

Standards of Current and Capacitance Based on Single-Electron Tunneling Devices

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Note: Published version has a mistake in Eq. 6. The demoninator of the final result should contain V instead of V_{90} .

1. INTRODUCTION

Single-electron tunneling (SET) devices provide a means of manipulating individual electrons and detecting the motion of these electrons with extraordinary precision. Their potential impact on metrology and fundamental constants was recognized early in the development of the field during the 1980s. By the early 1990s, several SET devices had demonstrated the ability to detect charges much smaller than e and to transfer individual charges from one electrode to another. In the last few years, the performance of these devices has been pushed to the levels needed for fundamental standards and high-precision measurements: SET electrometers can detect $\sim 10^{-5} e$ in a 1 Hz bandwidth; electron traps can store individual charges for hours; electron pumps can transfer hundreds of millions of individual electrons with an uncertainty of $\sim 10^{-9}$. Fundamental standards based on these capabilities are being pursued at several national measurement institutes throughout the world, and SET devices are at the threshold of making important contributions to practical and fundamental metrology.

Flensburg *et al.* [1] recently reviewed SET metrology with a focus on a detailed discussion of mechanisms that limit performance. In this paper I review the current status of SET standards for current and capacitance, which has changed significantly since the previous review, and I emphasize their potential impact on our knowledge of fundamental constants and on our system of units. I begin with a description of the essential physics of SET devices in Section 2. Section 3 describes various schemes for making capacitance and current standards based on SET devices. Section 4 covers the relevance of SET standards for fundamental constants and the *Système International d'Unités*, and Section 5 contains conclusions.

Standards based on SET devices are being pursued in areas other than electrical metrology. I will not discuss them in detail, but will briefly mention two of them here and point the reader to some of the recent literature. An absolute thermometer based on SET effects has been demonstrated and is now available as a commercial product [2]. Regulated sources of single photons based on SET effects have been proposed in two types of semiconductor systems: etched or self-assembled quantum dots [3] and one-dimensional channels subject to surface acoustic waves [4].

Of the various potential applications for SET standards of current and capacitance, the one that best illustrates the impact of manipulating individual electrons on metrology and fundamental constants is the realization of the quantum metrology triangle. The idea, first proposed by Likharev and Zorin in 1985 [5], is illustrated in a simplified form in Fig. 1. A frequency source traceable to an atomic clock is used to drive a Josephson voltage source that produces a voltage $U_J = f/K_J$ and an SET pump that produces a current $I_{\text{SET}} = Q_X f$, where theory predicts that $K_J = 2e/h$ and $Q_X = e$. The ratio U_J/I_{SET} defines a resistance that can be compared with the quantum Hall resistance $R_H = R_K$, where theory predicts that $R_K = h/e^2$. The triangle is essentially an application of Ohm's law, $U = IR$, to the quantities produced by the three quantum effects [6]. If an actual realization of the triangle can be shown to close with an uncertainty of $\sim 10^{-8}$, it will enhance our confidence in the three relations above involving e and h . If the triangle does not close within experimental uncertainty, it will stimulate a search for which of the three relations is not valid. I will discuss the quantum metrology triangle in more detail near the end of this review.

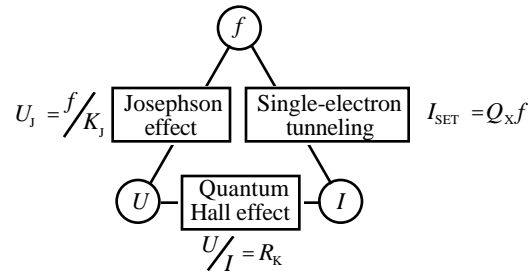


Fig. 1 The quantum metrology triangle. Current from an SET device and voltage from a Josephson effect device are combined to create a resistance that is compared to the quantum Hall resistance.

2. BASIC PHYSICS OF SET DEVICES

Consider two tunnel junctions defining a metallic island of total capacitance C , as shown in Fig. 2a. The basic condition needed for the observation and exploitation of SET effects is that the system must "notice" when the number of electrons on the island changes by one, despite the fact that there may be millions of electrons on the island. This condition is fulfilled when two requirements are met. First, the electrostatic energy needed to place a single extra electron onto the island, $e^2/2C$, must be large compared to the energy of thermal fluctuations, $k_B T$. Second, the tunnel barriers must be sufficiently opaque that the wavefunction of an extra electron on the island is well localized there. In other words, the lifetime associated with tunneling must be sufficiently long that the corresponding uncertainty in energy is small compared to the single-electron charging energy $e^2/2C$. This requirement is met when the tunnel resistance R is large compared to h/e^2 . In a device that satisfies these two requirements, the number of excess electrons on the island is an integer and there is a significant energy barrier for changing this number. If a voltage is applied across the two junctions in Fig. 2a, current will not flow through the system unless the voltage source can supply enough energy to overcome this energy barrier. This phenomenon, which is readily observed in real systems, is known as the Coulomb blockade of electron tunneling.

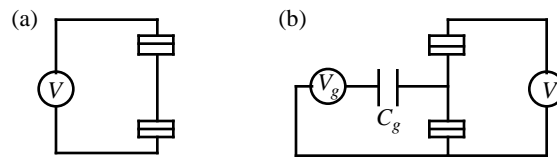


Fig. 2 Basic building blocks of SET devices. (a) A single island of total capacitance C is defined by two tunnel junctions (double box symbols). Charge can flow on or off the island only in discrete units of e . The voltage source V must supply the charging energy $e^2/2C$ in order for current to flow through the two junctions. (b) A single-electron tunneling transistor is created by adding a gate capacitor and voltage source that polarize the island with a continuous charge and modulate the charging energy barrier.

To make useful devices based on the ideas just described, there must be a way of manipulating the charging energy barrier so that tunneling can be turned on and off at will. This is accomplished by adding a third electrode to the system, as shown in Fig. 2b. A voltage applied to this electrode polarizes the island with a charge that can be varied continuously [7]. When this polarization charge is $e/2$, the energy of the system with 0 and 1 extra electrons on the island is the same, and the system does not "notice" if one extra electron tunnels on or off the island, *i.e.*, the charging energy barrier has been reduced to zero. The third electrode that allows the energy barrier to be manipulated in this way is called a gate electrode, and the complete three-terminal device in Fig. 2b is called a single-electron tunneling transistor (SETT).

Controlled transfer of individual charges can be accomplished by putting several junctions and islands in series, along with a gate electrode for each island, to form a device called an electron pump, as shown in Fig. 3a. When the gate voltages are pulsed in the sequence shown in Fig. 3b, the charging energy barrier is lowered to allow tunneling through one junction at a time. Essentially, the sequence of gate voltages creates a polarization wave that moves along the chain of islands, and an extra electron tunnels at each junction in order to neutralize this wave and thus minimize the energy of the entire system. At the end of the sequence, one electron has passed through and the pump has returned to its initial state.

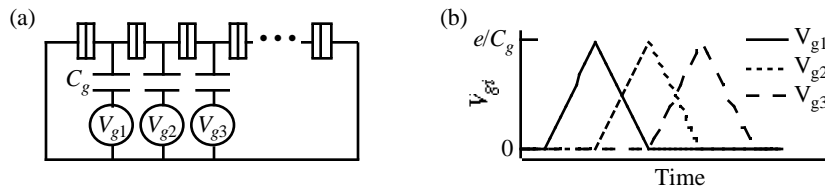


Fig. 3 Controlled transfer of electrons through an SET device. (a) Schematic of the SET pump, which is essentially several SETTs in series. (b) Sequence of gate voltage pulses used to operate the pump. Each pulse causes an electron to tunnel across one junction, and after the sequence is completed, one electron has been passed along the entire chain.

