EFFICIENT CALIBRATION STRATEGIES FOR LINEAR, TIME INVARIANT SYSTEMS

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ABSTRACT

An efficient strategy for accurately characterizing the frequency response of linear, time invariant systems is presented. The approach, based on circuit modeling, test point selection, and parameter estimation, optimizes calibration confidence with respect to test effort. The analytic tools and methodology needed for designing the strategy are included, together with experimental results. The approach can be particularly beneficial in volume testing of devices such as amplifiers, attenuators and filters, or systems whose frequency response is dominated by such devices.

INTRODUCTION

Linear, time invariant (LTI) systems are completely characterized by either their transfer function, $H(s)$, or their frequency response, $H(j\omega)$. This paper presents the basic tools and methodology needed for designing a calibration strategy for LTI systems which optimizes calibration confidence with respect to test effort. As a result, $H(j\omega)$ can be estimated for all ω within the bandwidth of interest, with known confidence and minimum test effort in terms of number of test points required.

For a large class of electronic measurement instrumentation, the flatness of frequency response is a vital performance measure, and in many instances, frequency response errors are the major limitations of performance. Instruments of this class include ac analog and digital voltmeters, oscilloscopes, waveform recorders, programmable filters, spectrum analyzers, and many other instruments whose useful bandwidth spans many decades of frequency. To a good approximation, instruments of this class, or at least their signal conditioning front ends, can be modeled as LTI circuits.

Unfortunately, accurate frequency response measurements can be both difficult and tedious to perform. In addition, if the instrument is merely treated as a "black box", as is often the case, then measurements must be made at numerous frequencies spanning the bandwidth of interest in order to gain some reasonable degree of confidence in the overall response of the instrument. For example, a few measurements made only at the extremes of the instrument's useful frequency band could entirely miss significant midband errors due to a pole-zero mismatch in a crossover network. Without a more sophisticated calibration strategy, measurement confidence is gained at considerable expense in terms of test time and effort.

To maximize the confidence of frequency response characterization for a given test effort, two related approaches have been developed. Both methods follow the same basic steps. The first step is careful circuit modeling, which results in a functionally complete, analytic description of the system. Once the model has been established, the calibration problem is basically one of parameter estimation. Therefore, step two involves the selection of test points, i.e., input frequencies, which give maximum sensitivity to the model's parameters. Based on this selection of test points, the actual calibration experiment is performed and test data obtained. Finally, the actual parameter values are estimated from the test data by a (linear or nonlinear) least squares solution, from which the actual transfer function is calculated. The first two steps, modeling and test point selection, are computationally the most difficult; however, they are only performed once for a specific design or device type, and the results may be applied to all subsequent devices of the same type. The basic approach and tools for implementation are, in principle, applicable to any LTI system.

MODELING

The model for any LTI system can be represented in the form of a transfer function in the s-plane.

However, such a model will in general, be mathematically nonlinear, and therefore difficult to work with. For this reason, two modeling approaches have been explored. In the first, the actual transfer function is derived from the circuit description with the aid of a symbolic circuit analysis program [1]; in the second, a linear approximation is derived in the form of a component sensitivity matrix using a more conventional circuit analysis program [2]. While the first method results in a more accurate model, the size of the circuits which can be handled is limited to about 20 nodes, and the subsequent steps are computationally more costly.

Either model is developed from a lumped element circuit representation, where elements such as transistors and operational amplifiers have been approximated by appropriate lumped element models. The parameters for these approximations can usually be obtained from the manufacturer's specifications for the device.

Transfer Function Model

The transfer function can be expressed in several forms which are mathematically equivalent. The pole-residue representation is given by

$$R(\omega) = K_0 + \sum_{i=1}^{n} (K_i) / (P_i + \omega), \qquad (1)$$

where ω is the angular frequency and the K's and P's are complex values. The transfer functions that arise from normally realizable electrical components have additional constraints on their form. For instance, any complex poles or zeros will always appear in complex conjugate pairs. Thus, the amplitude of the transfer function, or gain, can be expressed as

$$G(\omega) = \left| R(\omega) \right| = K_0 + \sum_{i=1}^{J} a_i / (P_i + \omega)$$

$$+ \sum_{k=1}^{m} (a_k + b_k\omega) / (c_k + d_k\omega + \omega^2), \qquad (2)$$

where all the parameters are real. The first sum is over the poles on the real axis and the second sum is over the complex conjugate pole pairs.

