SIM.EM-K1, 1 Ω
SIM.EM-K2, 1 GΩ
SIM.EM-S6, 1 MΩ

RMO COMPARISON FINAL REPORT

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2006 - 2007 Resistance standards comparison between SIM Laboratories
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1. Introduction

The Working Group for Electricity and Magnetism of the Inter-American Metrology System (SIM) initiated the key and supplemental regional comparisons SIM.EM-K1-K2-S6 to provide the first internationally recognized comparisons of precision resistance measurements for nations of the western hemisphere. These comparisons include the official metrology institutes of six members of SIM and follow the guidelines for key comparisons under the 1999 CIPM Mutual Recognition Arrangement (MRA). The National Institute of Standards and Technology (NIST) provided the comparison standards and acted as the pilot laboratory in the SIM.EM-K1-K2-S6 comparisons, which began in January 2006 and were completed in September 2007.

The MRA states that its technical basis is a set of results obtained in a course of time through key comparisons carried out by the Consultative Committees of the CIPM, the BIPM and the Regional Metrology Organizations (RMOs). As part of this process, the CIPM Consultative Committee for Electricity and Magnetism (CCEM) has carried out the key comparisons CCEM-K1 of 1 ohm resistance standards [1] and CCEM-K2 of resistance standards at 10 MΩ and 1 GΩ [2, 3]. Both NIST and the Canadian National Research Council (NRC) participated in CCEM-K1 and CCEM-K2. In parallel with these key comparisons, the BIPM has conducted a bilateral comparison of resistance scaling to 1 Ω, 100 Ω and 10 kΩ with NIST, using a transportable quantum Hall effect (QHE) system [4], and recently conducted a bilateral comparison with NIST at the 1 Ω resistance level, BIPM.EM-K13a [5].

By means of procedures for linking key comparison data [6 – 9], the SIM.EM-K1-K2 comparisons will help to provide assurance of equality in measurements between the nations organized in SIM and the participants in the CCEM key comparisons. The analysis included in this report (Appendix A) specifically provides methods for calculating the degrees of equivalence and their uncertainties between the national measurement standards of the participating laboratories (Appendix B). A subsequent analysis (Appendix G) provides methods and results linking this regional comparison to CCEM key comparison CCEM-K2 and a recent BIPM-NIST bilateral comparison at the 1 Ω level.

2. Traveling standards

2.1 Description of the standards

1 Ω

The 1 Ω resistance level is widely used as a reference in current comparator bridge scaling, and is important either in direct measurement or as a means of assuring ratio accuracy for measurements of resistance below 10 kΩ. Two traveling standards were used:

Thomas type wire-wound resistors. The resistance elements are manganin alloy wire hermetically sealed in double-walled metal containers. The four resistor terminations of the standards are screw-type terminations. The resistors have undisturbed drift rates of less than ±0.10 (μΩ/Ω)/year, and were used as traveling standards for the previous 10 years in the NIST Measurement Assurance Program where they have demonstrated good transport behavior.

The curve describing the temperature coefficient of resistance (TCR) has been measured at various times over many years, most recently in 2005, and has remained constant. The pressure coefficient of resistance (PCR) of these resistors has also been measured, most recently in 2005. The TCRs of the 1 Ω standards are well determined and repeatable, allowing accurate transfer of resistance values using oil bath temperatures in a range of at least 20 °C to 25 °C.

1 MΩ

Comparison at this intermediate resistance level provides assurance of accurate mid-range scaling and measurement techniques. Two traveling standards were used:

Air enclosure film resistors (Fluke* 742A-1M). The commercial-design resistor elements are sealed in a shielded enclosure provided with a grounding termination. Two screw-type resistor terminations are provided, useable with spade lug or banana-plug connectors. The resistors had drift rates of less than ±0.5 (μΩ/Ω)/year in the year prior to the comparison, and almost negligible TCR. The absence of any detectable voltage dependence has been demonstrated in these standards.