The fundamental speed limit for manipulating electrons through a conventional SET device is set by the time RC . After the energy barrier for a desired tunneling process has been lowered, the probability that the tunneling event has not occurred falls exponentially with a time constant of order RC . Calculations show that in order to make this probability of missing a desired tunneling event $\sim 10^{-9}$, one must wait $\sim 100RC$ for a typical SET device [8]. As described above, R must be larger than h/e^2 , and typically $\sim 100 \text{ k}\Omega$ is needed for metrological performance. The value of C depends on the method used for fabricating the junctions. Currently, electron beam lithography is routinely used to make junctions from vacuum-deposited Al films with $C \sim 0.1 \text{ fF}$. Junctions with $C \sim 0.01 \text{ fF}$ are possible with this technique, but with dramatically reduced yield. Other techniques and materials have yielded junctions of poor quality or with poorly controlled parameters. Thus the fastest electron transfer rate one could expect from optimized Al SET devices is $\sim 1/\{100(100 \text{ k}\Omega)(0.1 \text{ fF})\} = 1 \text{ GHz}$, which corresponds to a current of $\sim 100 \text{ pA}$. The fastest transfer with metrological uncertainty that has been demonstrated with Al devices is 5 MHz [9], but a true optimization for

high speed operation has not been attempted. In Section 3.2 I will describe other schemes for electron transfer that are expected to provide larger currents.

An important practical, but not truly fundamental, limitation on SET devices is the presence of background charges that polarize each island of the device. These charges may arise from defects in the substrate, in the material of the islands themselves, or in the tunnel barriers. The net polarization from the charges surrounding a particular island is random and can vary from $-e/2$ to $+e/2$. Furthermore, the defects responsible for the background charges apparently are not stable, because the island charges fluctuate in time, even in devices that have been kept at temperatures below 100 mK for weeks. As a consequence, a dc voltage must be applied to each island in an SET device to cancel the effect of the background charges, and this voltage must be readjusted each time the background charges change significantly. Although this presents a practical problem for all applications of SET devices, the performance of electron pumps has shown that it can be solved.

There is a wealth of interesting SET physics beyond the brief discussion above. Interested readers can find accessible reviews in refs. [10–12].

3. METROLOGICAL APPLICATIONS OF SET DEVICES

This section contains a snapshot of the current status of several SET devices that may provide new fundamental standards in the next decade or so. Rapid progress is occurring on several fronts, and some of the performance figures I give here will certainly be surpassed before this review is published. Nevertheless, they are useful indicators of the potential performance of various schemes [13].

3.1 SET capacitance standard

In defining capacitance, we consider a transfer of charge between two conductors. The transferred charge Q causes a potential difference V between the conductors, and the capacitance is simply $C = Q/V$. Since nature provides abundant electrons having identical quantized charges, it is natural to use them as the basic unit of charge in implementing the definition of capacitance. This is precisely what is done in the SET capacitance standard. An SET pump is used to place N electrons onto one electrode of a cryogenic, three-terminal, vacuum-gap capacitor [14,15], then the resulting V is measured, and the capacitance is given by $C = Ne/V$. The same cryogenic capacitor can also be measured in a conventional bridge configuration so that its value can be transferred to an artifact at room temperature for convenient comparison with other capacitors. A prototype of such a standard is in operation at NIST, and details about its operation can be found in a recent publication [16]. In anticipation of the discussion of SET current standards in the next section, it is important to note that the device used to place the electrons onto the capacitor must be able to start and stop very precisely.

The performance of the NIST prototype SET capacitance standard is summarized in Fig. 4. Several values of C obtained by counting electrons, spanning a period of about 24 hours, are shown in Fig. 4a. The mean of these values is compared with the value obtained using a commercial 1 kHz capacitance bridge in

Fig. 4b. The uncertainty bars represent the (Type A) relative standard uncertainty of the mean electron counting value (1×10^{-7}) and the estimated (Type B) relative standard uncertainty of the calibration of the bridge (1×10^{-6}). Thus the SET capacitance standard has demonstrated an irreproducibility of 1×10^{-7} and it agrees with a conventional measurement of the same capacitor within the uncertainty of the conventional measurement. Preliminary tests have also shown that the value of C is independent of voltage and frequency [16,17].

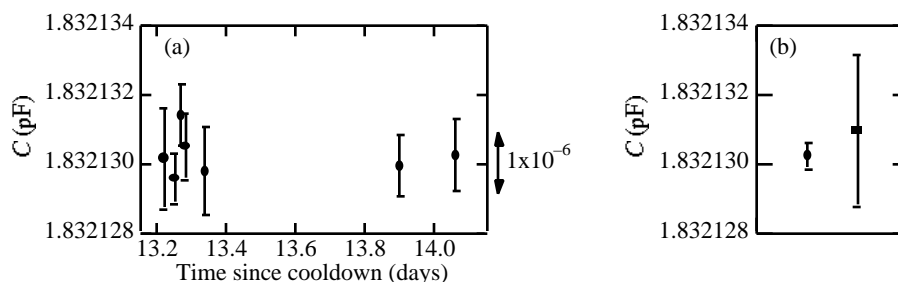


Fig. 4 Capacitance determined by counting electrons. (a) Several values from one day's operation of the NIST prototype SET capacitance standard. The scale bar at the right shows a relative change of 1×10^{-6} . (b) Comparison of the values of a cryogenic capacitor measured by counting electrons (circle) and by a capacitance bridge traceable to NIST's primary capacitance standard (square).

The development of the NIST prototype is proceeding along three paths. First, since the SET standard currently resides at the Boulder site, while NIST's primary capacitance standard, a calculable capacitor [18], is located at the Gaithersburg site, a transportable version of the prototype is being constructed to allow a direct comparison between the two. This will allow the uncertainty of the bridge value of C to be reduced to $\sim 10^{-8}$ through the use of custom bridges that cannot be transported to the Boulder site. Second, a detailed uncertainty analysis of all aspects of the standard is being conducted, with special emphasis on possible differences between the values obtained by counting electrons and by bridge techniques. An extensive list of uncertainty components has been generated [19] and experimental tests are in progress. The results to date have not revealed any major obstacles to a total uncertainty of $\sim 10^{-8}$. The third thrust is the design of a robust and automated system with computer control of as many functions as possible. This is important because the SET standard may be useful as a practical standard, even at the uncertainty level of the existing prototype, if it can be made easy to use. For example, it could provide a stable basis for the calibration of artifact standards in large industrial labs, thus eliminating the need to send these artifacts to a national measurement institute (NMI) for periodic calibration.

A project by the name of "Count" has recently begun in Europe, with participation from several NMIs, and one of its aims is to develop an SET capacitance standard similar to the NIST design. Thus there should be at least two working prototypes in the next few years, which will make intercomparisons possible and help establish a level of international confidence in this approach.

3.2 SET current standard

From a practical point of view, there is no pressing need for a fundamental current standard at the moment because we already have excellent fundamental standards of voltage and resistance. However, experiments such as the quantum metrology triangle require a current based on the electron charge and the second. Passing electrons one-by-one is the most direct way of obtaining such a current, and there are several schemes for implementing this idea. The uncertainty of most quantum metrology triangle experiments envisioned thus far is directly limited by the small SET current because the voltage produced by passing this current through a quantum Hall device is very small. The maximum current that can be produced with the required uncertainty is thus the primary criterion for comparing different SET current standards.

3.2.1 SET turnstile

The SET turnstile of Geerligs *et al.* [20] was the first device to demonstrate the transfer of electrons locked to an external frequency. It consists of a chain of junctions with an rf gate coupled to the island in the middle of the chain and a voltage bias across the chain of junctions to define the direction of current flow, as shown in Fig. 5. (There are also gates coupled to the other islands, not shown in Fig. 5, but they are used only to apply dc voltages to compensate for background charges.) A voltage of order e/C_g applied to the middle gate lowers the energy barrier for tunneling through the left half of the chain, and an extra electron becomes trapped on the middle island. When the gate voltage is returned to zero the extra electron tunnels through the right half of the chain to complete the cycle. The bias voltage, which is typically of order e/C , ensures that tunneling occurs only in the left half during the first step and only in the right half during the second step.

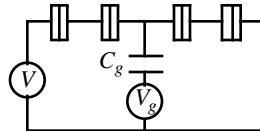


Fig. 5 Schematic of a 4-junction SET turnstile, whose operation is described in the text.

