehat{w}] \quad (4.7)$$

the optimal rejection probability is [7]

$$p_\varnothing = \begin{cases} 1 & if \quad c \le w^* \\ 0 & if \quad c > w^* \end{cases} \quad (4.8)$$

where

$$w^* = \frac{\breve{w} + (K-1)\widehat{w}}{K} \quad (4.9)$$

Assuming $c > w^*$, the optimal routing discipline selects route $r_k \in F$ with probability $p_k = 1/K$. The optimal adversarial strategy [7] selects route $r_k \in F$ with probability $p_k = 1/K$, then assigns $w_k = \breve{w}$ and $w_i = \widehat{w}$, $\forall i \neq k$. Thus, assuming $c > w^*$, the random route selection reduces the length of the route comparatively to deterministic route selection by

$$\Delta = \widehat{w}_* - w^* = \frac{\widehat{w} - \breve{w}}{K} \quad (4.10)$$

Consider a general case (4.1). Define the set of "acceptable" routes $F^* \subseteq F$ to be routes, which can be minimum length, subject to constraints (4.1). It is obvious that routes $r \in F \setminus F^*$ can be excluded from consideration, and thus we assume that all routes $r \in F$ are acceptable. It can be shown that the optimal admission strategy is given by (4.8) where

$$w^* = \left[\sum_k \prod_{i \neq k} (\widehat{w}_i - \breve{w}_i)\right]^{-1} \left[\prod_i (\widehat{w}_i - \breve{w}_i) + \sum_k \breve{w}_k \prod_{i \neq k} (\widehat{w}_i - \breve{w}_i)\right] \tag{4.11}$$

Assuming $c > w^*$, the optimal probabilities π_k of route $r_k \in F$ selection can be found from the system of linear algebraic equations. It can be shown that in a particular case of $K = 2$ feasible routes these probabilities are

$$\pi_1 = \frac{\widehat{w}_2 - \breve{w}_1}{\widehat{w}_1 - \breve{w}_1 + \widehat{w}_2 - \breve{w}_2} \tag{4.12}$$

and $\pi_2 = 1 - \pi_1$, assuming that $c > w^*$, where the length of the "shortest random walk" is

$$w^* = \frac{\widehat{w}_1 \widehat{w}_2 - \breve{w}_1 \breve{w}_2}{\widehat{w}_1 - \breve{w}_1 + \widehat{w}_2 - \breve{w}_2} \tag{4.13}$$

The gain (3.4) from allowing randomized routing is

$$\Delta = \min\{\widehat{w}_1, \widehat{w}_2\} - \frac{\widehat{w}_1 \widehat{w}_2 - \breve{w}_1 \breve{w}_2}{\widehat{w}_1 - \breve{w}_1 + \widehat{w}_2 - \breve{w}_2} \tag{4.14}$$

5 Application to Robust Traffic Engineering

Consider a network with performance characterized by the following penalty function:

$$H = \sum_l h_l(\lambda_l) \tag{5.1}$$

where the total flow carried on a link l is

$$\lambda_l = \sum_{r:l \in r} x_r \le c_l, \ \forall l, \tag{5.2}$$

the flow carried on a route r is x_r, the capacity of link l is c_l, and function $h_l(\lambda_l)$ characterizes penalty associated with carrying average load λ_l on link l. Traffic flows $x = (x_r)$ satisfy the following conservation conditions:

$$\sum_{r \in F_{ij}} x_r = \mu_{ij}, \ x_r \ge 0, \ \forall r \in F_{ij} \ \ \forall (i, j), \tag{5.3}$$

where the set of feasible routes with origin-destination (i, j) is F_{ij}, and the matrix of external demands is $\mu = (\mu_{ij})$. Given $\mu = (\mu_{ij})$, the optimal vector of traffic flows $x = x^*$ minimizes the total penalty (1):

$$\min_x H \tag{5.4}$$

subject to constraints (5.1)-(5.3) [1]. We assume that functions h_l are monotonously increasing and convex:

$$d_l(\lambda) = dh_l(\lambda)/d\lambda > 0, \tag{5.5}$$

$$d_l'(\lambda) = d^2 h_l(\lambda)/d\lambda^2 > 0, \tag{5.6}$$

$\forall \lambda \in [0, \infty)$, and problem (5.1)-(5.4) has at least one feasible solution. These assumptions imply that the optimization problem (5.1)-(5.4) has unique optimal solution x^*, no other locally optimal solution exists, and solution x^* can be characterized in terms of the link costs (5.5) as follows [1]. A set of path flows is optimal if and only if flows are positive only on paths of minimum cost, where the cost of a path is a sum of the costs of the links comprising this path:

$$d_r = \sum_{l \in r} d_l(\lambda_l) \tag{5.7}$$

This characterization also implies that at the optimum, the paths along which the input flow is split must have equal to each other costs and equal to the minimum cost of all feasible paths with the same origin-destination (equal cost multi-path [5]).