The transfer function obtained from the circuit description must be checked to ensure that it is not significantly over or under specified. If the model is under specified, then there will likely be features of the true transfer function which the model cannot fit. This is determined by fitting the model to data from candidate circuits. The residuals of the fits should be random and of an amplitude close to the expected measurement error. If not, the model should be reanalyzed and appropriate parameters added to it. If the model is over specified, there may be little or no sensitivity to some of the parameters, or in the case of interdependence, the sensitivity to two or more parameters will be nearly identical. In these cases, the additional parameters can be fixed in value, or two or more can be combined into one variable parameter.

Linear Model

An alternative to generating an analytic expression of the transfer function is to generate a linear approximation, evaluated at a relatively large number of discrete frequencies spanning the frequency range of interest. This can be accomplished with more commonly used nodal analysis programs, either using the adjoint circuit method [3] for calculating component sensitivities, or by directly calculating the changes caused when perturbing the components a small amount. The resulting linear model is given by

$$A = S \times \epsilon, \qquad (3)$$

where A is an n-dimensional vector of transfer function amplitude errors, with n being the number of candidate frequencies at which the transfer function will be estimated, S is an $n \times m$ sensitivity matrix with m the number of independent circuit components in the model, and ϵ is the vector of circuit component errors, i.e., the proportional deviations from the nominal design values.

Independence among the circuit components is assured by applying a QR Decomposition method [4] to the sensitivity matrix. This method, applied to the transpose of S, is described below for test point selection.

TEST POINT SELECTION

Test points should be selected from a candidate set using a criterion which gives maximum sensitivity to the parameters of the model. When using the explicit transfer function model, the optimal design is chosen by calculating and globally minimizing (with respect to test point location) the prediction standard deviation of the frequency response amplitude function. (In actual practice, this calculation is approximated by minimizing the determinant of the covariance matrix of the parameter estimates.) The calculated prediction standard deviations are later used to provide estimates of calibration confidence for all frequencies within the candidate set. This is accomplished by first linearizing equation (2) with a first-order Taylor expansion, and then using a "D-optimality" criterion [5] to design an experiment to estimate the parameters of the linearized model.

For the linear model, test points are selected by performing a QR Decomposition (QRD) on the transpose of the sensitivity matrix. With this approach, S^T is effectively reduced to the product of two square matrices, one orthogonal

(Q) and one right triangular (R). The decomposition process selects and orders the columns of **Q** following a modified Gram-Schmidt orthogonalization, first choosing the column of largest norm, orthogonalizing all remaining columns to it, next choosing the column of largest norm of these remaining, and so on, until the norms of all remaining columns are negligibly small. The frequencies corresponding to those columns of significant norm are the selected test points. The ordering, therefore, selects the test points which are maximally independent and thus robust.

Measurements are made at the test frequencies selected by one of these two processes. To further minimize errors due to measurement noise, or to provide checks for model errors, additional points beyond the minimum required by the rank of the model are usually added.

PARAMETER ESTIMATION

Having taken the measurements at the designated frequencies, it remains to fit the model to the calibration data. For the explicit transfer function model, this becomes a problem of finding the parameters a, b, c, d, K and P, of equation (2). For convenience, the entire set of parameters is denoted by β, and p is the number of parameters to be estimated, i.e., the length of the vector β. Assume that N observations are made and that the observations y_i are subject to random errors. That is,

$$y_i = G(\omega_i, \beta) + e_i, \quad i = 1, \ldots, N \tag{4}$$

where e_i has mean 0 and variance σ^2. The object is to find the parameters β which solve

$$\min_{\beta} \sum_{i=1}^{N} (y_i - G(\omega_i, \beta))^2. \tag{5}$$

This, of course, is a nonlinear least squares problem. The method used to solve the problem is based on Moré's implementation of the Levenberg-Marquardt algorithm (see, e.g., [6]).

