Noise Properties of Microwave Signals Synthesized with Femtosecond Lasers

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Abstract: The excess noise associated with the process of coherent optical-to-microwave frequency division was measured. This was accomplished by referencing two mode-locked Ti: sapphire lasers to the same stable CW laser and extracting microwave signals at the harmonics of pulse repetition rate. The spectral density of the excess phase noise was found to be close to $-140\,dBc/Hz$ at $100\,Hz$ offset from a $10\,GHz$ carrier.

Key words: *femtosecond lasers, frequency stabilisation, phase-locked loop, interferometry*

1. Introduction

Mode-locked femtosecond lasers are a key element of an optical frequency synthesizer enabling coherent division of optical frequencies to microwave domain [1-3]. If the frequency synthesis process is noise free, one can take advantage of high-Q optical cavities and atomic resonances in order to produce microwave signals of extreme spectral purity. For example, assuming that a $10\,GHz$ signal is coherently derived from a $563\,THz$ cavity stabilised laser with a sub-hertz linewidth [4], its phase noise spectral density would be approximately $-110\,dBc/Hz$ at Fourier frequency $f = 1\,Hz$, which is almost $40\,dB$ better than the phase noise of a high quality quartz oscillator multiplied to $10\,GHz$. Unfortunately, photodetection of the ultra-short light pulses is not a noise-free process. It is accompanied by excess phase noise originating from power fluctuations of a femtosecond laser [5] due to the phenomenon, termed below as a power-to-phase conversion. There are also other noise mechanisms, such as a shot noise, which influence the fidelity of frequency transfer from optical to microwave domain. This work is mainly concerned with the various techniques for high resolution phase noise measurements of microwave signals extracted by demodulation of femtosecond light pulses.

1. Amplitude-to-Phase Conversion in Two-Oscillator Noise Measurement Systems

Extremely sensitive measurement systems, capable of strong discrimination between phase and amplitude fluctuations, are required for accurate characterization of noise properties of microwave signals synthesized from the high precision lasers. Conventional two-oscillator noise measurement systems based on double-balanced mixers (DBM) are not suitable for such applications due to their relatively poor phase sensitivity. The latter is affected by intrinsic voltage fluctuations of the DBM, as well as by the excess phase noise resulting from the amplitude-to-phase (AM-to-PM) conversion in the phase-locked loop (PLL) which references one oscillator to another.

The origin of AM-to-PM conversion in the PLL was linked to the imperfections of real mixers. This was confirmed by modeling the responses of an imperfect mixer (consisting of amplitude detectors with dissimilar conversion efficiencies) to the different types of signal modulation. In particular, we found that the AC signal induced at the mixer output by AM-modulation of the microwave signal does not vanish when mixer DC voltage is set to zero. This means that a closed phase-locked loop based on an imperfectly balanced mixer has no immunity to amplitude fluctuations.
To deal with the issue of AM-to-PM conversion, a few approaches can be suggested. The simplest solution would be to make the PLL mixer operate at the offset from the zero-crossing point which could be achieved by injecting an offset voltage into the PLL, for example, with a sum device introduced between the mixer and loop filter.

2. Properties of Interferometric Two-Oscillator Noise Measurement Systems

The most effective way of eliminating the AM-to-PM conversion in the PLL is related to application of interferometric signal processing. The latter also offers a possibility of oscillator noise measurements with uncertainty approaching standard thermal noise limit [6].
A schematic diagram of the interferometric two-oscillator noise measurement system is shown in Fig. 1. It features a microwave interferometer followed by a low-noise amplifier and non-linear mixing stage. Phase synchronization of a "slave" oscillator results in destructive interference of the microwave signals at the "dark port" of the interferometer. The residual fluctuations of the difference signal (with strongly suppressed carrier) from the "dark port" are amplified and down-converted to DC where their spectral density is measured with the FFT spectrum analyser.

