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Improved 1 kHz Capacitance Calibration Uncertainty

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***Abstract* – Uncertainties for 1 kHz capacitance calibrations have been decreased at the National Institute of Standards and Technology (NIST). The improvements are based on frequency-dependence characterization from 1592 Hz to 1000 Hz. The relevant measurements and the traceability procedures from the calculable capacitor to the customer calibration are described.**

***Keywords* – calibration, capacitance, farad, fused-silica capacitor, impedance, measurement, uncertainty**

I. INTRODUCTION

For the past few years, customers of the NIST impedance calibration laboratory have been requesting lower uncertainties. This has been driven by the stability of commercial fused-silica capacitors which is at several parts in 10^7, meaning that the capacitors can support measurements at lower uncertainties than are given by our calibration laboratory. The uncertainty of commercial ac bridges used to measure these capacitors is also limited by these uncertainties since the uncertainty of the bridge is linked to measurements of capacitors measured at NIST.

At NIST, the calculable capacitor and associated measurements, which form the basis for the capacitance unit, were developed at a frequency of 1592 Hz [1, 2]. This frequency was chosen because the calculable capacitor was also the basis for the resistance unit, and at that frequency a convenient connection between the capacitance and resistance can be made. The connection between resistance and capacitance is made by matching the impedance of a 1000 pF capacitor to the impedance of a 100 kΩ resistor in an ac bridge. At 1592 Hz, these impedances are equivalent. Both of these values were available as stable precision standards and therefore were a good choice to make the connection.

The capacitance unit is obtained at 1592 Hz from the calculable capacitor and is transferred to the calibration laboratory only at that frequency. The capacitance calibrations, however, are performed only at 100 Hz, 400 Hz, and 1000 Hz, and not at 1592 Hz. The calibration laboratory has used the capacitance value at 1592 Hz for the measurements at the other frequencies. Since the frequency dependence is unknown, a large uncertainty has been assigned to the measurement. The relative uncertainty for the measurement of a 10 pF fused-silica capacitor at 1000 Hz has been 1.5×10^{-6}, which is much larger than the relative uncertainty of the measurement from the calculable capacitor, which is 2×10^{-8}.

Recently, we have been characterizing the frequency dependence of the 10 pF transfer capacitors using the calculable capacitor. The frequency dependence of the 10 pF fused-silica capacitor is now known between 1592 Hz and 1000 Hz. This has allowed us to reduce the calibration laboratory relative uncertainty from 1.5×10^{-6} to 0.5×10^{-6}. This paper describes the measurement of the calculable capacitor at 1000 Hz and all the re-evaluations that were necessary. Similar work will be required at 100 Hz and 400 Hz.

II. MEASUREMENTS

In order to determine the change in frequency between 1592 Hz and 1000 Hz of the 10 pF standards used in the transfer of the unit, we needed a capacitance standard whose frequency change is known between these frequencies. The calculable capacitor is based on a theory by Thompson and Lampard [3] that shows that its capacitance can be found from a single length measurement. The calculable capacitor can be used at frequencies other than 1592 Hz since the theory is frequency-independent. What is required is that we make a measurement with the calculable capacitor at 1000 Hz along with the auxiliary measurements that are usually made with this measurement. All uncertainty evaluations at 1592 Hz that would change with frequency will also have to be re-evaluated. This will allow us to determine the 10 pF transfer capacitor at both 1592 Hz and 1000 Hz in terms of the calculable capacitor.

The chain of measurements that connects the calculable capacitor with the capacitance calibration laboratory is shown in Figure 1. Usually, the calculable capacitor measurement is done twice a year. The capacitor measured against the calculable capacitor is then used to transfer the SI unit of capacitance to a bank of 10 pF capacitors that maintain the unit between calculable capacitor measurements. The unit is then transferred to the calibration laboratory by another standard. These measurements must be repeated at 1000 Hz along with all the auxiliary measurements that are required to obtain an uncertainty of parts in 10^8. We will describe the process required for this re-evaluation. There are three main phases to obtaining the unit: measurement of the 10 pF capacitor with

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the calculable capacitor; measurement of the 10 pF bank and other capacitor with the 10 pF transfer standards; and transfer of the capacitance unit from the 10 pF bank to the calibration laboratory.

