

# CORRECTION OF SYSTEMATIC ERRORS DUE TO THE VOLTAGE LEADS IN AC JOSEPHSON VOLTAGE STANDARD

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## **Abstract**

NIST recently reported the first application of a quantum ac Josephson Voltage Standard (ACJVS) for calibration of thermal transfer standards in the 1 kHz to 10 kHz frequency range. This paper describes preliminary work on extending its frequency calibration range up to 100 kHz by correcting systematic errors due to voltage leads.

## **Introduction**

During the last twelve years, the quantum-based pulse-driven ACJVS has progressed from a concept to a calibration instrument, [1], [2]. The operating principle of such a standard was described in detail in a series of papers [1-6]. Very substantial progress has been achieved recently by identifying and reducing the systematic errors, extending the voltage operating range to almost 300 mV, and extending the frequency range to 100 kHz [6].

The standard ac voltage is produced by either one or two arrays of Josephson junctions, excited by a synchronized combination of a 15 GHz sine wave and a two-level digital bit stream, clocked at 10 GHz. In response to the bipolar drive signal, a properly excited and biased array (“on margins”) generates a bipolar output pulse train, in which the area of each pulse is determined with quantum accuracy. The spectrum of the output pulse train contains low frequency component(s), a sine wave or an arbitrarily shaped time-dependent voltage constructed from harmonics. The desired analog waveforms are converted to a digital binary representation using a delta-sigma modulator algorithm.

The voltage is exactly *calculable* from knowledge of the modulating code, the pulse frequency and the number of Josephson junctions in the array. The voltage appearing at the input of a calibrated instrument (DUT) differs from this calculable value by systematic errors introduced by connections between the junctions and between the arrays and the DUT. The former errors, due, for example, to the inductance of superconductive connections between

the junctions, were investigated in [6]. In the following, we present preliminary investigations of systematic errors due to signals on the voltage leads between the array and the DUT. These frequency-dependent errors can be neglected at 10 kHz and below [2], but must be taken into consideration at higher frequencies.

## **Experimental Test System**

The source of the quantized ac voltage was a chip containing two independent arrays of 5120 junctions, located inside a liquid helium Dewar. Each array could generate a maximum low-frequency rms ac voltage of 110 mV. Two separate twisted pair lines (TPL) made of copper magnet wire, connected the arrays to the cryoprobe room-temperature output terminals. Outputs of the two arrays could be connected in series, doubling the rms output voltage to 220 mV. The calibrated DUT was a commercial, amplifier-aided, ac-dc thermal transfer standard (TTS.) Its input was connected to the output of the cryoprobe through a coaxial cable. The total length of the cable, partially wound on a magnetic core to form a coaxer, was 1.68 m.

**Voltage lead parameters.** The two TPLs, both of an approximate length of 1.30 m, were not identical, probably due to an unequal twisting. Their average parallel capacitance and series inductance, measured at 20 kHz, were 141 pF and 0.63  $\mu$ H. An average series resistance of the TPL, when the array was immersed in the liquid helium was 0.37  $\Omega$ , less than half of its room temperature resistance. Catalog equivalent parameters of the coaxial cable are 101 pF/m, 0.25  $\mu$ H/m, and 0.049  $\Omega$ /m.

## **Results of Simulations and Tests**

**Output Spectrum.** Due to the digital nature of the ACJVS voltage synthesis, the useful low-frequency output is accompanied by high frequency digitization harmonics. These harmonics are significantly attenuated by the on-chip, low-pass filters (LPFs) [7] and the voltage leads. Spectra measured at the cryoprobe output show no digitization harmonics greater than -55 dBm at frequencies below 250 MHz. Fortunately, the ac/dc transfer standard is less

Table 1. SPICE simulations; voltage rise on voltage leads loaded by a TSS.  $\delta V1$ : single array, TPL only;  $\delta V2$ : single array, TPL + coaxial cable;  $\delta V3$ : single array, LPF + TPL + coaxial cable;  $\delta V4$ : two arrays in series, TPLs + coaxial cable.

Frequency kHz	$\delta V1$ $\mu V/V$	$\delta V2$ $\mu V/V$	$\delta V3$ $\mu V/V$	$\delta V4$ $\mu V/V$
10	0	1	-5.1	1.5
20	1.5	4	-5.8	6
50	10	23	5.6	37
70	19	46	21.7	72
100	39	94	58.9	147

sensitive to high frequencies. Its frequency characteristic above 1 MHz corresponds approximately to a first-order low-pass filter with a 10 MHz, -3 dB corner frequency.

In the most accurate measurements below 10 kHz an additional 3 MHz Butterworth LC filter was used [4]. However, in the tests reported here we either used no filter or a first order RC LPF (-3 dB at 15 MHz).

**SPICE simulations.** The systematic errors associated with the voltage leads were evaluated using SPICE simulations. The TPLs and the coaxial cable were modeled by five lumped-element T-sections of the R-L-C-L type. The TTS was modeled as a parallel RC circuit whose parameters were experimentally determined including the frequency-dependent input resistance. The results of simulations are shown in Table 1. The column marked  $\delta V1$  shows a relative voltage rise on a TPL loaded by the TTS.  $\delta V2$ ,  $\delta V3$ , and  $\delta V4$  show a voltage rise at the end of the coaxial cable for a single array, with no LC or RC LPF, and for two arrays connected in series with no LPF. The validity of these simulations was confirmed by measurements on a physical model of the leads.

**Experimental results.** Table 2 shows results of ac-dc transfer calibrations of a TSS at 100 mV on a 220 mV range. Results obtained without and with a 15 MHz LPF at the output of the ACJVS, are in the columns marked  $\delta 1$  and  $\delta 2$ . The estimated expanded uncertainty is in column Uc1. The largest component of the uncertainty is the determination of the voltage lead systematic errors, namely  $\delta V2$  and  $\delta V3$  in Table 1. It can be significantly decreased by careful measurements of the lead parameters. On the other hand, no influence of the digitization harmonics was included in the error budget. This TSS was previously calibrated at the National Research Council Canada using a thermal voltage converter (TVC); the results are shown in columns  $\delta 3$  and Uc2. Comparison of the two methods of TSS calibration, i.e. using a TVC or an ACJVS, shows very good

Table 2. Measured ac-dc difference  $\delta$  of a TTS, after applying leads corrections; 100 mV on 220 mV range.  $\delta 1$ : ACJVS, no LPF;  $\delta 2$ : ACJVS, RC LPF;  $\delta 3$ : TVC calibrations at NRC; Uc1: expanded uncertainty of the series of measurements; Uc2: NRC “best uncertainty.”

Freq kHz	$\delta 1$ $\mu V/V$	$\delta 2$ $\mu V/V$	$\delta 3$ $\mu V/V$	Uc1 $\mu V/V$	Uc2 $\mu V/V$
2.5	10.2	10.2	9.3	2	5
10	-10.9	-12.4	-13.3	3	6
20	3.3	1.7	0.7	3	6
50	19.6	15.8	16	9	7
100	15.4	26.7	31	20	7

agreement, particularly when the digitization harmonics are suppressed by a LPF. When two arrays were connected in series to double the rms voltage to 200 mV, there was a disagreement at frequencies above 10 kHz between the expected voltage lead systematic error,  $\delta V4$ , and the measured error. The reasons for this disagreement are under investigation.

### Conclusions

Long leads between a quantum ACJVS and a DUT introduce systematic voltage calibration errors. The paper shows that these errors can be efficiently calculated from the voltage lead parameters and corrected, extending the calibration range of the ACJVS up to 100 kHz.

### References

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