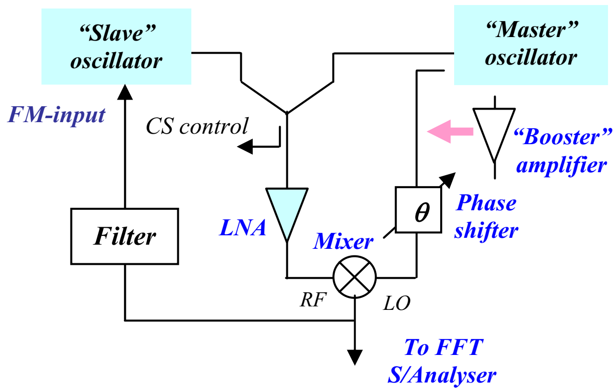

Fig. 1. Two-oscillator noise measurement system with interferometric signal processing.

When dealing with weak microwave signals, like those extracted from high-speed photodetectors, there may not be sufficient power for driving the mixer's LO port. Introducing a "booster" amplifier into the mixer's LO arm solves the above problem with almost no detrimental effect on the sensitivity of spectral measurements (as long as level of carrier suppression in the interferometer is sufficiently high).

If the amplitude of one of the oscillators is modulated, this induces an AC signal at the mixer output with the amplitude proportional to $cos\left(\theta + \Delta\varphi_{21}\right)$, where θ is the reference phase shift and $\Delta\varphi_{21}$ is the phase shift between two signals at the inputs of the microwave interferometer. On the other hand, phase modulation results in AC signal with the amplitude proportional to $sin\left(\theta + \Delta\varphi_{21}\right)$. This means that by varying the reference phase shift θ one can cancel the amplitude sensitivity of the two-oscillator interferometric measurement system while maximizing its phase sensitivity.

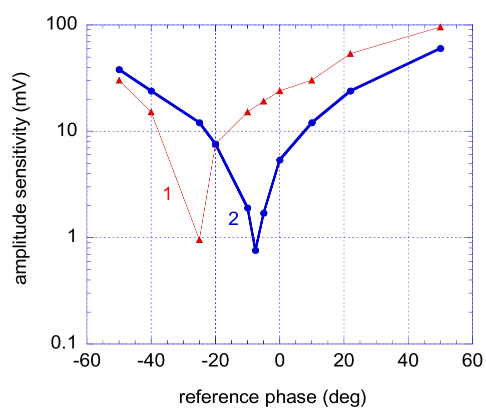

Fig.2. Amplitude sensitivity of a $1GHz$ interferometric two-oscillator noise measurement system at signal power $P_s \approx -12.5\,dBm$: "slave" oscillator amplitude is modulated (curve 1), "master" oscillator amplitude is modulated (curve 2). AM-modulation index is 1%.

Fig. 2 shows the experimentally observed amplitude sensitivities of a $1GHz$ interferometric two-oscillator noise measurement system. Curve 1 corresponds to the case when amplitude of the "slave" oscillator is modulated. Curve 2 shows the measurement system response to AM-modulation of the "master" oscillator. Each response function has a deep minimum, but these minima do not coincide. The phase offset between the minima of two sensitivity functions is partially due to the mixer imperfections and also due to spurious PM-modulation which accompanies the AM-modulation of any frequency synthesizer.

According to [6] the phase sensitivity of the two-oscillator noise measurement system improves as a square root of signal power. To find out how the amplitude sensitivity of the interferometric two-oscillator measurement system varies with power, first, the measurement system was tuned to be AM-insensitive at a given level of power ($P_s = -18\,dBm$). Then, powers of both input signals were varied synchronously to keep the carrier of the difference signal at the "dark port" of the interferometer suppressed. The results of these measurements are summarized in Fig. 3.