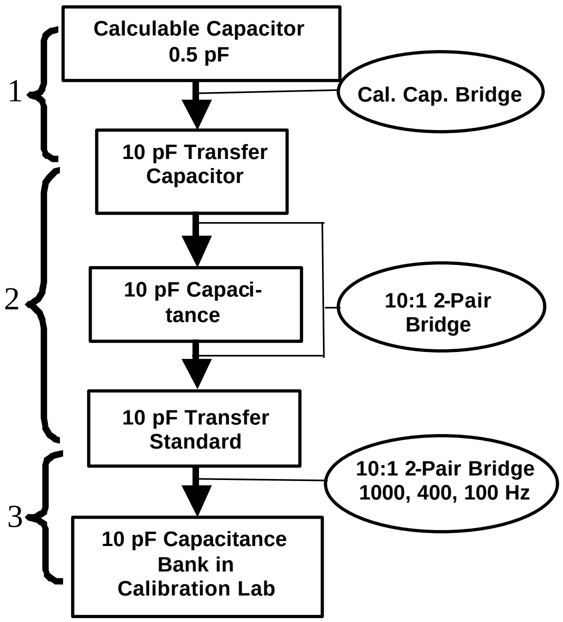

Figure 1. Sequence of measurements from the calculable capacitor to the calibration laboratory. The sequence can be grouped into the following 3 phases, which are identified in the figure: 1. Measurement of the 10 pF capacitor with the calculable capacitor; 2. Transfer of capacitance unit to 10 pF bank; 3. Transfer of capacitance unit from the 10 pF bank to the calibration laboratory.

A. Measurement of the 10 pF capacitor with the calculable capacitor

The calculable capacitor is a special capacitor that links the capacitance unit with the SI unit of length. It is nominally 0.5 pF in value and its value is found from the measurement of the length of its electrodes. The length measurement is done with a Fabry Perot interferometer and at 1592 Hz the uncertainty of the measurement is 2×10^{-8}. A description of the calculable capacitor is given in [4].

The 10 pF transportable fused-silica reference capacitor is compared with the calculable capacitor configured at two different values, 0.2 pF, and 0.7 pF. The difference between the two measurements is used to assign a value to the 10 pF standard. The bridge used in this comparison has a special transformer, which supplies a voltage ratio of 200 V to 14 V and 200 V to 4 V at 1592 Hz. This corresponds to the ratio of 0.7 pF to 10 pF and 0.2 pF to 10 pF. The voltage of 200 V is applied to the calculable capacitor and 14 V or 4 V to the 10 pF capacitor depending on the position of the calculable capacitor. The bridge is balanced by injecting the necessary current at the detector through a second 10 pF capacitor and an adjustable voltage divider. Quadrature adjustment is provided by injection through a suitable conductance and another voltage divider. The bridge is two-terminal-pair, so measurements are necessary to account for the effects of capacitances and inductances in the cables and the calculable capacitor bars.

A few changes were necessary for the measurements at 1000 Hz. The same transformer was used but at a lower voltage of 100 V. At 1000 Hz, the transformer saturates at 200 V. This meant voltage dependence measurements were necessary to ensure that voltage dependence did not enter in to the measurement of the frequency dependence of the calculable capacitor. The rest of the bridge was the same except for the conductance used in the quadrature adjustment.

A special conductance is used whose circuit diagram is shown in Figure 2. The capacitance-to-ground is adjusted so that the phase angle of the resistor is small enough that it does not introduce a real component into the measurement. The phase angle of the conductance is adjusted by comparison with another resistor whose phase angle is known to be sufficiently low. The resistor with the known phase angle is traceable to the straight wire resistor designed by Haddad [5] whose phase angle can be calculated at different frequencies. Since the conductance is only adjusted for 1592 Hz, a new one was built and adjusted at 1000 Hz.

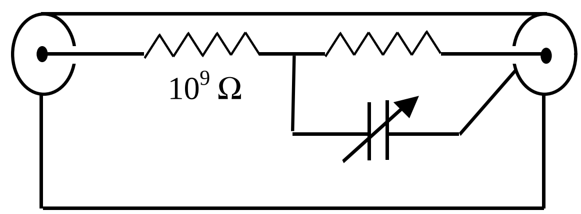

Figure 2. Circuit diagram of conductance used in calculable capacitor bridge

Typically the calculable capacitor measurement must be completed in a week. In order to make sure the transfer capacitor does not change its value between the measurement against the calculable capacitor and against the 10 pF bank, measurement against the bank is done before and after the calculable capacitor measurement, usually on following Mondays. This leaves about 4 days to make the calculable capacitor measurement. Each measurement takes about an hour and a half. The measurement is repeated approximately five times to get a relative standard deviation of 2×10^{-9}. Usually the five measurements can be obtained in one day.

In order to determine the frequency dependence of the 10 pF transfer standard, the measurements at 1000 Hz and 1592 Hz must be done around the same time, which means within the four-day period. The 1000 Hz measurements are noisier because of the lower voltage used, and more data points are typically required. The measurements at 1000 Hz required at least 2 days.