Characterization of the optimal routing in terms of the link cost has important implications for assigning link weights in Open Shortest Path First (*OSPF*) routing protocol [5]. It is natural to assign *OSPF* link weights w_l to be equal to the link costs (5.5):

$$w_l = d_l(\lambda_l) \tag{5.8}$$

Link weight assignment (5.8) can be used for adaptive *OSPF* implementation, with link loads λ_l estimated from the measurements. If the demand matrix $\mu = (\mu_{ij})$ is fixed and known, off-line implementation of *OSPF* can be based on pre computed "optimal" link weights $w_l^* = d_l(\lambda_l^*)$, where the average link l loads $\lambda_i^* = \sum_{r:l\in r} x_r^*$ and the optimal traffic assignment x^* is given by the solution to the optimization problem (5.1)-(5.4).

In many practical situations available information on the demand matrix $\mu = (\mu_{ij})$ can be more reliably quantified in term of the "confidence region" $\mu \in M$ rather than point estimate $\mu \approx \tilde{\mu}$. Following [8] we approximately assume that

$$M = \left\{ \sum_{s=1}^{S} \gamma_s \mu^s \middle| \sum_{s=1}^{S} \gamma_s = 1, \gamma_s \geq 0 \right\}. \tag{5.9}$$

We will refer to μ^s as scenarios, and interpret polyhedron (5.9) as a mixture of these scenarios with weights $\gamma = (\gamma_s)$. In a situation of uncertain external demands $\mu \in M$ a routing protocol is not capable of controlling the flows $x = (x_r)$, but hopefully capable of controlling the fractions of the offered load μ to be carries on feasible routes $\xi = (\xi_r)$, where

$$\xi_r = x_r / \mu_{ij} \text{ , } r \in F_{ij} \tag{5.10}$$

Consider penalty (5.1) as a function of the fractions ξ and the external demands μ: $H = H(\xi|\mu)$. The loss (regret) resulted from optimization of the routing algorithm for scenario $s = j$ while the actual scenario is $s = i$ can be characterized by

$$\Phi_{ij} = H(\xi^j | \mu^i) - H(\xi^i | \mu^i), \tag{5.11}$$

$i, j = 1,..,S$, where fractions $\xi^s = (\xi_r^s)$, $\xi_r^s = x_r^s / \mu_{ij}$, $r \in F_{ij}$ are optimized for scenario $s = 1,..,S$. Consider a two player, zero sum game of the routing algorithm attempting to minimize loss (5.11) by selecting $j = 1,..,S$, and adversarial environment attempting to maximize loss (5.11) by selecting $i = 1,..,S$. Let α_j be the optimal, generally mixed, strategy for the routing algorithm. It is natural to interpret the weighted sum

$$\xi = \sum_j \alpha_j \xi^j \tag{5.12}$$

as robust load allocation scheme guarding against the worst case mixture $\sum_j \beta_j \mu^j$ of scenarios $\mu^i, i = 1,..,S$, where β_i is the optimal, generally mixed, strategy for the environment in this game.

Allocation (5.12) requires ability to arbitrarily split traffic among feasible routes. In practice it can be achieved in *MPLS* network by randomization of the routing decisions at the packet level. However, it is often desirable to allocate a single route for the entire flow. According to the routing optimality principle, the load should be carried on a minimum cost routes. The following game theoretic interpretation G provides natural extension of this optimality principle to a situation of uncertain external demands. Consider a non-cooperative game G of all origin-destination pairs (i, j) and the adversarial environment. Each pair (i,j) attempts to minimize the excessive, relatively to the minimum, cost of the route

$$V_{ij} = \left[\sum_{l \in r} d_l \left(\sum_{r':l' \in r'} x_{r'} \right) - \min_{r'' \in R_{ij}} \sum_{l \in r''} d_l \left(\sum_{r':l' \in r'} x_{r'} \right) \right] \mu_{ij} \tag{5.13}$$

by selecting a feasible strategy $r \in F_{ij}$. The adversarial environment attempts to maximize the aggregate excessive cost

$$U_\Sigma = \sum_{(i,j)} V_{ij} \tag{5.14}$$

by selecting a feasible strategy $\mu \in M$. Note that a mixed routing strategy in this and following games can be interpreted as traffic split at the flow as well as packet level.

Game G can serve as the starting point for developing off-line as well as on-line decentralized, robust traffic engineering schemes. Given fractions (5.10), uncertainty in the expected demands $\mu \in M$ induce uncertainty in the link weights $w_l \in [\breve{w}_l, \widehat{w}_l]$, where

$$\breve{w}_l = \min_{\mu \in M} d_l \left(\sum_{(i,j)} \mu_{ij} \sum_{r:l \in r \subseteq F_{ij}} \xi_r \right), \tag{5.15}$$

$$\widehat{w}_l = \max_{\mu \in M} d_l \left(\sum_{(i,j)} \mu_{ij} \sum_{r:l \in r \subseteq F_{ij}} \xi_r \right), \tag{5.16}$$

and function $d_l(\lambda)$ is given by (5.5).