In the initial experiments with 4-junction SET turnstiles the current through the turnstile was measured directly using essentially conventional electronics. When the gate was driven at about 10 MHz, a plateau in current vs. bias voltage was observed that was flat within the noise of the measurement (relative standard deviation of 10^{-3}). The average current on the plateau agreed with the expected value $I = ef$ within the estimated experimental uncertainty of 3×10^{-3} . Later experiments by the same research group used turnstiles with 4 and 6 junctions of various designs and had a similar experimental uncertainty [21]. Measurements with smaller uncertainty have not been reported, and interest in the turnstile faded after an SET pump with 5 junctions was demonstrated to have an uncertainty of less than 10^{-6} [22].

3.2.2 Conventional SET pump

The SET pump differs from the SET turnstile in that a high-frequency voltage (as well as a dc voltage for compensation of background charges) is applied to each island. Furthermore, since the direction of current through the pump is determined by the order in which the gates are pulsed, a bias voltage across the chain is not needed. Tunneling events in the pump can be controlled more precisely than in the turnstile (in particular, they occur closer to threshold and thus there is less unwanted dissipation), and this is reflected in better performance.

The SET pump was first demonstrated by Pothier *et al.* [23] at CEA-Saclay and has since been studied extensively at NIST. This effort has been comprehensive in scope, encompassing detailed theoretical calculations of basic error mechanisms, experiments designed to test specific theoretical predictions, and careful design of experimental apparatus to ensure that the optimal operating conditions prescribed by theory are realized in practice. Particular attention has been paid to the last point [24]. The layout of islands and gates is optimized to minimize cross capacitance effects and a custom circuit cancels the remaining effects. Triangular gate pulses (see Fig. 3b) provide a more ideal operating sequence than sinusoidal gate voltages. Cryogenic needle switches provide multiple measurement configurations that allow each pump to be completely characterized in a single cooldown.

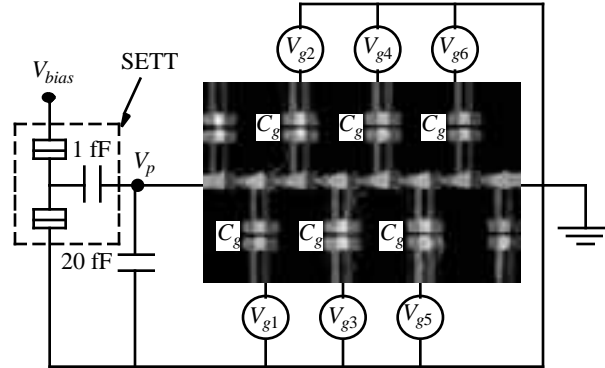


Fig. 6 Circuit for characterizing an SET pump. An SETT fabricated on the same chip as the pump monitors the charge on one end of the pump through a gate capacitance of 1 fF. Even with a stray capacitance of 20 fF, each electron passing through the pump is easily detected by the SETT. This allows the measurement of individual error and leakage events, as shown in Fig. 7.

Since direct current measurements that can test controlled charge transfer with the uncertainties required for metrology are difficult, a different measurement technique has been used for the NIST pumps. As shown in Fig. 6, each SET pump is coupled to an SETT that can resolve a single electron after it passes through the pump. The SETT is locked in a feedback circuit (using an auxiliary gate not shown in Fig. 2) to

ensure that it operates with maximum sensitivity. The pump is operated in a shuttle mode where one electron is pumped to and from the island between the pump and the SETT. When this is done slowly, as in Fig. 7a, the SETT easily detects each pumped electron. When this is done at the normal operating frequency of about 5 MHz, well beyond the SETT bandwidth of about 1000 Hz, the SETT responds only to the average charge on the external island because the individual changes of $+e$ and $-e$ happen too quickly. The average charge changes only when the pump fails to transfer e during a given cycle, so the SETT records only the errors, as in Fig. 7b. The relative error is simply the number of errors observed divided by the number of pumping cycles during the measurement, which for Fig. 7b is 4×10^{-9} (the best result obtained to date). The rate of leakage events in the hold mode, when the gates are not pulsed, can also be measured in this way. Fig. 7c shows a typical result, where the average time between individual leakage events is 10 minutes. Although detailed studies of pump errors have been done only for the shuttle mode, the performance of the SET capacitance standard shows that the pump also works well when pumping many electrons in one direction.

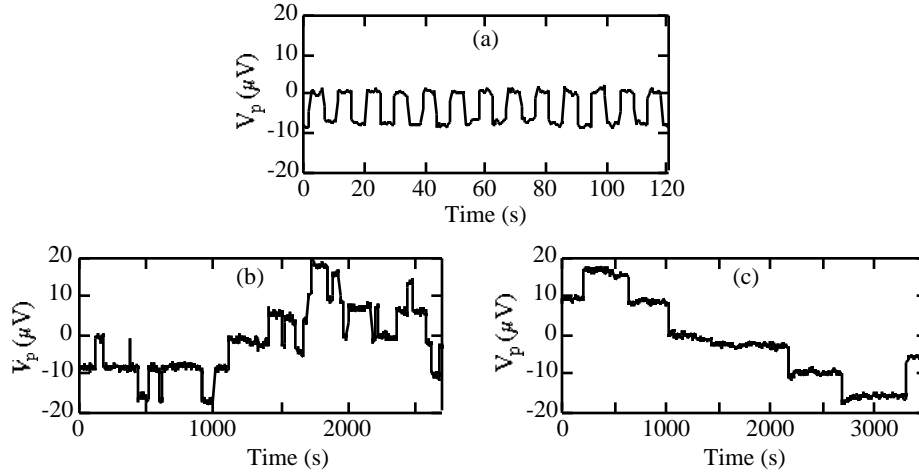


Fig. 7 Detection of individual error and leakage events in an SET pump. The voltage in each plot is V_p as measured with an SETT (see Fig. 6). (a) Pumping slowly, where each pumped electron can be resolved as a change of about $7.5 \mu\text{V}$. (b) Pumping at a few MHz, where only the errors can be resolved. (c) The hold mode, where the pump is turned off and one electron leaks through every few minutes.

Since feedback between theory and experiment has been particularly important for the successful development of the SET pump, it is worth reviewing the current status of our understanding in this area. The analysis of Jensen and Martinis [25] predicted that a 5-junction SET pump with readily achievable parameters should be capable of a relative error $< 10^{-9}$ and a leakage rate of 10^{-6} s^{-1} . When such pumps were tested [22], they showed a relative error of 5×10^{-7} and a leakage rate of 0.1 s^{-1} . While a "brute force" approach of making 7-junction pumps has achieved the performance needed for electrical metrology [9], the discrepancy between theory and experiment continues to be pursued. A thorough characterization of a

particular 7-junction pump allowed a direct comparison between experiment and theory with no adjustable parameters [26,27]. This comparison revealed that theory and experiment agree at high temperatures (140 mK) where pumping errors and leakage are dominated by thermal activation, but differ by orders of magnitude at low temperatures (40 mK). This result indicated that the standard theory of the SET pump was incomplete, *i.e.*, that the source of the errors and leakage at low temperature was not included in the standard theory. A likely mechanism that was missing from the standard theory was photon-assisted tunneling processes [28]. To investigate this issue, Covington *et al.* [29] exposed pumps with 4 and 6 junctions to a known source of microwave radiation and measured the resulting increase in leakage rate. They found that this increase could be explained by a straightforward modification of the standard SET theory to include photon-assisted tunneling. The success of this modified theory then led to the question of whether the errors and leakage in the absence of applied microwaves could be similarly explained. As a first attempt at such an explanation, Kautz, Keller, and Martinis [30] have recently proposed a model in which non-equilibrium fluctuating background charges give rise to a $1/f$ noise that extends to the microwave frequencies ($f \sim 50$ GHz) needed to cause photon-assisted tunneling in the NIST pumps. When the amplitude of this $1/f$ spectrum is adjusted until the model gives the observed error and leakage rates, it is found that amplitudes between about $3 \text{ nV/Hz}^{1/2}$ and $100 \text{ nV/Hz}^{1/2}$ at 1 Hz (equivalent to charge noise between about $2 \times 10^{-6} \text{ e/Hz}^{1/2}$ and $60 \times 10^{-6} \text{ e/Hz}^{1/2}$ at 1 Hz for a typical capacitance of 0.1 fF) reproduce the values measured in all pumps with 4, 5, 6, and 7 junctions measured at NIST. This range of noise amplitudes is remarkably similar to the range observed in direct noise measurements on SETTs at $f < 1 \text{ kHz}$, which give values between about $30 \text{ nV/Hz}^{1/2}$ and $3000 \text{ nV/Hz}^{1/2}$ (equivalent to charge noise between about $20 \times 10^{-6} \text{ e/Hz}^{1/2}$ and $2000 \times 10^{-6} \text{ e/Hz}^{1/2}$ at 1 Hz for a typical capacitance of 0.1 fF). In other words, if the noise typically measured at low frequencies in SETTs is extrapolated to microwave frequencies, the modified theory used to explain the results of Covington *et al.* can also explain the error and leakage for pumps in the absence of intentionally applied radiation. Although this extrapolation is still speculative, and a consistent microscopic model for the production of such noise by fluctuating defects may be problematic [31], the existence of a non-equilibrium state is supported by evidence of slow time dependence in the low frequency noise of SETTs [30].