Some difficulty was encountered with the 1000 Hz measurements. Initial measurements in June and August of 1999 had a type-A relative uncertainty of 0.004×10^{-6}. Measurements in August and September 2000 had higher relative uncertainties as large as 0.01×10^{-6}. For the same time difference, the relative difference between the 1000 Hz and 1592 Hz measurements went from 0.08×10^{-6} to anywhere from 0.01×10^{-6} to 0.05×10^{-6}. As can be seen in Figure 3, the scatter of the measurements greatly increased. The 1592 Hz measurements are also shown for comparison. We investigated whether this was due to shifts in value around a certain voltage (as is sometimes observed in fused-silica capacitors in our laboratory) or due to problems with the conductance box. None of these things appeared to be the problem. The 1592 Hz measurements, when made at the same voltage as the 1000 Hz measurements, did not show this scatter. We plan to continue this investigation at a later time. Since we could not determine the cause of the scatter, we decided to use the average of all the data and increase the uncertainty of the measurement to account for it.

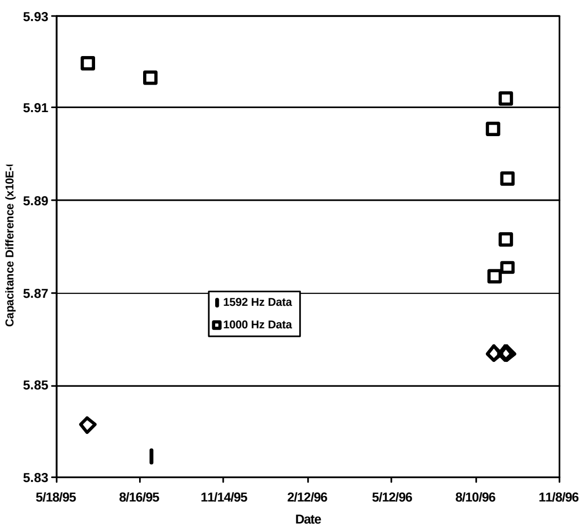

Figure 3. Calculable capacitor measurements of 10 pF transfer capacitor. Vertical axis is difference from nominal value of capacitor C112 ($\times10^{-6}$) with respect to the Calculable Capacitor.

The voltage difference on the 10 pF capacitor between the 200 V and 100 V measurements is very small (14 V and 7 V in the 0.7 pF position and 4 V and 2 V in the 0.2 pF position.) However, a change in the capacitance measurement of the 10 pF standard was observed between 2 V and 4 V. The relative change is 0.01×10^{-6} and is thought to be due to a change in the 10 pF standard. We have studied the voltage dependence of the calculable capacitor and not found any. In addition, the same voltage dependence was observed in measurements made independent of the calculable capacitor, where the 10 pF capacitor was compared with capacitors designed to have very small voltage dependencies. This voltage dependence is removed from the frequency dependence measurements of the transfer capacitor.

B. Auxiliary Measurements

There are several auxiliary measurements required for the calculable capacitor measurement. These include transformer ratio measurements, loading correction, choke corrections, as well as measurements generally associated with any ac bridge [4]. These include linearity of the real and quadrature bridge adjustments (dials) and the phase defect. Many of these measurements are very stable and need to be performed only every few years. For this work, only the transformer ratio and loading corrections at 1000 Hz were measured. The other measurements were not repeated at 1000 Hz at this time since we do not expect them to significantly change between 1000 Hz and 1592 Hz. The corresponding relative uncertainties for these measurements, which are on the order of 0.003×10^{-6}, were increased since they were not repeated at 1000 Hz. It is expected that when we further reduce the uncertainty at 1000 Hz, these uncertainties will be measured.

The transformer ratio was determined by using capacitors with known values in the same ratios used when comparing the calculable capacitor to the 10 pF capacitor. Transformer ratios are generally stable, and for some NIST bridge transformers the ratios have changed by only 0.001×10^{-6} or 0.002×10^{-6} over a period of 20 years [4]. Therefore, we do not expect to repeat this measurement for some time.

Description	**DC/C_{cc}(´ 10^{-9})** 1592 Hz	**DC/C_{cc}(´ 10^{-9})** 1000 Hz
Inductance in adjacent ground bars	- 17.36	- 6.78
Guard tube self inductance	+ 4.26	+ 2.22
Inductance in line and detector bars	+ 1.81	+ 0.72
Mutual inductance in opposite bars (+ guard tube)	- 2.28	- 0.90
Mutual inductance in adjacent bars	- 0.75	- 0.03
Net cable correction	+ 2.34	+ 0.92
Total	**- 11.98**	**-3.85**

Table 1. Loading corrections applied to the calculable capacitor measurements at 1592 Hz and 1000 Hz.

