Correspondence

Probability Density Function of Power Received in a Reverberation Chamber

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Abstract—**This correspondence provides a new derivation for the power received by an antenna in a reverberation chamber.**

Index Terms—**Maximum entropy, probability density function (PDF), received power, reverberation chamber.**

The probability density function (PDF) for both the power received P_r by an arbitrary antenna [1] and the square of a rectangular field component [2] in a reverberation chamber has been shown to be exponential. In the two cases, the derivation involved showing that the real and imaginary parts of the complex current received by an antenna or a complex rectangular field component are Gaussian, and that the square is χ^2-distributed [3] with 2 DOF. Hence, the PDF is exponential [3]. The purpose of this correspondence is to provide a more direct, maximum-entropy [4] derivation of the PDF that deals only with real received power.

We begin with the known result for the mean power received P_r by an arbitrary antenna in a reverberation chamber [1], [5]

$$P_r = \frac{1}{2}\frac{E_0^2}{\eta}\frac{\lambda^2}{4\pi} \equiv m \tag{1}$$

where E_0^2 is the mean-square electric field, η is the free-space impedance, λ is the free-space wavelength, and the mean value m is introduced to simplify the following derivation. The physical interpretation of (1) is that the mean received power is equal to the power density times the effective area of an isotropic antenna times a polarization mismatch factor of 1/2 [6]. For simplicity we assume that the receiving antenna is 100% efficient and perfectly impedance matched, but loss and mismatch can be accounted for [6].

To implement the maximum-entropy derivation [4], we define the information entropy as [4]

$$-\int_0^\infty f(P_r)\ln[f(P_r)]\mathrm{d}P_r \tag{2}$$

where $f(P_r)$ is the PDF of the received power. The lower limit of the integral in (2) is 0 because the received power must be positive [$f(P_r) = 0$ for $P_r < 0$]. The PDF must satisfy the usual integral constraint

$$\int_0^\infty f(P_r)\mathrm{d}P_r = 1. \tag{3}$$

The mean value m is given by

$$\int_0^\infty P_r f(P_r)\mathrm{d}P_r = m. \tag{4}$$

The maximum-entropy method maximizes the entropy (uncertainty) in (2) subject to the constraints in (3) and (4). This can be done by the method of Lagrange multipliers. The physical reasoning behind this procedure is that our only initial information about the power received in a reverberation chamber is the mean value in (1) or as rewritten in (4). Therefore, we wish to determine $f(P_r)$ in a way that does not introduce any unjustified (or biasing) information.

We begin by writing the Lagrangian L in the following form [4]:

$$L = -\int_0^\infty f(P_r)\ln[f(P_r)]\mathrm{d}P_r - (\lambda_0 - 1)\left[\int_0^\infty f(P_r)\mathrm{d}P_r - 1\right] - \lambda_1\left[\int_0^\infty P_r f(P_r)\mathrm{d}P_r - m\right] \tag{5}$$

where λ_0 and λ_1 are unknown constants (Lagrange multipliers). An extremum (in this case, a maximum) of L can be obtained from the following derivative relation:

$$\frac{\partial L}{\partial f(P_r)} = 0. \tag{6}$$

If we substitute (5) into (6) and perform the differentiation, we obtain

$$\ln[f(P_r)] = -\lambda_0 - \lambda_1 P_r. \tag{7}$$

Equation (7) can be rewritten in exponential form

$$f(P_r) = \exp(-\lambda_0 - \lambda_1 P_r). \tag{8}$$

We can solve for λ_0 and λ_1 by substituting (8) into the constraints (3) and (4)

$$\lambda_0 = \ln(m) \quad \text{and} \quad \lambda_1 = \frac{1}{m}. \tag{9}$$

Substitution of (9) into (8) yields the desired PDF

$$f(P_r) = \frac{\exp(-P_r/m)}{m}. \tag{10}$$

The exponential PDF in (10) agrees with results in [1], [2], and [4]. It has also been confirmed experimentally [5], [7].

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