The close interplay between theory and experiment has been important for the development of the electron pump, and it continues to improve our understanding of the basic error mechanisms in all SET devices. Analogous benefits can be anticipated as a similar level of effort is applied to other schemes for creating an SET current standard.

3.2.3 R-SET pump

The R-SET pump differs from the conventional SET pump by the addition of resistors at one or both ends of the chain of junctions (see Fig. 8). The purpose of these resistors is to suppress a certain class of unwanted tunneling events. These events are called cotunneling events because they involve simultaneous and coherent tunneling of electrons at two or more junctions. For example, the leakage of one electron through all N junctions in a pump can occur *via* N thermally activated real transitions, or *via* one N -junction cotunneling transition involving the virtual occupation of $N - 1$ intermediate states. At typical operating

temperatures for SET devices, the cotunneling rate dominates and thus cotunneling generally sets the ultimate theoretical limit on device performance (neglecting photon-assisted processes).

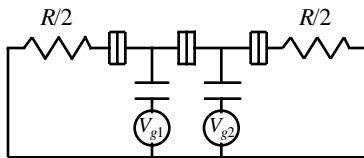


Fig. 8 Schematic of a 3-junction R-SET pump, whose operation is described in the text.

The resistors at the ends of the R-SET pump suppress cotunneling by modifying the electromagnetic environment seen by the pump. The importance of the interaction between an SET device and its environment is reviewed in [11]. Each tunneling event in an SET device involves a redistribution of charge throughout the device and its electromagnetic environment. In other words, the tunneling electrons are coupled to the electromagnetic modes of the environment. When the device and its environment are treated as a single quantum system, it can be shown that tunneling rates are affected by the environmental impedance $Z_e(\omega)$ for frequencies up to $\omega_c = (e^2/2C)/\hbar$. In particular, when $Z_e(\omega)$ is comparable to $h/e^2 \approx 26 \text{ k}\Omega$ for $\omega < \omega_c$, tunneling rates will be strongly suppressed compared to the case of $Z_e(\omega) = 0$. At frequencies comparable to ω_c , which is typically between 10 GHz and 100 GHz, the impedance seen by a conventional SET pump is only $\sim 100 \Omega$ because of stray capacitance in the metallic leads, and tunneling rates under typical conditions are only slightly suppressed. However, if a special effort is made to construct small resistors with $R \sim h/e^2$ and place them close to the pump, the stray capacitance can be made small enough to have $Z_e(\omega) \sim h/e^2$ for $\omega < \omega_c$, and tunneling rates can be significantly suppressed.

Both desired and undesired tunneling rates will be affected by the environmental resistors in Fig. 8, but fortunately the undesired cotunneling rates will be affected more strongly. This can be seen from the charge redistribution illustrated in Fig. 9. Consider the point in the pumping cycle when a polarization is applied to the island labeled B through its gate. In the desired event where an electron tunnels from A to B, the charge that flows through R is e/N . In the undesired event where an electron tunnels from C to B through the other $N - 1$ junctions, the charge that flows through R is $e(N - 1)/N$. Since more charge is redistributed through the electromagnetic environment for the undesired event, it will be more strongly suppressed by the effect described in the previous paragraph. The difference in the amount of suppression between desired and undesired events is greatest when the number of junctions is large.

Experimental tests of the R-SET pump have recently begun at PTB as part of the Count project [32]. Although these experiments have not yet used the direct method of counting errors and leakage events described in Section 3.2.2, the initial results are promising. Indirect measurements of leakage indicate that a 4-junction array with resistors has a leakage rate roughly comparable to that of a 7-junction array without resistors [33]. Indirect evaluation of relative pumping error based on the shape of the current vs. voltage curve while pumping gives an estimate of $\sim 10^{-4}$ [34,35]. One important question is whether the environmental resistors can really improve performance in the presence of the photon-assisted tunneling that

appears to dominate in the NIST pumps. Direct measurement techniques and other refinements to these experiments are planned, and they should reveal whether the environmental resistors truly produce the intended benefits. If so, it may be possible to achieve the performance needed for metrology with perhaps as few as 3 junctions, which would make practical operation of the pump somewhat simpler by reducing the complexity of the custom electronics.

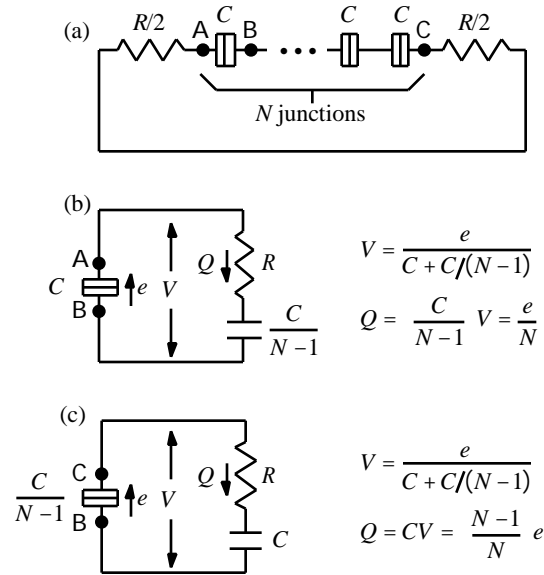


Fig. 9 Charge redistribution in an R-SET pump. (a) Simplified schematic of an N -junction R-SET pump. (b) Equivalent circuit for the desired tunneling event across the single junction between points A and B. The $N-1$ junctions where tunneling does not occur are represented as an ordinary capacitor. When an electron tunnels from A to B, its charge must be redistributed throughout the rest of the circuit so that the same voltage appears across C and the series combination $C/(N-1)$. The charge that must flow through the environment to accomplish this is e/N . (c) Equivalent circuit for the undesired cotunneling event across the $N-1$ junctions between points B and C. The junctions involved in the cotunneling process are represented as a single junction of capacitance $C/(N-1)$. In this case the charge that must flow through the environment is $eN/(N-1)$.

3.2.4 Superconducting pump

An array of Al junctions can be used to pump individual Cooper pairs if the junctions are superconducting and the Josephson coupling energy E_J is comparable to E_c . Such a device is known as a single-Cooper-pair tunneling (SCPT) pump. In the first experiments on an SCPT pump, using a direct current measurement technique as for the SET turnstile, a flat plateau was not found [36]. Because of this poor initial result, and more importantly because the theory of the SCPT pump is more complex and not as

fully developed as the theory of the SET pump, relatively little attention has been paid to the possibility of a superconducting current standard. This is beginning to change for reasons I will describe shortly.

The advantage of the SCPT pump is that, in principle, it can operate faster than the SET pump without an increase in relative error. The tunneling of Cooper pairs is a coherent process, in contrast to the incoherent and stochastic tunneling of electrons, and thus the RC time of the junctions does not set a speed limit for the SCPT pump. However, this same coherence presents a problem because it allows a supercurrent to leak through the entire device. In order to make a useful SCPT device, this supercurrent must be suppressed without inhibiting the process of transferring Cooper pairs across individual junctions. In essence, what is needed is an array of junctions that is globally insulating but locally superconducting. In arrays of junctions with $E_J < E_C$, an excess Cooper pair takes the form of a soliton [37], and the supercurrent is suppressed when the array is much longer than the soliton length. An SCPT pump has not been tried with such an array, and the number of junctions and gates required for metrology may be impractical, but an SCPT turnstile based on a long array is being pursued as a test of the general idea [38]. There is recent progress in calculating errors and leakage in the SCPT pump [39,40] to guide experiments in this area.