1 GΩ

This resistance level is the highest value measured in the CCEM-K2 key comparison and checks high resistance measurements including scaling, guarding, and linearity. Two traveling standards were used:

NIST type film resistors. These resistors have identical design and components to those used in CCEM-K2 key comparison, and were constructed along with those standards in 1996. A new determination of the drift and voltage coefficient of resistance (VCR) has been completed in 2005. The resistance elements are hermetically sealed in metal

* Certain commercial equipment, instruments, or materials are identified in this report to specify the experimental procedure adequately. Such identification is not intended to imply recommendation or endorsement by the National Institute of Standards and Technology, nor is it intended to imply that the materials or equipment identified are necessarily the best available for the purpose.

containers. The two resistor terminations of the standards are coaxial British Post Office connectors mounted on grooved PTFE Teflon circular plates on the top panel of the enclosures. The resistor containers are electrically isolated from the enclosures and electrically connected to the shield of one of the coaxial connectors. This allows the sealed inner container of the standard resistor element to be operated either in floating mode, a grounded mode, or driven at a guard potential.

2.2 Description of the transport package

A heavy plastic shipping container was bolted to a dense plastic pallet, and filled with polyurethane foam for protection of the comparison standards.

The transport case contained the following items:

- Two 1 Ω standard resistors: Leeds & Northrup model 4210
  Serial Number 1779882 (NIST #1002), Serial Number 1779885 (NIST #1005)
- Two 1 MΩ standard resistors: Fluke model 742A-1M
  Serial Number 8409006, Serial Number 8409008
- Two 1 GΩ standard resistors: NIST type HR
  Serial Number HR9104, Serial Number HR9105
- 4 BPO-BNC adapters
- 2 cable assemblies for reading 10 kΩ thermistors installed in 1 GΩ standards
- 2 ambient conditions recorders, Telatemp Micro8000. These recorders were used to monitor the temperature of the standards during transport.
- Instruction manual

3. Comparison strategy

3.1 Measurement instructions

Pre-conditioning: The 1 Ω standards should be installed in a thermostatic mineral oil bath, regulated at the chosen working temperature, at least 48 h before starting the measurements. Air-type standards should be conditioned to air-bath or ambient laboratory conditions for at least 24 h.

Measurand: The resistance value of the traveling standards should be measured at DC, expressed in terms of the conventional value of the von Klitzing constant $R_{K-90} = 25812.807\ \Omega$.

Measurements: The measurements should be repeated at least twice each week during the period allocated to the participating laboratory, approximately three to four weeks. The average value and standard deviation of each set of measurements should be recorded, along with the environmental parameters at the time of measurement.

Preliminary data: At the end of each week of the comparison, the preliminary results should be transmitted electronically (preferably by e-mail) to the coordinators at the pilot laboratory. For each measurement set, the participant's preliminary data sheet should include the date, serial number of the standard, average value, standard deviation, and values of relevant measurement parameters.

3.2 Method of measurement

The measurement method was not specified. It is assumed that every participant laboratory has used its best normal measurement process. The method and the traceability scheme were described in the laboratory's measurement report.

3.3 Quantities to be measured at the time of each test

Standards are identified by the numeral *i*. The test number is identified as x.

1 Ω

$\boldsymbol{R_{i,x}}$ - Resistance of the 1 Ω standard at the following conditions:
$\boldsymbol{J_{i,x}}$ - test current: 50 mA $\leq \boldsymbol{J_{i,x}} \leq$ 100 mA;
$\boldsymbol{T_{i,x}}$ - stirred oil bath temperature (22.5 ± 2.5) °C (T_0 = 25.000 °C);
$\boldsymbol{P_{i,x}}$ - ambient barometric pressure *including the pressure exerted by the oil above the resistor*, (P_0 = 101.325 kPa).

1 MΩ

$\boldsymbol{R_{i,x}}$ - Resistance of the 1 MΩ standards at the following conditions:
$\boldsymbol{V_{i,x}}$ - test voltage: 10 V $\leq \boldsymbol{V_{i,x}} \leq$ 33 V, (V_0 = 33 V);
$\boldsymbol{T_{i,x}}$ - ambient or air bath temperature: (23.0 ± 2.0) °C, (T_0 = 23.00 °C);
$\boldsymbol{RH_{i,x}}$ - ambient relative humidity: (35 ± 20) %.