For the linear model, the parameters of equation (3), i.e., the component errors ϵ, are found by reapplying the QRD algorithm. In this case, the decomposition is performed on a truncated, reordered matrix, **S'**, whose rows correspond to the chosen test frequencies. Equation (3) then becomes

$$\mathbf{A'} = \mathbf{S'} \times \epsilon, \tag{6}$$

where **A'** is the vector of amplitude errors measured at the selected test frequencies, and ϵ is the vector of independent circuit component errors as before. A least squares estimate for vector ϵ is easily obtained by using one more QRD.

Having estimated the actual parameters of the model based on the test data, it is a simple matter to calculate the frequency response at all other candidate frequencies by substituting the parameter values into the appropriate model, i.e., equation (2) or (3).

APPLICATION AND EXPERIMENTAL RESULTS

The techniques described above have been applied to the calibration of an amplifier/attenuator circuit which comprises the front end of a wide band sampling wattmeter developed at NBS. The circuit to be modeled is shown schematically in figure 1, and the final lumped element model, for one switch position, is given in figure 2. The integrated circuit operational amplifier was modeled based on the manufacturer's parametric description. From this circuit model, two analytic models were derived in the manner

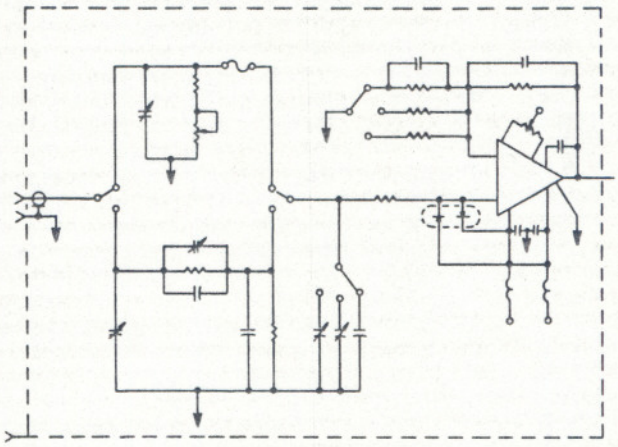

Fig. 1 Schematic diagram of example (amplifier/attenuator) circuit.

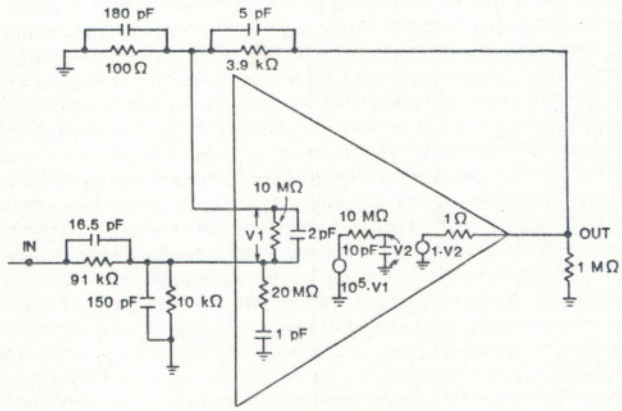

Fig. 2 Lumped element model of example circuit.

discussed above. The resulting explicit transfer function model is adequately described in the frequency range of interest (dc to 1 MHz) with seven parameters: a constant term together with three poles and their residues. For the linear model, the 16 circuit components reduced to 11 independent components, following the decomposition process. The frequency dimension of the sensitivity matrix was chosen to be 41, representing 41 candidate frequencies uniformly distributed in log space from 10 Hz to 1 MHz.

Based on these two models, test points were chosen from the same candidate set of 41 frequencies using the methods previously outlined. The locations of the two sets of test frequencies are given by the dots in figures 3 and 4, respectively. Figure 3 shows the calculated prediction standard deviation versus frequency, based on the first model, using 15 test points. The solid and dashed curves give the prediction standard deviations, respectively, for the optimal design, and for an intuitive design based on uniform log spacing. The prediction standard deviation is normalized to the standard deviation of a single observation. For the optimal design, its maximum value is about 0.9, compared with 1.2 for the intuitive design. The prediction standard deviations based on the 15 test points selected by QRD, are plotted in figure 4. Note that the chosen locations are similar to those shown in figure 3, and the maximum prediction standard deviation is also about the same. The prediction standard deviations can be used to estimate the confidence (or random uncertainty) in predicting the circuit's frequency response at any frequency of interest within the range of candidate frequencies.