Another possible solution to the conflicting requirements of the SCPT pump is to exploit the influence of the environmental impedance that was described in Section 3.2.3 [41]. For a leakage event the charge that must flow through the environmental impedance is $2e$, while for a desired tunneling event at a single junction it is only $2e/N$. An analysis of the N -junction SCPT pump with a purely resistive environmental impedance R shows that when

$$\frac{N}{N-1} \frac{h}{(2e)^2} < R < N^2 \frac{h}{(2e)^2}, \quad (1)$$

both leakage through N junctions and pumping errors due to cotunneling through $N-1$ junctions are strongly suppressed, while the desired tunneling events are only slightly suppressed. For a pump with $N=7$, this range is from about 10 k Ω to 300 k Ω . Resistors with values in this range and with small stray capacitance have already been used in several SET experiments, including the R-SET pump, so this approach seems feasible. Further quantitative analysis of this scheme is needed to show that errors due to unpaired quasiparticles, occupation of excited states due to Zener tunneling, and photon-assisted tunneling can be made adequately small. Nevertheless, careful engineering of the electromagnetic environment appears to provide a promising route to a practical implementation of the SCPT pump.

3.2.5 SETSAW current standard

There have been two approaches to combining SET effects with surface acoustic wave (SAW) effects to produce a current source. The first approach is to place an SET or SCPT pump on a piezoelectric substrate and use a traveling SAW to accomplish the sequence of gate voltages needed to pump electrons or Cooper pairs [42,43]. The potential advantages of this approach are that it would be relatively simple to drive a large number of gates and that an operating frequency > 1 GHz would be possible (the latter would probably be useful only with an SCPT pump). When this approach was tried with an SET pump on GaAs, the measured current was only about 1/3 of the expected value [42,43]. Although there was reason to expect

improved performance with changes to device design, further experiments were not pursued, possibly because a conventional SET pump with a relative error $< 10^{-6}$ was demonstrated at about the same time.

The second approach to a SETSAW current source involves an SET system that is quite different from the tunnel junctions I have described thus far [44,45]. The starting material for this approach is a two-dimensional electron gas (2DEG) in a heterostructure of GaAs/AlGaAs similar to those used to make quantum Hall devices. The 2DEG is confined, by etching or by depletion from surface gates, to a channel $\sim 0.1 \mu\text{m}$ wide and $\sim 1 \mu\text{m}$ long between two wide reservoirs, as illustrated in Fig. 10a. This channel is squeezed beyond the point where current can flow through it, so that there is an energy barrier for electrons to travel from one reservoir to the other. When the SAW is excited by a transducer far to the left of the channel, it creates a modulation potential moving from left to right that is superposed on the energy barrier. If the amplitude of the SAW is large enough, each period of the modulation potential creates a small well that can carry one electron over the energy barrier, as illustrated in Fig. 10b. For a SAW wavelength of $\sim 1 \mu\text{m}$, these wells are small enough that the repulsion between electrons ensures that only one electron can occupy each well. The current induced through the channel by a SAW of frequency f is then $I = ef$. The speed limit for this approach is not precisely known, but is likely to be $\sim 10 \text{ GHz}$ or higher.

While the SETSAW devices described here may be able to deliver an accurate steady-state current, they cannot deliver a certain number of electrons with an uncertainty of $\sim 10^{-8}$ because the SAW cannot be switched on and off quickly enough. Thus they are not suitable replacements for the SET pump that is used in the capacitance standard described in Section 3.1.

The SETSAW current standard in a 2DEG has been pursued for several years at the University of Cambridge, with more recent involvement by NPL. The current due to the SAW is usually plotted as a function of the voltage that squeezes the channel, and a plateau like that shown in Fig. 10c is observed. To provide a useful current standard, the plateau must be flat over a finite range of voltage (and of temperature, SAW amplitude, and other parameters) and it must occur at the value $I = ef$. The most impressive plateau reported to date has a region where I is constant at 0.5 nA to within the noise of $\sim 0.05 \text{ pA}$, corresponding to a relative statistical uncertainty of $\sim 1 \times 10^{-4}$, extending over a range of 0.5 mV in the squeezing voltage [46]. The total uncertainty of the current on this plateau was 7×10^{-3} , the nominal relative uncertainty of the commercial ammeter that was used. Recent measurements have been done at NPL with a total uncertainty of 3×10^{-5} , but none of the devices used for this work displayed a truly flat plateau [47]. Defining the SETSAW current as the inflection point of the current versus squeezing voltage curve, it was found that this current differed from ef by about 1.5×10^{-4} . The NPL measurements showed that this difference becomes larger as the plateau becomes less flat, implying that a device with a truly flat plateau should produce the expected value of current.

Researchers at NPL have also explored the effect of heating in SETSAW devices [47]. The results indicate that the temperature of the electrons in the 2DEG is elevated well above the cryostat temperature of about 1 K , possibly as high as 20 K in the smallest devices. Although some of this heating comes from losses at the transducers and can be reduced by better impedance matching, the dominant effect occurs within the SETSAW device itself and will be difficult to reduce. One potential solution is to fabricate a composite

structure in which the 2DEG material is placed on top of a substrate with a much stronger piezoelectric effect than GaAs, for example LiNbO₃. The stronger coupling between the lattice and the 2DEG would allow a SAW with a smaller amplitude, and thus less loss, to provide the required modulation of the energy barrier in the 2DEG channel.