1 GΩ

$\boldsymbol{R_{i,x}}$ - Resistance of the 1 GΩ standards at the following conditions:

$\boldsymbol{V_{i,x}}$ - test voltage: $\boldsymbol{V_{i,x}} \le 100$ V, ($V_0 = 100$ V);
$\boldsymbol{T_{i,x}}$ - ambient or air bath temperature: (23.0 ± 2.0) °C, ($T_0 = 23.00$ °C);
$\boldsymbol{RH}_{\mathrm{i,x}}$ - ambient relative humidity: (35 ± 20) %.

3.4 Comparison of measurement results

The results were adjusted to account for the differences from T_0, P_0, and V_0 using the TCR, PCR, and VCR of each standard as determined by the pilot laboratory.

1 Ω
The TCR and PCR coefficients and equation of each 1 Ω standard are given below.

TCR for Serial Number 1779882 (NIST #1002):
$\alpha_{1779882} = 2.182 \times 10^{-6}$ per °C
$\beta_{1779882} = -0.5429 \times 10^{-6}$ per °C^2
PCR for Serial Number 1779882 (NIST #1002):
$\gamma_{1779882} = 0.00575 \times 10^{-6}$ per kPa

TCR for Serial Number 1779885 (NIST #1005)
$\alpha_{1779885} = 2.0542 \times 10^{-6}$ per °C
$\beta_{1779885} = -0.5467 \times 10^{-6}$ per °C^2
PCR for Serial Number 1779885 (NIST #1005)
$\gamma_{1779885} = 0.00600 \times 10^{-6}$ per kPa

$$R_{Lix}(T_0, P_0, V_0) = R_{ix} \cdot \left(1 - \alpha_i (T_{ix} - T_0) - \beta_i (T_{ix} - T_0)^2 - \gamma_i (P_{ix} - P_0)\right)$$

1 MΩ
The TCR coefficients and equation of each 1 MΩ standard are given below.

TCR for Serial Number 8409006
$\alpha_{8409006} = 0.020 \times 10^{-6}$ per °C
$\beta_{8409006} = -0.020 \times 10^{-6}$ per °C^2

TCR for Serial Number 8409008
$\alpha_{8409008} = 0.036 \times 10^{-6}$ per °C
$\beta_{8409008} = -0.008 \times 10^{-6}$ per °C^2

$$R_{Lix}(T_0, P_0, V_0) = R_{ix} \cdot \left(1 - \alpha_i (T_{ix} - T_0) - \beta_i (T_{ix} - T_0)^2\right)$$

1 GΩ
The TCR and VCR coefficients and equation of each 1 GΩ standard are given below.

TCR for Serial Number HR9104

$\alpha_{HR9104} = -22 \times 10^{-6}$ per °C

$\beta_{HR9104} = 0.76 \times 10^{-6}$ per °C^2

VCR for Serial Number HR9104

$\delta_{HR9104} = -0.034 \times 10^{-6}$ per V

TCR for Serial Number HR9105

$\alpha_{HR9105} = -30 \times 10^{-6}$ per °C

$\beta_{HR9105} = 0.54 \times 10^{-6}$ per °C^2

VCR for Serial Number HR9105

$\delta_{HR9105} = -0.042 \times 10^{-6}$ per V

$$R_{Lix}(T_0, P_0, V_0) = R_{ix} \cdot \left(1 - \alpha_i (T_{ix} - T_0) - \beta_i (T_{ix} - T_0)^2 - \delta_i (V_{ix} - V_0)\right)$$

3.5 Uncertainty budgets

All of the participants agreed to submit an uncertainty budget for each level of resistance measured, and to use a common set of uncertainty terms. The participants were allowed to submit a revised uncertainty budget for the second round of measurements at their laboratory. The uncertainty budgets were used to calculate the Type A and Type B standard uncertainty terms given in Section 6 of this report and used in the analysis procedure. The definitions of the uncertainty terms are given below. The participants all agreed that some uncertainty terms would be calculated by the pilot laboratory, and these terms are noted below. Where complete knowledge of the degree of freedom for one or more terms is lacking, the participants agreed to assign the total standard uncertainty an infinite degree of freedom.