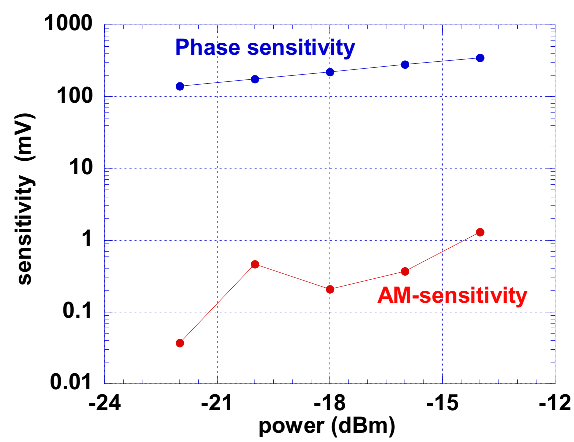

Fig.3. Phase sensitivity in mV/rad and amplitude sensitivity in mV of two-oscillator noise measurement system with interferometric signal processing as a function of signal power. Powers of both input signals vary synchronously.

During these measurements the amplitude sensitivity of the interferometric measurement system does not exceed $1\,mV$ within the range of powers from $-22\,dBm$ to $-14\,dBm$. For comparison, the phase sensitivity (within the same range of signal power) varies from $160\,mV/rad$ to $350\,mV/rad$ which corresponds to the ratio of PM and AM-sensitivities of ~ 500.

The immunity of the interferometric noise measurement system to amplitude fluctuations proved to be much greater than that of a digital PLL. For example, we measured amplitude sensitivity of the digital PLL based on Analog Devices chip AD9901KP operating at $150\,MHz$. The reference signal was derived from a microwave synthesizer followed by a frequency divider by 8. With the amplitude of microwave signal modulated, the highest ratio of PM and AM-sensitivities of the digital PLL was only a factor of 30.

Phase noise floors of various $1\,GHz$ readout systems measured at $P_s = -10\,dBm$ are shown in Fig. 4.

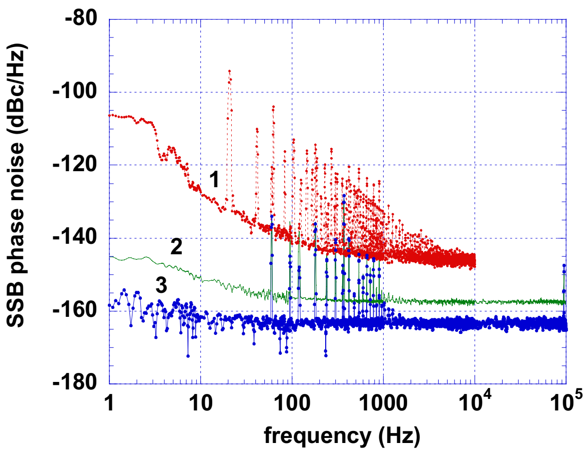

Fig.4. Phase noise floors of simple phase bridge (curve 1); single-channel interferometric noise measurement system (curve 2) and dual-channel interferometric noise measurement system with cross-correlation signal processing (curve 3). Signal frequency is $1\,GHz$, signal power $P_s = -10\,dBm$. The noise floors (curves 2, 3) were measured with the resistive power combiner having $6\,dB$ insertion loss. They could be lowered by another $3\,dB$ by using reactive power splitters.

Curve 1 corresponds to the phase noise floor of a simple phase bridge. Curve 2 shows the phase noise floor of the interferometric noise measurement system. At the first glance it may seem strange, that at low Fourier frequencies the spectral resolution of the interferometric measurement system is still limited by the 1/f-noise, as if carrier was not sufficiently suppressed at the input of the low-noise amplifier. This limitation results from the presence of a strong microwave signal at the mixer LO port (one can attempt to reduce the 1/f-noise limit by making use of a modulation-demodulation technique, similar to that of the Pound frequency discriminator). Curve 3 corresponds to the phase noise floor of a dual-channel two-oscillator noise measurement system with interferometric signal processing. The spectral resolution of the latter system is limited neither by intrinsic fluctuations in the microwave amplifiers nor by the Nyquist thermal noise (at least in theory) [7, 8]. In practice, the lack of isolation between the channels results in much less than expected improvement in the phase noise floor of a dual-channel readout system relative to the single-channel one (typically ~ $10...12\,dB$).