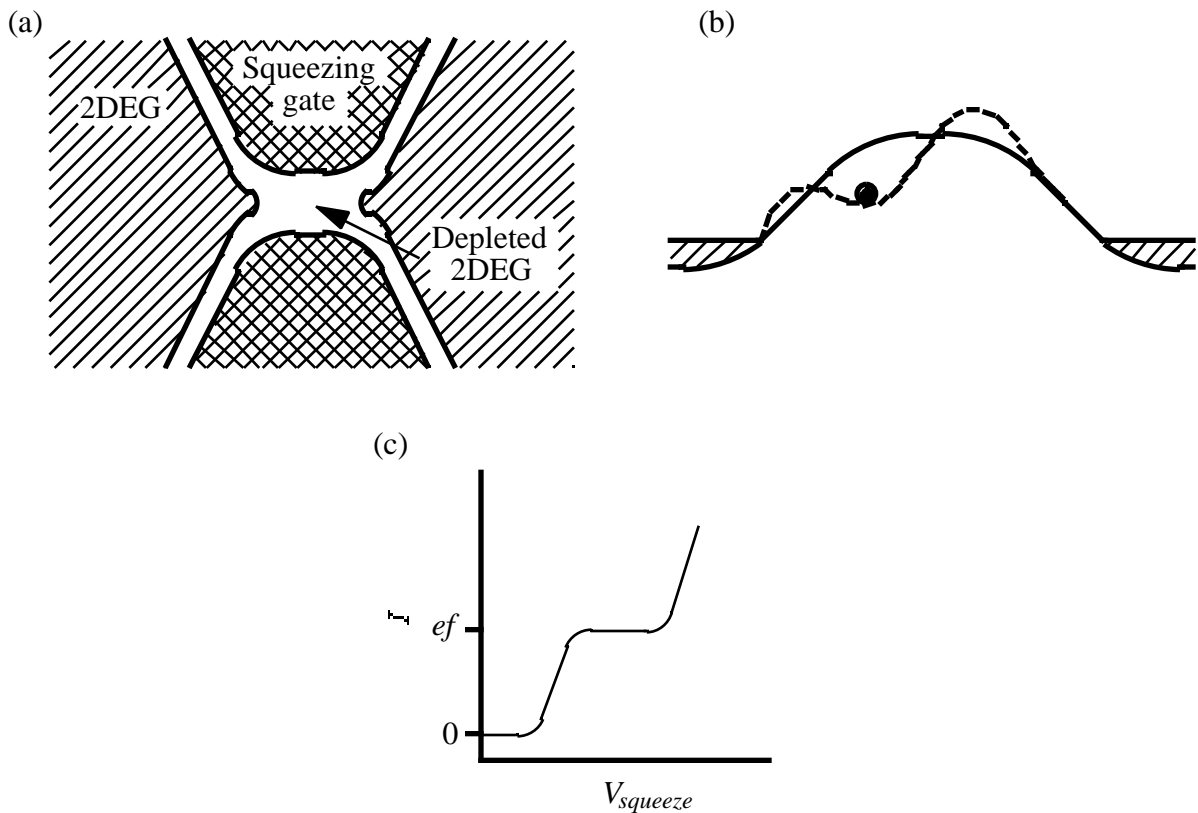


Fig. 10 Passing electrons through a SETSAW device. (a) Schematic geometry of the device, showing the narrow channel of depleted 2DEG through which the current flows. (b) Schematic energy profile along the horizontal axis of the channel in (a). Without the SAW (solid line), there is a large energy barrier between the 2DEG reservoirs. With the SAW (dotted line), a travelling well carries one electron over this barrier for each period of the SAW. (c) Schematic curve of current vs. squeezing voltage.

SETSAW projects have recently begun at PTB [48] and at DFM, and the expanding effort should further our understanding of the potential for a current source in the range of 1 nA with the uncertainty required by metrological applications.

3.2.6 RF-SETT for passive electron counting

The devices described previously in this section are designed to *generate* an accurate current, but it may also be possible to simply *observe* individual charges as they flow through a device. A promising system for such a scheme is a long array of tunnel junctions in which charge flows in the form of solitons

with regular spacing [49]. An electrometer can detect the passage of each soliton if it is fast enough and has low enough noise. Fast enough means that it must have a bandwidth larger than the average rate at which solitons pass. Since $1 \text{ pA} \approx 6 \times 10^6 \text{ e/s}$, the bandwidth should clearly be at least 10 MHz. The noise requirement can be calculated as follows. For an electrometer with an input charge noise spectral density S_q and a bandwidth B , the standard deviation of the input charge is $\sqrt{S_q B}$. The electrometer must not be coupled too strongly to the array, to avoid perturbing the solitons, so I assume the signal from a passing soliton is $\sim 0.1e$. Assuming that the probability of missing a soliton has a normal distribution, a standard uncertainty of 1×10^{-8} in the measured current requires that the signal be ~ 5.5 times larger than the standard deviation, *i.e.*, $0.1e/\sqrt{S_q B} \approx 5.5$. To passively count every electron in a current of 1 nA with an uncertainty of $\sim 10^{-8}$ thus requires a device with $B \sim 6 \text{ GHz}$ and $\sqrt{S_q} \sim 2 \times 10^{-7} \text{ e}/\sqrt{\text{Hz}}$.

An ordinary SETT is limited to a bandwidth $1/2 RC \sim 1 \text{ kHz}$, where $R \sim 100 \text{ k}\Omega$ is the tunnel junction resistance and $C \sim 1 \text{ nF}$ is the stray capacitance of the lead running to the first voltage amplifier at room temperature. This has been improved to $\sim 1 \text{ MHz}$ using various cryogenic amplifier schemes [50–52], but the real hope for passive electron counting lies in a more radical modification shown in Fig. 11. Borrowing ideas from SQUID technology, Schoelkopf *et al.* [53] have integrated the SETT into a resonant circuit to create a device called the RF-SETT. Instead of measuring the output current or voltage, the signal from the RF-SETT is detected by monitoring the damping of the resonant circuit. The first version of this device demonstrated a bandwidth of more than 100 MHz and a noise of $\sqrt{S_q} \sim 10^{-5} \text{ e}/\sqrt{\text{Hz}}$ at 1 MHz [53]. An optimized version is expected to have a bandwidth of $\sim 1 \text{ GHz}$ and a noise of $\sqrt{S_q} \sim 10^{-6} \text{ e}/\sqrt{\text{Hz}}$, which would allow $\sim 0.1 \text{ nA}$ to be measured with an uncertainty of $\sim 10^{-8}$. There are other questions that I have not addressed, such as possible errors due to the statistical nature of the soliton transport itself, but the RF-SETT is certainly a promising step toward an SET current standard based on passive electron counting. This approach is also part of the European Count project.

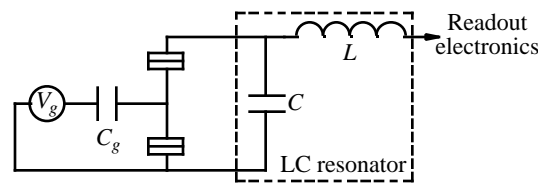


Fig. 11 The RF-SETT, whose operation is described in the text.

3.3 Parallelization of SET devices

In most quantum metrology triangle schemes proposed to date, the current required to enable an uncertainty of 10^{-8} is roughly 1 nA to 10 nA. Aside from the SCPT pump and the SETSAW current source, which have yet to demonstrate an uncertainty of 10^{-8} , such a current can be achieved only by putting many SET current sources in parallel. The technical difficulty of this approach can be illustrated by considering the challenges involved in the case of the conventional SET pump.

Assuming that individual 7-junction pumps could operate 10 times faster and deliver 10 pA each, an array of 100 in parallel could deliver 1 nA. The first challenge is to fabricate 700 working junctions with

reasonably uniform parameters, along with gate lines to each of 600 islands. The NIST fabrication process currently has a yield of about 50% for chips with 9 junctions (one pump and one electrometer), so this would require a tremendous improvement in fabrication. This daunting task is surely the main reason why parallel SET devices have not been pursued to date. However, it is instructive to continue under the assumption that this can be done because if it is done there will be other issues to confront.

The second challenge is to adjust the dc voltage on each of the 600 gate lines in order to cancel the background charges described in Section 2. As was mentioned there, the background charges fluctuate in time, and even after one month at temperatures below 100 mK it is still necessary to tune the gate voltages about once a day. For the NIST pumps, these voltages are tuned by increasing the bias on a single gate until the error rate increases noticeably, decreasing the bias on the same gate until the error increases again, and then using the average of the two threshold values as the optimal setting for that gate [24]. There are several technical aspects to this challenge, and experience with the NIST pumps suggests that tuning an array of 100 pumps is feasible with somewhat optimistic assumptions, but tuning an array of 1000 pumps does not appear to be feasible. The main difficulties are detecting single error events from individual pumps in the array and the total time required to tune the array.

If SETSAW devices or R-SET devices with 3 junctions can achieve the required uncertainty, some of the difficulties mentioned above would be reduced. It is important to note that the SET capacitance would benefit from even a modest increase in current, so parallelization on a modest scale may be worth pursuing even if the currents required by the quantum metrology triangle seem unlikely to be reached.

4. IMPACT ON FUNDAMENTAL CONSTANTS AND THE SI

Fundamental electrical metrology is in a rather unusual situation at the moment because it effectively has two sets of units. This fact, which is not widely recognized outside the metrology community, is crucial to understanding how SET devices can contribute to our knowledge of fundamental constants and potentially affect the *Système International d'Unités* (SI). Thus I begin this section with a brief explanation of the current situation in order to provide the necessary background for the rest of the discussion.