- **a. Scaling / traceability** describes the influence of the traceability of the values of reference standards.
- **b. Reference standard(s)** describes the influence of the stability and predictability of the reference standard(s).
- **c. Measuring apparatus** describes the influence of the apparatus used to measure the resistance of the comparison standard.
- **d. Ambient conditions** describes the uncertainty of results for temperature, barometric pressure, current and voltage at the time of measurement.
- **e. Standard deviation** describes the statistical measure of standard deviation for measurements that are repeated under identical measurement conditions.
- **f. Repeatability** (calculated by pilot) describes the influence of stability of the standard, estimated from the data reported by the participant in each period and by comparison with models of drift behavior due to thermal shock.
- **g. Corrections applied** (calculated by pilot) describes the uncertainty of corrections for different measurement conditions.

4. Organization

Criteria for participation in the comparison included the availability of adequately trained staff and high-level measurement apparatus and procedures. The expected levels of uncertainty for the participants at each resistance level were: less than 0.5 μΩ/Ω at 1 ohm, less than 5 μΩ/Ω at 1 MΩ, and less than 50 μΩ/Ω at 1 GΩ. The participating institutes are listed in the following table.

Country	Institute	Acronym
Argentina	Instituto Nacional de Technologia Industrial	INTI
Brazil	National Institute of Metrology Standardization and Industrial Quality	INMETRO
Uruguay	Administración Nacional de Usinas y Transmisiones Eléctricas	UTE
Canada	National Research Council	NRC
Mexico	Centro Nacional de Metrologia	CENAM
United States	National Institute of Standards and Technology	NIST