In contrast to a single-channel readout system the dual-channel one could be made immune to amplitude fluctuations of both oscillators. Indeed, each channel of the dual-channel noise measurement system can be tuned to be insensitive to amplitude fluctuations of one particular oscillator. In such a case, amplitude fluctuations detected by the individual channels would be independent and their contribution to the measurement system noise floor reduced, when a cross-spectral density of voltage fluctuations of the two channels is calculated.

3. Phase Fluctuations of Microwave Signals Extracted from Optical Pulse Trains

An optical frequency divider is implemented by phase locking a femtosecond laser to a stable optical source, such as a cavity stabilised laser. This process requires a joint operation of two phase-locked loops stabilising pulse repetition rate f_R and offset frequency of the femtosecond comb f_o (it is assumed that spectrum of a femtosecond laser consists of equidistant spectral lines: $f_n = n\,f_R + f_o$, where n is a large integer).

Signal at frequency f_o (first beat note) is extracted by mixing a frequency-doubled infrared part of the optical comb with the green part [9, 10]. The phase of this beat note is locked to a stable RF synthesizer by controlling the pump power of the femtosecond laser.

Second beat note is derived from the Interaction between the cavity stabilised laser and the closest spectral component of the femtosecond comb. The phase of the second beat note is locked to another stable RF synthesizer by varying the pulse repetition rate of the femtosecond laser with a PZT transducer.

With both frequencies f_R and f_o stabilized, the spectral density of the pulse repetition rate fluctuations can be expressed as

$$S_\varphi^{rep} \approx \frac{S_\varphi^{optical}}{n^2} + \frac{S_\varphi^{offset}}{n^2} + \frac{\tilde{S}_\varphi^{rep}}{(1+\gamma)^2} + S_\varphi^{det} + S_{shot}\,, \qquad (1)$$

where $S_\varphi^{optical}$ is the phase noise spectral density of the cavity stabilised laser, S_φ^{offset} is the spectral density of offset frequency fluctuations (reduced by the gain of the PLL controlling the f_o) and $\tilde{S}_\varphi^{rep}$ is the spectral density of repetition rate fluctuations of a free-running femtosecond laser. Parameter γ is the gain of the PLL stabilizing the phase of the second beat note. The last two terms in (1)

S_φ^{det} and S_{shot} describe, respectively, the phase noise arising form the power-to-phase conversion in a photodetector and laser shot noise.

An estimate for the first term in (1) can be obtained by considering an ultra-stable $532\,THz$ laser developed at NIST [4]. Allan deviation of its fractional frequency fluctuations was measured to be $4 \cdot 10^{-16}$ over $1...10\,s$ of averaging, which translates into: $S_\varphi^{optical}(f) \approx 0.03/f^3\ \left(rad^2/Hz\right)$. The noise-free optical frequency division by $n = 53\ 200$ would result in a $10\,GHz$ signal with the phase noise: $S_\varphi^{optical}/n^2 \approx -109\,dBc/Hz$ at $f = 1\,Hz$. This is at least $40\,dB$ better than the phase noise performance of the state-of-the-art quartz oscillator frequency multiplied to $10\,GHz$.

Having implemented a digital PLL stabilising the offset frequency f_o we measured $S_\varphi^{offset}(1\,Hz) \approx -60\,dBc/Hz$. This amounts to the additional phase noise in the spectrum of $10\,GHz$ signal with power spectral density $S_\varphi^{offset}(1\,Hz)/n^2 \sim -154\,dBc/Hz$.