Assume that vectors $\breve{w} = (\breve{w}_l)$ and $\widehat{w} = (\widehat{w}_l)$ are fixed, and consider a problem of robust selection of the shortest feasible path $r \in F_{ij}$ in a weighted graph $\Gamma^N(\breve{w}, \widehat{w})$ with uncertain link weights $w_l \in [\breve{w}_l, \widehat{w}_l]$. It is natural to formalize this problem as a game g_{ij} of the routing algorithm attempting to minimize the excessive, relatively to the minimum, route cost

$$L_{ij}(w,r) = \sum_{l \in r} w_l - \min_{r' \in F_{ij}} \sum_{l \in r'} w_l \tag{5.17}$$

by selecting a feasible strategy $r \in F_{ij}$, and the adversarial environment attempting to maximize cost (18) by selecting a feasible strategy (15).

In a case of off-line routing the target fractions (5.10) in (5.15)-(5.16) are determined off line, for example, using one of the procedures described in the previous section of the paper. In a case of on-line, adaptive routing bounds (5.10) are based on the real-time measurements. Given bounds $\breve{w} = (\breve{w}_l)$ and $\widehat{w} = (\widehat{w}_l)$, the robust traffic split for origin-destination (i,j) is determined by the optimal, generally mixed, routing strategy in the game g_{ij}. Note that this optimal routing solution is based on two metrics per link $\breve{w}_l$ and $\widehat{w}_l$, and thus can be implemented with *OSPF-OMP* routing protocol [9].

In conclusion, briefly discuss stability of the routing resulted from solutions to the games g_{ij}. If the optimal solution to the load allocation problem (5.1)-(5.4) does not split traffic, this optimal solution can be implemented with *OSPF* routing protocol based on the corresponding "optimal" link weights. If, however, the optimal solution splits traffic among feasible routes, a situation of equal cost multi-path occurs due to the route optimality principle. This situation is typical for moderately and heavily loaded networks with multiple feasible routes since the minimum cost routing increases load on the minimum cost route until the admission strategy takes over or a situation of equal cost multi-path occurs. It is usually assumed that *OSPF* splits traffic equally among minimum cost feasible routes. Routing instability (route flapping) due to abrupt changes in the load allocation resulted from small changes in the link weights in a situation of equal cost multi-path presents a serious problem for adaptive *OSPF*. From the game theoretic perspective the route flapping instability can be viewed as an attempt of the minimum cost routing algorithm to solve the corresponding game in pure strategies or strategies describing equal split among some routes. The game theoretic framework provides a natural guiding principle for regularization of the otherwise ill-conditioned problem of route cost minimization in a situation of equal cost multi-path [7], [10]. We currently investigate a problem of global stability of the *OSPF-OMP* routing protocol splitting traffic according to the optimal mixed routing strategy $p_r(\breve{w}, \widehat{w})$ in games g_{ij} for a case when information on the current fractions ξ and feasible demands M is available to the routers. In this case performance of the corresponding *OSPF-OMP* routing protocol can be described by equations (5.15)-(5.16) supplemented with equations describing the optimal routing strategies $p_r(\breve{w}, \widehat{w})$ in games the g_{ij}:

$$\xi_r = p_r(\breve{w}, \widehat{w}) \tag{5.18}$$

Other directions of future research include relation between centralized and decentralized game schemes, as well as developing computationally feasible algorithms for solving corresponding games. Solutions for some particular cases have been obtained in [7], [11].

References

1. D. Bertsekas and R Gallager, Data Networks, Prentice-Hall, New Jersey, 1992.
2. D. Bertsekas and J. Tsitsiklis, Neuro-Dynamic Programming, Athena Scientific, Belmont, Massachusetts, 1996.
3. F.P. Kelly, "Routing in Circuit-Switched Networks: Optimization, Shadow Prices and Decentralization," *Adv. Appl. Prob*., 20 (1988) 112-144.
4. R.A. Guerin and A. Orda, "QoS Routing in Networks with Inaccurate Information: Theory and Algorithms," *IEEE/ACM Trans. on Networking*, 7 (1999) 350-364.
5. C. Huitema, Routing in the Internet, Prentice Hall, 2000.
6. V. Marbukh, "Minimum Regret Approach to Network Management under Uncertainty with Applications to Connection Admission Control and Routing," *First International Conference on Networking (ICN2001),* 309-318, Colmar, France, 2001.
7. V. Marbukh, “Randomized Routing as a Regularized Solution to the Route Cost Minimization Problem,” *6th World Multiconf. On Systems, Cybernetics and Informatics (SCI2002),* Orlando, Florida, 2002.
8. B. Fortz and M. Thorup, "Optimizing OSPF/IS-IS weights in a changing world," *IEEE JSAC,* Vol. 20, No. 4, May 2002, 519-528.
9. Villamizar, "MPLS Optimized Multi-Path (MPLS-OMP)," Internet draft, February 1999.
10. V. Marbukh, "Minimum Cost Routing under Adversarial Uncertainty: Robustness through Randomization," *Int. Symp. on Information Theory (ISIT2002)*, Lausanne, Switzerland, 2002.
11. V. Marbukh, "QoS Routing under Adversarial Binary Uncertainty," *Int. Conf. on Communications (ICC2002),* New York, 2002.