Prior to 1990, the SI units of voltage and resistance were maintained at the various NMIs by means of artifact standards that were compared with realizations of the SI volt and ohm. Realization experiments, which must strictly conform to the definitions of the SI [54], are difficult and time consuming for these quantities, and thus are performed only occasionally. When quantum standards based on the Josephson effect and the quantum Hall effect became available in the 1970s and 1980s, it was realized that the artifact standards were not nearly as stable and reproducible as the new quantum standards, and in fact inter-NMI comparisons of artifacts revealed significant discrepancies in some cases. In 1990 the constants K_J and R_K were assigned conventional values, thus establishing a conventional unit of voltage based on the Josephson effect and a conventional unit of resistance based on the quantum Hall Effect [55,56]. The conventional values, denoted K_{J-90} and R_{K-90} , were chosen in order that the new units (V_{90} and Ω_{90}) would be as close as possible to the SI volt and ohm (V and Ω) [57]. The values of K_{J-90} and R_{K-90} are *exact*, and the possible

differences from the SI values of K_J and R_K are expressed as uncertainties in the ratios V_{90}/V and Ω_{90}/Ω , as shown in Eq. 2.

$$\begin{aligned} K_{J-90} &= 483\,597.9 \frac{\text{GHz}}{\text{V}} & \frac{V_{90}}{V} &= 1 \pm 4 \times 10^{-7} \\ R_{K-90} &= 25\,812.807 \Omega & \frac{\Omega_{90}}{\Omega} &= 1 \pm 2 \times 10^{-7} \end{aligned} \quad (2)$$

Conversions between the two sets of units are easily accomplished using the relations $V_{90}/V = K_{J-90}/K_J$ and $\Omega_{90}/\Omega = R_K/R_{K-90}$.

Since 1 January 1990, all NMIs have used V_{90} and Ω_{90} as practical units of voltage and resistance for their calibrations. It is important to note that the definitions of the SI volt and ohm have not changed in any way, nor have these units been replaced by the 1990 volt and ohm. The 1990 agreement simply created a second set of units for voltage and resistance in order to facilitate dramatic improvements in the consistency of inter-NMI comparisons of these quantities. Consistency with the other units of the SI, in particular the equivalence of electrical and mechanical power, was provided at the level of the uncertainties in Eq. 2, but the irreproducibilities of the 1990 volt and ohm are considerably smaller than these uncertainties.

4.1 Choice of units for SET standards

Although the adoption of V_{90} and Ω_{90} was motivated by practical concerns, namely a desire for better international consistency in measurements of voltage and resistance, it also created the potential for more far-reaching impacts. Consider the combination

$$\frac{2}{K_{J-90} R_{K-90}} = 1.602\,176\,491\,6\dots \times 10^{-19} \text{ C } e_{90}, \quad (3)$$

which shows that the exact values adopted for K_{J-90} and R_{K-90} imply an exact value for the electron charge. We can define a 1990 coulomb, related to the SI coulomb by $C_{90}/C = e/e_{90}$, and this is clearly a natural unit of charge to be used with SET standards. As I will now describe, SET standards (like other standards based on fundamental constants) can provide a value in either SI or 1990 units, and some interesting relations can be found when this flexibility is exploited.

Consider an SET current standard that has a negligible experimental uncertainty, say $< 10^{-9}$. The current is $I_{\text{SET}} = ef$, where f is in Hz and e must be given in a particular system of units. In SI units, we have $I_{\text{SET}}^{(\text{SI})} = ef$ with $e = 1.602\,176\,462(63) \times 10^{-19} \text{ C}$ from the 1998 adjustment (with a relative uncertainty of 3.9×10^{-8}) [56]. In 1990 units, we have $I_{\text{SET}}^{(1990)} = e_{90} (C_{90}/C) f$ with e_{90} given by Eq. 3 (with no uncertainty).

Taking for example a frequency $f = 10 \text{ MHz}$, we find

$$\begin{aligned} I_{\text{SET}}^{(\text{SI})} &= 1.602\,176\,462(63) \text{ pA} \\ I_{\text{SET}}^{(1990)} &= 1.602\,176\,491\,6\dots \text{ pA}_{90} \end{aligned} \quad (4)$$

In other words, the current from the same SET current standard can be expressed either in A, with a nonexperimental uncertainty coming from the value of e , or in A_{90} , with zero nonexperimental uncertainty because e_{90} is a defined constant. Computing the relative difference between the two values we find

$$\frac{I_{\text{SET}}^{(1990)} - I_{\text{SET}}^{(\text{SI})}}{I_{\text{SET}}^{(1990)}} = 1.8 \times 10^{-8}. \quad (5)$$

Since this is smaller than the current uncertainty in $I_{\text{SET}}^{(\text{SI})}$, the SI and 1990 values do not differ significantly. However, if the uncertainty in e decreases by more than a factor of 2, as it almost certainly will, the existence of a current standard with sufficiently small experimental uncertainty would force a choice between units. Clearly it would be desirable to use A_{90} so that the value of the SET current standard would be consistent with the 1990 voltage and resistance standards, but this would mean moving the practical basis for yet another electrical quantity outside the SI.

Now consider an SET capacitance standard with an experimental uncertainty of $< 10^{-9}$. The capacitance is $C_{\text{SET}} = Ne/V$, where V is necessarily measured in terms of V_{90} because standards in terms of V with the required uncertainty are not available. To express the value in SI units, we express e in terms of the fine-structure constant α and K_J (assuming $K_J = 2e/h$) as $e = (4\alpha/\mu_0 c)(1/K_J)$, where μ_0 is the permeability of free space and c is the speed of light in vacuum. We also use the conventional notation $x = \{x\}_s X_s = \{x\}_{s'} X_{s'}$ to express the value of a quantity x as the product of a dimensionless numerical value $\{x\}$ and a unit X in any system of units s or s' . This yields

$$C_{\text{SET}}^{(\text{SI})} = \frac{Ne}{V^{(\text{SI})}} = \frac{Ne}{\{V\}_{\text{SI}} V} = \frac{N(4\alpha/\mu_0 c)}{\{V\}_{90} K_{J-90} V}. \quad (6)$$

with $\alpha = 7.297\,352\,533(27) \times 10^{-3}$ from the 1998 adjustment (with a relative uncertainty of 3.7×10^{-9}) and μ_0 and c defined constants [56]. In 1990 units, we have

$$C_{\text{SET}}^{(1990)} = \frac{Ne}{V^{(1990)}} = \frac{Ne_{90} (C_{90}/C)}{\{V\}_{90} V_{90}}. \quad (7)$$

Taking for example $N = 10^8$ and $\{V\}_{90} = 10$, we find

$$\begin{aligned} C_{\text{SET}}^{(\text{SI})} &= 1.602\,176\,456(6) \text{ pF} \\ C_{\text{SET}}^{(1990)} &= 1.602\,176\,491\,6\dots \text{ pF}_{90} \end{aligned} \quad (8)$$

Thus the capacitance from the SET standard can be expressed either in F, with a nonexperimental uncertainty coming from the value of α , or in F_{90} , with zero nonexperimental uncertainty because e_{90} is a defined constant. Computing the relative difference between the two values we find

$$\frac{C_{\text{SET}}^{(1990)} - C_{\text{SET}}^{(\text{SI})}}{C_{\text{SET}}^{(1990)}} = 2.2 \times 10^{-8}. \quad (9)$$

In this case the difference is about 6 times *larger* than the uncertainty in $C_{\text{SET}}^{(\text{SI})}$. Thus an SET capacitance standard with sufficiently small experimental uncertainty would force a choice between expressing the result in terms of F or F_{90} .

The situation for capacitance has another important component that does not play a role in the case of voltage, resistance, or current. When V_{90} and R_{90} were established, there were no SI standards of V and R with comparable uncertainty, and no such standards are expected in the foreseeable future. Thus the only way to allow voltage and resistance metrology to proceed with uncertainties of $\sim 10^{-9}$ is to adopt the conventional units. However, in the case of capacitance there exists a standard for the SI farad, the calculable capacitor, the NIST version of which has an uncertainty of 1.9×10^{-8} [58]. This is comparable to the relative difference in Eq. 9. Thus one can imagine a situation in the relatively near future where two units for capacitance can be realized with an uncertainty of say 1×10^{-8} but differ from each other by 2.2×10^{-8} . We

would then have two options for expressing the value of a particular capacitor. (1) We could use the 1990 farad, which is by definition consistent with other electrical measurements that can be done with an uncertainty of 1×10^{-8} , but which has no ongoing link to the SI that would allow it to reflect improvements in our knowledge of fundamental constants (significant improvements have already occurred since 1990). (2) We could use the SI farad, which is by definition consistent with the nonelectrical SI units but is not consistent with V_{90} and Ω_{90} .