The loading corrections refer to the effects on the measurement of capacitances-to-ground in the cables, and inductances and resistance in the cables and in the bars. These change with frequency and must be re-evaluated at 1000 Hz. There are five types of corrections: cable corrections, the effect of inductance in adjacent ground bars, the effect of inductance in the line and detector bars, the effect of the guard tube self-inductance, and mutual inductance in opposite bars. The capacitance-to-ground, inductance and resistance of all cables, connectors, and the four main electrodes must be measured.

The loading corrections at 1592 Hz and 1000 Hz are shown in Table 1.

C. Uncertainties

There are many uncertainties associated with the calculable capacitor measurement. These are shown in Table 2. Many of these, like the uncertainty associated with geometrical imperfection, are not expected to change with frequency. Those that changed are shown in bold. They include the variability of repeated observations (Type-A), which was previously discussed, the loading corrections, the voltage dependence, and the transformer ratio measurement. Not all the parameters needed to calculate the loading corrections were measured at 1000 Hz. The unmeasured parameters include the inductance of the guard tube, the measurement of which would require taking apart the calculable capacitor. The relative corrections are small (0.003×10^{-6}) and should decrease with frequency. However, the relative uncertainty was increased from 0.004×10^{-6} to 0.008×10^{-6} to account for any unknown factors. There are two voltage-dependence measurements associated with the calculable capacitor measurement of the 10 pF capacitor. One is the recently discovered change discussed earlier between 2 V and 4 V, which does not affect the 1592 Hz measurement. The other is the change in voltage on the 10 pF capacitor between the calculable capacitor measurement (2 V, 4 V, 7 V, and 14 V) and the measurement against the 10 pF bank (100 V). This has not been measured at 1000 Hz. We do not expect this to change with frequency but it has been a few years since the 1592 Hz measurement was done. Given that and considering that the 1000 Hz measurement requires an additional voltage-dependence measurement, we thought it was prudent to increase the relative uncertainty in the voltage dependence from 0.005×10^{-6} to 0.010×10^{-6}. The transformer ratio at 1000 Hz was done at a lower voltage than the 1592 Hz measurement because of saturation of the transformer at the higher voltages. This increased the noise of the measurements and so the relative uncertainty was increased 0.002×10^{-6} to 0.004×10^{-6}.

A. Source of uncertainty	**Relative std. unc. 1592 Hz**	**Relative std. unc. 1000 Hz**
Variability of repeated observations	$\mathbf{2 \times 10^{-9}}$	$\mathbf{50 \times 10^{-9}}$
A. Geometrical imperfections in the calculable capacitor	15×10^{-9}	15×10^{-9}
Laser/Interferometer alignment	3×10^{-9}	3×10^{-9}
Frequency (loading) corrections	$\mathbf{4 \times 10^{-9}}$	$\mathbf{8 \times 10^{-9}}$
Microphonic coupling	5×10^{-9}	5×10^{-9}
Voltage dependence	$\mathbf{5 \times 10^{-9}}$	$\mathbf{10 \times 10^{-9}}$
Drift between calibrations/ failure to close	6×10^{-9}	6×10^{-9}
Transformer ratio measurement	$\mathbf{2 \times 10^{-9}}$	$\mathbf{4 \times 10^{-9}}$
Bridge linearity and phase adjustment	3×10^{-9}	*3×10^{-9}
Detector uncertainties	2×10^{-9}	2×10^{-9}
Coaxial choke effectiveness	1×10^{-9}	*1×10^{-9}
Temperature corrections for 10 pF capacitors	2×10^{-9}	2×10^{-9}
Relative standard uncertainty	$\mathbf{19 \times 10^{-9}}$	$\mathbf{55 \times 10^{-9}}$

Table 2. Relative standard uncertainties associated with the calculable capacitor measurement of a 10 pF capacitor at 1592 Hz and 1000 Hz. Numbers in bold change with frequency. Asterisk denotes a 1592 Hz number but uncertainty is not expected to change with frequency.

D. Transfer to the 10 pF bank

After the 10 pF transportable fused-silica capacitor is measured against the calculable capacitor, it is measured against a bank of five 10 pF fused-silica standards [6]. These standards, which were designed and fabricated at NIST, are maintained in an oil bath at 25°C. They have an average drift rate of 20 aF per year, which allows them to be used to maintain the NIST realization of the farad for several months between calculable capacitor measurements. All comparisons are made by the sequential substitution of each of the 10 pF fused-silica standards. The transfer measurements are done with a two-terminal-pair, 10:1 transformer bridge using a 100 pF capacitor as a fixed reference.