Third term in (1) describes the repetition rate fluctuations of a free-running femtosecond laser reduced by the gain of the optical beat note PLL. For a broadband Ti: sapphire mode-locked laser operating at pulse repetition rate of $1\,GHz$ we measured $\widetilde{S}_\varphi^{rep} \approx 1/f^4\ \left(rad^2/Hz\right)$ [11]. Further requiring $\widetilde{S}_\varphi^{rep}/(1+\gamma)^2 \le S_\varphi^{optical}/n^2$, one obtains the PLL gain $\gamma \ge 80\,dB$ at $f = 1\,kHz$. Such a high gain may not be easy to achieve due to the low-frequency mechanical resonances of the PZT transducers (approximately $30...50\ kHz$ for present systems).

Power-to-phase conversion in a photodetector gives rise to additional phase noise in the spectrum of the extracted microwave signal with spectral density S_φ^{det}. This noise may severely limit the fidelity of frequency transfer from optical to microwave domain (at low Fourier frequencies), if power of the optical comb is not stabilised.

The spectral density of the shot noise S_{shot} is independent on frequency. For an optical comb with average power $5\,mW$ and microwave signal with power $P_s \approx 0.1\,mW$ it can be calculated that $S_{shot} \approx -154\,dBc/Hz$ [11].

The experimental setup for measuring the excess phase noise associated with the optical frequency division process included two mode-locked Ti: sapphire fs-lasers each referenced to a common frequency stabilized CW laser. Offset frequencies of both fs-lasers were phase stabilised and their pulse repetition rates were exactly matched (by adjusting frequencies of the RF synthesizers in the phase-locked loops). Phase noise spectra of microwave signals extracted at 1st and 10th harmonics of pulse repetition rate were measured with the interferometric readout systems. The results of these measurements are shown in Fig. 5.

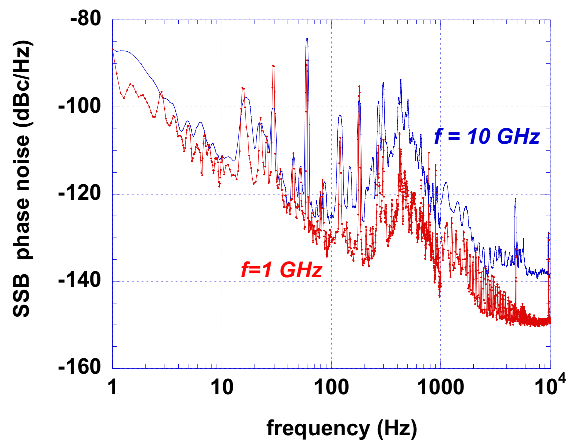

Fig.5. Typical spectra of excess phase fluctuations of microwave signals at 1st and 10th harmonics of pulse repetition rate (during the quiet operation of the optical frequency divider the phase noise $10\,dB$ lower, than typical, was observed at Fourier frequencies below $100\,Hz$)

At Fourier frequencies above a few hundred Hz the two noise spectra are offset by approximately 20dB, as expected from the scaling of the noise intensity associated with the frequency multiplication process. On the other hand, at low Fourier frequencies the vertical offset between the two noise spectra almost vanishes. This effect might be attributed to power-to-phase conversion in the photodetectors [5], in which case, it should be possible to suppress the low frequency excess noise by stabilizing the average power of the optical pulse train or reducing the beam-pointing instabilities of a femtosecond laser [5]. We have not tried these noise reduction techniques.

Data in Fig. 5 correspond to a typical measurement run. The lowest phase noise observed for a $10\,GHz$ signal was $-140\,dBc/Hz$ at $f = 100\,Hz$. This is almost $40\,dB$ better than the phase noise of the state-of-the-art quartz oscillator multiplied to $10\,GHz$.

4. Conclusion

We have shown that:

- imperfections of the real mixers, particularly the imbalance between the detectors constituting the mixer, is responsible for the spurious AM-to-PM conversion in the two-oscillator phase noise measurement systems;

- two-oscillator phase noise measurement system with interferometric signal processing exhibit the lowest sensitivity to oscillator amplitude fluctuations relative to other measurement systems. The AM-insensitive tuning of the interferometric readout system is achieved by adjusting the phase difference between the signals interacting at the mixer. Once achieved, the immunity to oscillator amplitude noise is maintained regardless of variations of signal power, as long as the carrier of the difference signal at the output of the interferometer remains suppressed.