In both examples, the test point selection process is based on models linearized at the design center, i.e., with the components set to their nominal values. In practice of course, the measurements will be made on circuits off of design center, so it is important to know the robustness of the models to component errors. This has been investigated for the linearized transfer function model, by changing the design centers by 10 and 50 percent, and recomputing the standard deviations based on the new model, using the previous test frequencies. The resulting prediction standard deviations are within 4 and 13 percent of those shown in figure 3, indicating the experimental design is robust to substantial changes in the components.

For the linear model, component errors can also directly affect parameter estimation in the same way, depending on the second and higher order sensitivities which the model ignores. So long as the component errors are reasonably small, however, e.g., < 5 percent, the resulting calibration uncertainties will be proportionately small in comparison.

comparison, the estimates based on the limited (15 point) data sets are plotted along with the

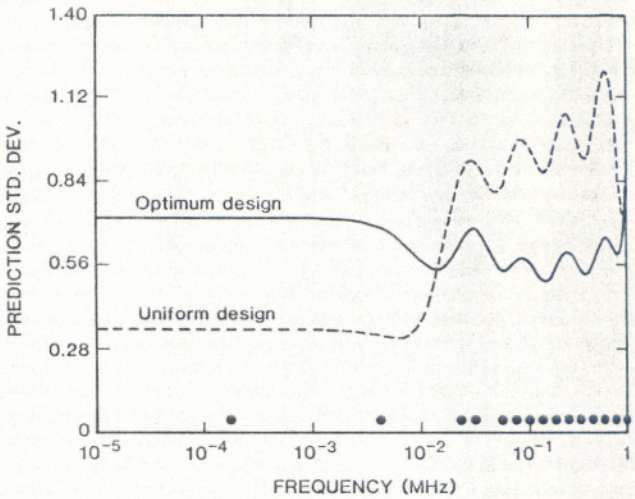

Fig. 3 Prediction standard deviation for D-Optimal design and intuitive design, based on transfer function model.

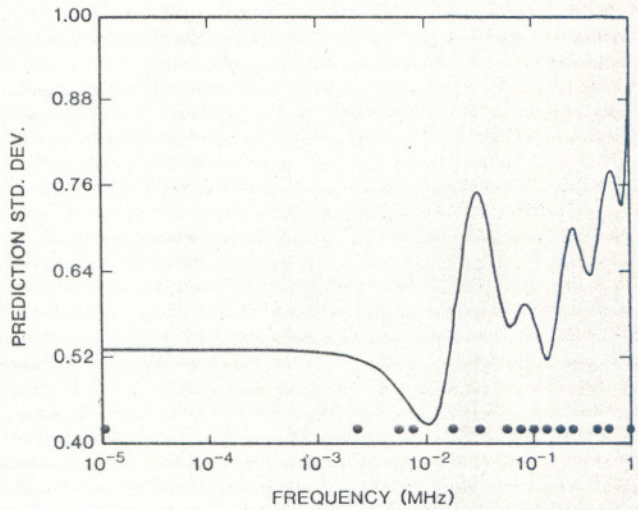

Fig. 4 Prediction standard deviation, based the transfer function model, but using the points selected by QRD.

deviations from the measurement data taken at all 41 candidate frequencies. Note that, in both cases, the differences between the predicted response and the full data set are quite small. The larger differences in figure 5 likely result from using only 7 model components versus 11 for the linear model of figure 6. On the otherhand, freedom.