Once the frequency change of the 10 pF transfer standards was known between 1592 Hz and 1000 Hz, this was used to determine the frequency dependence of the standards in the bank. This was done be comparing the standards to each other by substitution in a 10:1 bridge at 1592 Hz and 1000 Hz. The two-terminal bridge that is normally used for these measurements could not be used since it operates only at 1592 Hz. This increased the time to make these measurements since the four-terminal-pair bridge has many auxiliary balances. The transformer ratio of this bridge at 1000 Hz had to be measured and was evaluated using our usual method of the permutation of eleven 10 pF capacitors [7].

The relative change with frequency between 1592 Hz and 1000 Hz, A, is given by the equation $\gamma_{1000\ Hz} = \gamma_{1592\ Hz} + A$, where γ_{1000Hz} and γ_{1592Hz} are the deviations from nominal value for a 10 pF capacitor, C_{1592Hz} and C_{1000Hz}, at 1592 Hz and 1000 Hz, respectively, e. g. C_{1000Hz} = 10 pF $(1+\gamma_{1000Hz}\times10^{-6})$. Then, for capacitors C_{113} and C_{125} used to transfer the capacitance unit to the calibration laboratory, A is $0.41\pm0.06\times10^{-6}$ and $0.12\pm0.05\times10^{-6}$, respectively.

The previous uncertainty was based on measurements made by Cutkosky and Lee [6], which were not based on a standard with a known frequency dependence, the main contributor for the total uncertainty of 1.5×10^{-6}. The uncertainty assigned to the measurement frequency dependence, A above, is much smaller than the value previously used, 0.7×10^{-6}.

By comparing these measurements with ones made a few years ago, it appears the frequency dependence of the capacitors are fairly stable and that once a frequency dependence is determined it may not have to be re-evaluated for several years (see Table 3). This means that the capacitance unit can continue to be determined at 1592 Hz and the corrections applied to the capacitors to obtain their value at other frequencies.

Capacitor	A (8/2000)	A (10/1997)	A (1965*)
125	$0.12 x10^{-6}$	$0.13 x10^{-6}$	—
113	$0.41 x10^{-6}$	$0.41 x10^{-6}$	—
109	$0.17 x10^{-6}$	—	—
110	$0.30 x10^{-6}$	—	—
124	$0.04 x10^{-6}$	$0.06 x10^{-6}$	—
128	$0.22 x10^{-6}$	—	—
112	$0.04 x10^{-6}$	$0.04 x10^{-6}$	$0.08 x10^{-6}$

Table 3. Frequency dependence of other 10 pF capacitors

E. Transfer of unit to the calibration laboratory.

As mentioned above, transfer standard capacitors C_{113} and C_{125} are used to transfer the farad to the calibration laboratory. The calibration laboratory maintains a bank of 5 oil bath 10 pF fused-silica capacitors identical to the bank described above that is the NIST representation of the SI farad. This provides redundancy in the maintenance of the U.S. representation of the farad, as well as a reference from which to measure NIST check standards and customer capacitors.

The two transfer standards, C_{113} and C_{125}, are used as references against which the 5 oil bath capacitors are measured at 1 kHz using a two-terminal-pair capacitance bridge. Each of the oil bath fused-silica capacitors in the calibration laboratory is compared against each of the two known transfer standards. The transfer standards and the oil bath standards are constructed with resistance and capacitance terminals. The resistance terminals allow for the measurement of the temperature of the standard at the time of capacitance measurement. The temperature of the device is directly proportional to the resistance across the resistance terminals. All standards have a reference temperature from which a reference capacitance value can be computed for each measurement value. The reference values are used for monitoring the control of the capacitance characteristics. The average of four oil bath capacitance values is monitored (the fifth standard is used as a dummy reference).

The comparisons of the oil bath capacitors against the transfer capacitors, along with the SI correction of the farad (obtained from measurements of the transfer standards against the calculable capacitor) and the frequency corrections from 1592 Hz to 1 kHz, provide well-characterized reference standards and associated uncertainties with which to measure customer capacitance standards at 1 kHz.

III. RESULTS

The Type-B uncertainty component for the error due to frequency dependence has been reduced by an order of magnitude from 0.7×10^{-6} to 0.06×10^{-6}. Therefore, the overall expanded uncertainty (k = 2) for 1 kHz calibration of 10 pF and 100 pF fused-silica capacitors has been reduced from 1.5×10^{-6} to 0.5×10^{-6}.

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