A single-channel two-oscillator noise measurement system can not be tuned to be immune to amplitude fluctuations of both oscillators unless system components are perfectly balanced.
A dual-channel noise measurement system with a cross-correlation signal processing can be made insensitive to amplitude fluctuations of both oscillators

Optical frequency dividers can generate microwave signals with much lower phase-noise close to the carrier than existing microwave sources. More than $40\,dB$ improvement in the phase noise performance relative to the best commercial synthesizers was demonstrated at low Fourier frequencies within the range of $1\,Hz$ to $300\,Hz$ when extracting microwave signals at $10\,GHz$.

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References

1. T. Udem, J. Reichert, R. Holzwarth, and T. W. Hänsch, *"Accurate measurement of large optical frequency differences with a mode-locked laser,"* Optics Letters, vol. 24, pp. 881-883, 1999.

2. S. A. Diddams, D. J. Jones, J. Ye, T. Cundiff, J. L. Hall, J. K. Ranka, R. S. Windeler, R. Holzwarth, T. Udem, and T. W. Hänsch, *"Direct link between microwave and optical frequencies with a 300 THz femtosecond laser comb,"* Phys. Rev. Letters, vol. 84, pp. 5102-5105, 2000.

3. L. Hollberg, C. W. Oates, E. A. Curtis, E. N. Ivanov, S. A. Diddams, T. Udem, H. G. Robinson, J. C. Bergquist, R. J. Rafac, W. M. Itano, R. E. Drullinger, and D. J. Wineland, *"Optical frequency standards and measurements,"* IEEE Journal of Quantum Electronics, vol. 37, pp. 1502-1513, 2001.

4. B. C. Young, F. C. Cruz, W. M. Itano, and J. C. Bergquist, "Visible lasers with subhertz linewidths," *Phys. Rev. Letters*, vol. 82, pp. 3799-3802, 1999.

5. E. N. Ivanov, S. A. Diddams, and L. Hollberg, *"Analysis of Noise Mechanisms Limiting the Frequency Stability of Microwave Signals Generated with a Femtosecond Laser,"* IEEE Journal of Selected Topics in Quantum Electronics, vol. 9, pp. 1059-1065, 2003.

6. E. N. Ivanov, M. E. Tobar and R. A. Woode, Applications of interferometric signal processing to phase noise reduction in microwave oscillators, IEEE Transactions on MTT, vol. 46, N10, October pp.1537-1545, 1998.

7. E. Rubiola, V. Giordano and J. Groslambert, *"Improved Interferometric Method to Measure Near-Carrier AM and PM Noise",* IEEE Trans. on Instrum. and Meas., vol. 48, no. 2, pp. 642-646, April 1999

8. E. N. Ivanov and F. L. Walls, "*Interpreting anomalously low voltage noise in two-channel measurement systems*," IEEE Trans. on UFFC, vol. 49, n. 1, January 2002, pp. 11-19.

9. R. Holzwarth, Th. Udem, and T.W. Hänsch, J. C. Knight, W. J. Wadsworth, and P. St. J. Russell, *" Optical Frequency Synthesizer for Precision Spectroscopy"*, Phys. Rev. Letters, vol. 85, no. 11, pp. 2264-2267, 2000

10. T. M. Ramond, S. A. Diddams, L. Hollberg and A. Bartels *"Phase coherent link from optical to microwave frequencies by means the broadband continuum from a 1 GHz Ti: Sapphire femtosecond oscillator,"* Optics Letters, vol. 27(20), pp. 1842-1844, 2002

11. E. N. Ivanov, L. Hollberg and S. A. Diddams, *"Experimental study of noise properties of a Ti: sapphire femtosecond laser",* IEEE Trans. on UFFC, vol. 50, no. 4, pp. 355 – 361, April, 2003.