Of course, the difference between 1990 and SI units is not fundamental. It is simply a consequence of the fact that the conventional values adopted in 1990 were not perfect, as well as the remarkable progress in fundamental constants and SET standards that may soon make that imperfection apparent in real experiments. The difference between F_{90} and F (and between A_{90} and A) could be temporarily eliminated by choosing new conventional values for K_J and R_K in order to make the value of the electron charge in Eq. 3 equal to the accepted SI value at some point in time. In other words, we could do the same thing for the Josephson and quantum Hall standards that we have always done for artifact standards: when we notice that their values no longer agree with SI values, we adjust their values. In the case of artifacts, the need for adjustment is due to drifts in the artifacts, while in the case of quantum standards it is due to evolution in our knowledge of the fundamental constants and the SI values of quantities that involve these constants. Such adjustments should not be made unless circumstances make it necessary, or at least clearly advantageous, but it appears that the SET capacitance standard may bring about such circumstances within the next decade.

4.2 The quantum metrology triangle

The three quantum electrical standards can be described by the following relations:

$$U_J = \frac{n_J f_J}{K_J} \quad (10)$$

$$R_H = \frac{R_K}{i} \quad (11)$$

$$I_{SET} = Q_X f_{SET}, \quad (12)$$

where n_J and i are integers describing the step number and plateau number of the Josephson and quantum Hall devices, respectively. When focusing on fundamental considerations, it is important to note that these are empirical relations, *i.e.*, we observe that, for example, voltage from Josephson effect devices obeys Eq. 10 with a phenomenological constant K_J that appears to be universal. Of course, theory predicts that $K_J = 2e/h$, $R_K = h/e^2$, and $Q_X = e$ on quite general grounds, and to date no experiments have contradicted these predictions. However, from a fundamental point of view the relations between the phenomenological constants and h and e are still open to question. This issue can be divided into two parts, which I will discuss for the case of K_J . The cases of R_K and Q_X are analogous.

We can first ask "Is K_J really identical to $2e/h$, *i.e.*, can we rewrite Eq. 10 as $U_J = (n_J f_J) / (2e/h)$?" There are two types of information relevant to this question. The first type is tests of the universality of K_J . These include comparisons of Josephson effect devices with different parameters and comparisons of complete metrology systems [55]. They tell us that any deviation from universality is no larger than 1×10^{-10} and possibly as small as 3×10^{-19} . This universality is consistent with $K_J = 2e/h$, but does not prove it. The

second type is the test made by Taylor and Cohen in 1991 [59], which used the results of various measurements of fundamental constants to calculate the best *independent* values of K_J and $2e/h$ in SI units. This test showed a possible discrepancy between K_J and $2e/h$ at the level of 2×10^{-7} , but the significance of this result is unclear because the set of input data for the calculation was not very robust [59]. It would perhaps be useful to repeat this test on the more recent input data used for the 1998 adjustment of the fundamental constants.

We can also ask "What is the value of K_J in SI units?" There are two types of information relevant to this question as well. The first type is experiments that give a direct SI value of K_J , where "direct" means without assuming $K_J = 2e/h$. The second type is experiments that give a value of $2e/h$ in SI units, combined with the assumption that $K_J = 2e/h$. These values of K_J are referred to as "indirect". Examples of experiments that give direct and indirect SI values of K_J are described in [55].

The quantum metrology triangle can contribute information relevant to the questions above in two distinct ways, depending on what assumptions are made [60]. In the first case, we assume that the theoretical predictions $K_J = 2e/h$, $R_K = h/e^2$, and $Q_X = e$ are correct. Then the quantum triangle experiment gives [61]

$$\begin{aligned}
 U_J &= R_H I_{\text{SET}} \\
 \frac{n_J f_J}{2e/h} &= \frac{h/e^2}{i} e f_{\text{SET}} \\
 \frac{f_{\text{SET}}}{f_J} &= \frac{n_J i}{2}
 \end{aligned} \tag{13}$$

Since the frequencies used to drive the SET and Josephson effect devices can be known with negligible uncertainty, this equation is equivalent to $1 = 1$. If a real experiment shows that the quantum metrology triangle closes with an uncertainty of $\sim 1 \times 10^{-8}$, it will enhance our confidence in the three theoretical predictions (neglecting the possibility of cancelling errors). On the contrary, if a real experiment clearly shows that the quantum triangle is not closed, we will suspect that one of the predictions is not valid, but of course we will not know which one is wrong without further tests.

In the second case, we do not assume the three predictions but simply retain the phenomenological constants in the equations. Then the quantum triangle experiment gives

$$\begin{aligned}
 U_J &= R_H I_{\text{SET}} \\
 \frac{n_J f_J}{K_J} &= \frac{R_K}{i} Q_X f_{\text{SET}} \\
 K_J R_K Q_X &= \frac{n_J f_J i}{f_{\text{SET}}}
 \end{aligned} \tag{14}$$

This amounts to a direct value of the product $K_J R_K Q_X$, with the important feature that the value is dimensionless and thus the same in any system of units. This expression could be useful as an additional distinct "observational equation" in the type of analysis conducted by Taylor and Cohen [59].

It is also possible to create a quantum metrology triangle using ac impedances, which one might call the ac version of Ohm's law. For example, consider an experiment in which the value of a capacitor is determined using an SET capacitance standard and a capacitance derived from the quantum Hall effect. The quantum Hall resistance can be related to an ac resistance at 1 kHz either by operating the quantum Hall

device at ac or by using a resistor with a calculable ac/dc difference. This ac resistance can be compared to the impedance of a capacitor using a quadrature bridge, so that we can define $C_{\text{QHE}} = i/\omega R_K$. Comparing this with C_{SET} using a capacitance bridge would give [61]

$$\begin{aligned} C_{\text{SET}} &= \frac{NQ_X}{U_J} = C_{\text{QHE}} = \frac{i}{\omega R_K} \\ NQ_X \frac{K_J}{n_J f_J} &= \frac{i}{\omega R_K} \\ K_J Q_X R_K &= \frac{in_J f_J}{\omega N} \end{aligned} \quad (15)$$

This is a quantum metrology triangle in the same sense as Eq. 14 and offers the same information but with quite different systematic uncertainties. In particular, this scheme is not directly limited by the small current produced by the SET pump, since the charge from the pump is placed onto a capacitor where it generates a large voltage.

Finally, other interesting relations can be found by combining two or more fundamental metrology experiments in various ways [60]. If we determine the value of a capacitor using both an SET capacitance standard and a calculable capacitor, and if we use the phenomenological constant Q_X in Eq. 12, we find

$$\begin{aligned} C_{\text{SET}} &= \frac{NQ_X}{U_J} = C_{\text{calc}} = \frac{\epsilon_0 \ln 2}{\pi} L \\ NQ_X \frac{K_J}{n_J f_J} &= \frac{\epsilon_0 \ln 2}{\pi} L \\ K_J Q_X &= \frac{\epsilon_0 \ln 2}{\pi} L \frac{n_J f_J}{N} \end{aligned} \quad (16)$$

If we assume that $K_J = 2e/h$ and $Q_X = e$, this expression gives a value of $K_J Q_X = 2e^2/h = 4/\mu_0 c$ in SI units, which is equivalent to a determination of μ_0 [62]. If we do not assume these things, the expression gives a direct determination of the product $K_J Q_X$ in SI units, as well as another distinct observational equation [59]. Also, if we combine Eq. 16 with Eq. 14 we find

$$R_K = \frac{n_J f_J i}{f_{\text{SET}}} \frac{\pi}{\epsilon_0 \ln 2} \frac{N}{Ln_J f_J}, \quad (17)$$

which gives a direct value of R_K in SI units.

5. CONCLUSION

It is difficult to predict which of the various schemes described here will ultimately succeed in having significant impacts on metrology and fundamental constants. This is not surprising, since the physics of single-electron effects has received widespread attention only for the past 15 years, and SET devices first demonstrated the performance needed for fundamental metrology only about 5 years ago. There is still much room for improvement in our basic understanding of the limits on performance of these devices that attempt to transfer individual electrons as fast as possible and with as few mistakes as possible. Given the relative immaturity of SET metrology, the progress reported here is impressive. Furthermore, with the goal

of realizing a quantum metrology triangle as an incentive to push SET metrology to its limits, the promise for future progress is great.

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