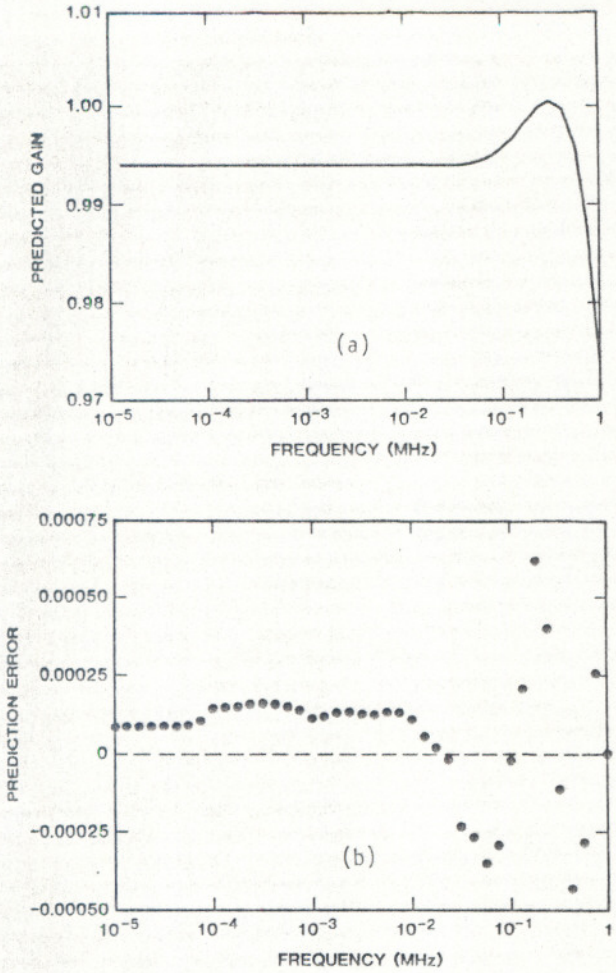

Fig. 5 Estimated frequency response based on transfer function model with measurements made at the 15 D-Optimal design points (a). Deviations of estimated response from full data set (b).

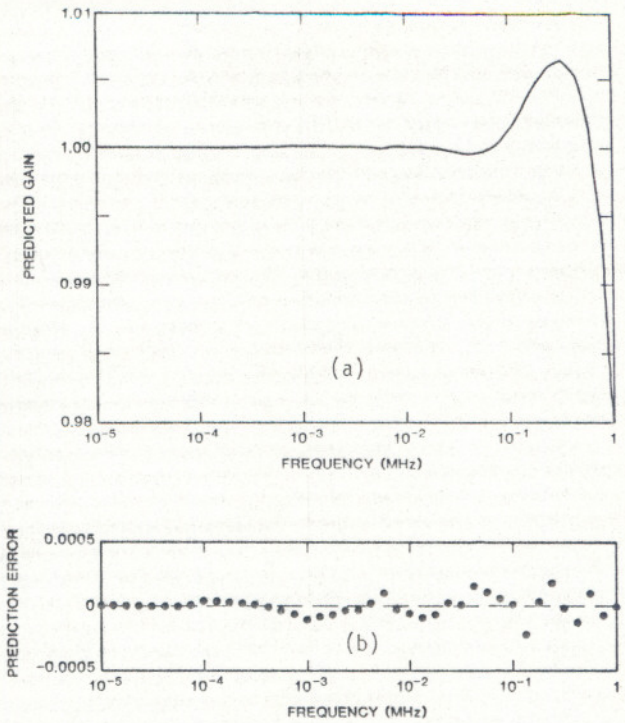

Fig. 6 Estimated frequency response based on linear model and 15 test points selected by QRD (a). Deviations of estimated response from full data set (b).

CONCLUSION

Based on the results plotted in figures 5 and 6, it can be seen that both calibration strategy approaches can lead to accurate estimates of the overall frequency repsonse of an LTI circuit, using limited test data. Furthermore, the methods provide a resonable basis for assigning confidence estimates to the results, and for detecting any serious defects in the model, which show up as evident patterns in the residuals.

The first approach can be expected to yield more accurate estimates for cases where the circuit is substantially off of design center. However, in practice this situation usually means that the device is out of specification, obviating the need for a very accurate calibration. Therefore, for most applications, the linear model approach can be expected to give good results at less computational expense. Furthermore, this approach is more generally adaptable to other testing problems using the same basic software, since the model always has the same form. For example, calibration strategies for switched resistance networks (D/A converter ladder networks in this case) have been developed using the same general methods and software [7]. Optimal code states rather than test frequencies were the calibration variables or test points to be selected. When using the linear model approach for such problems, the only post testing computations to be performed are a simple linear least squares solution to estimate the parameters, and a matrix multiplication to compute the calibration results for all desired input states.

As an area for future work, the application of the testing strategies to the problem of fault diagnosis will be explored. In this case, test point selection will include physical test nodes as testing variables, with the goal of eliminating ambiguity groups amoung the components of the model.

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