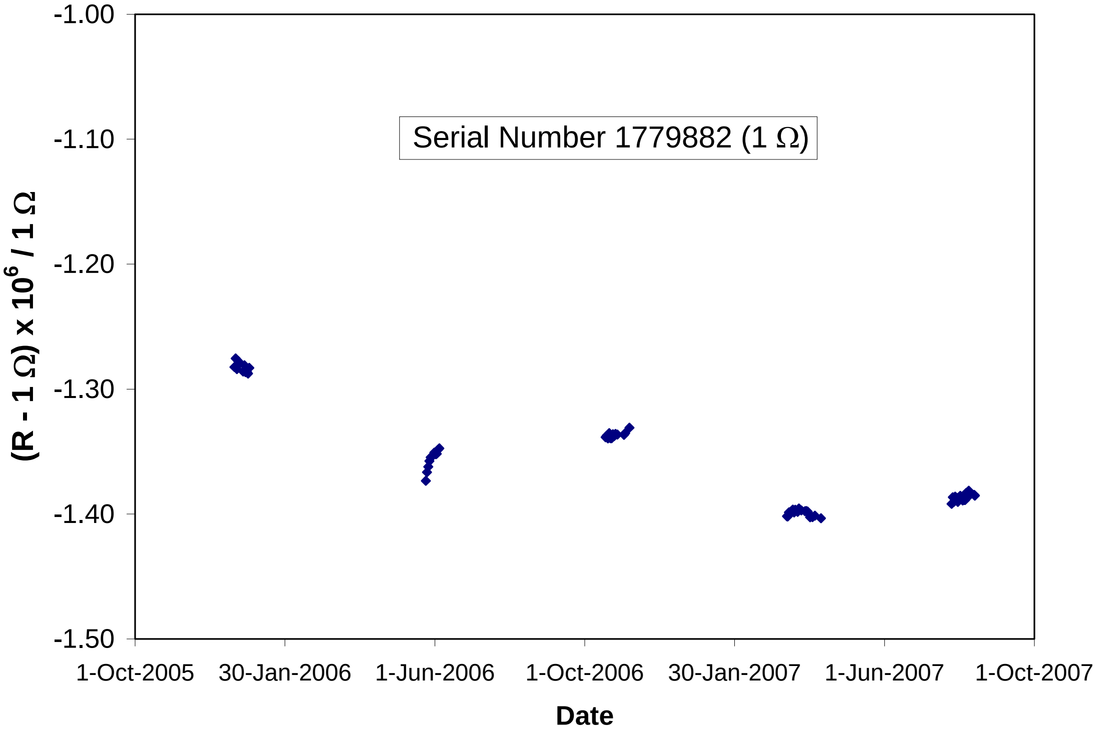
Table 1: Participants

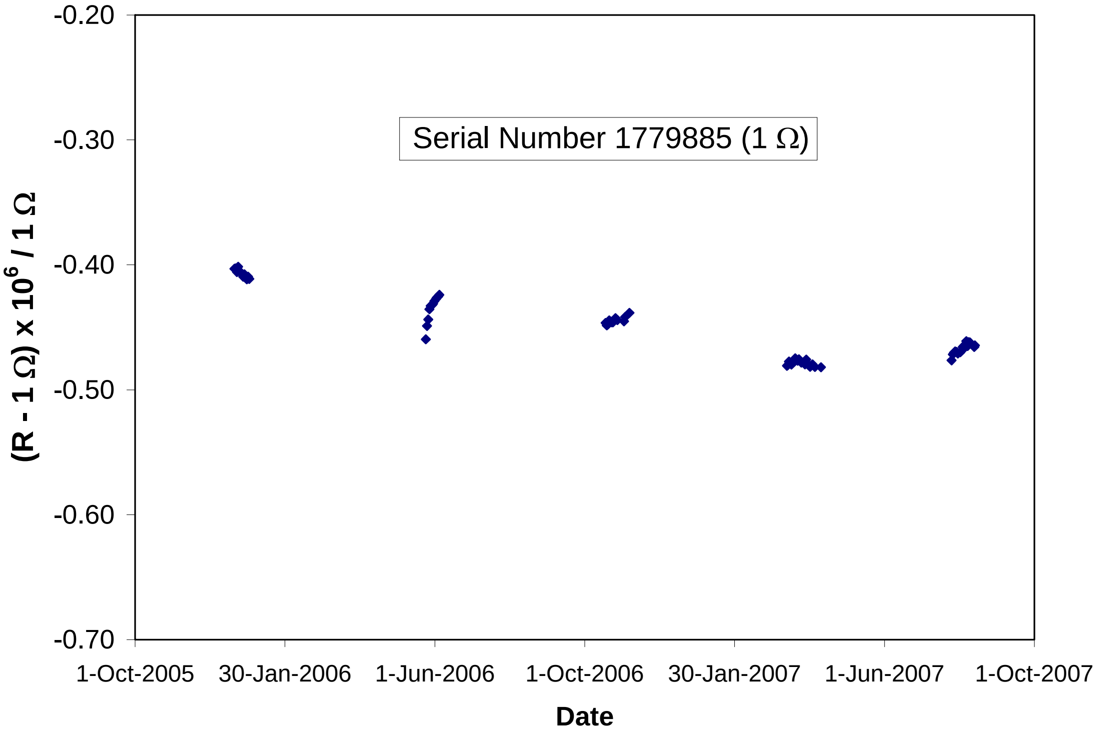
NIST made the initial measurements of the comparison in December 2005. In order to minimize shipping over the great distances between the NMIs, the participants in Argentina, Brazil, and Uruguay comprised one group, and completed their first measurements between January 2006 and April 2006. After NIST measurements in May and June 2006, the participants in Canada and Mexico received the transport package and conducted measurements between June and September 2006. The comparison's first round was completed with NIST measurements in October 2006.

The second round measurements repeated those of the first in the same order, except that the UTE in Uruguay agreed not to participate, due to delays encountered in customs in the first round. NIST shipped the transport package to INTI in November 2006. The package returned to NIST from INMETRO in March 2007. The resistors were shipped to NRC in April 2007, and returned from CENAM in July 2007. The comparison was completed with closing measurements at NIST in July and August 2007.

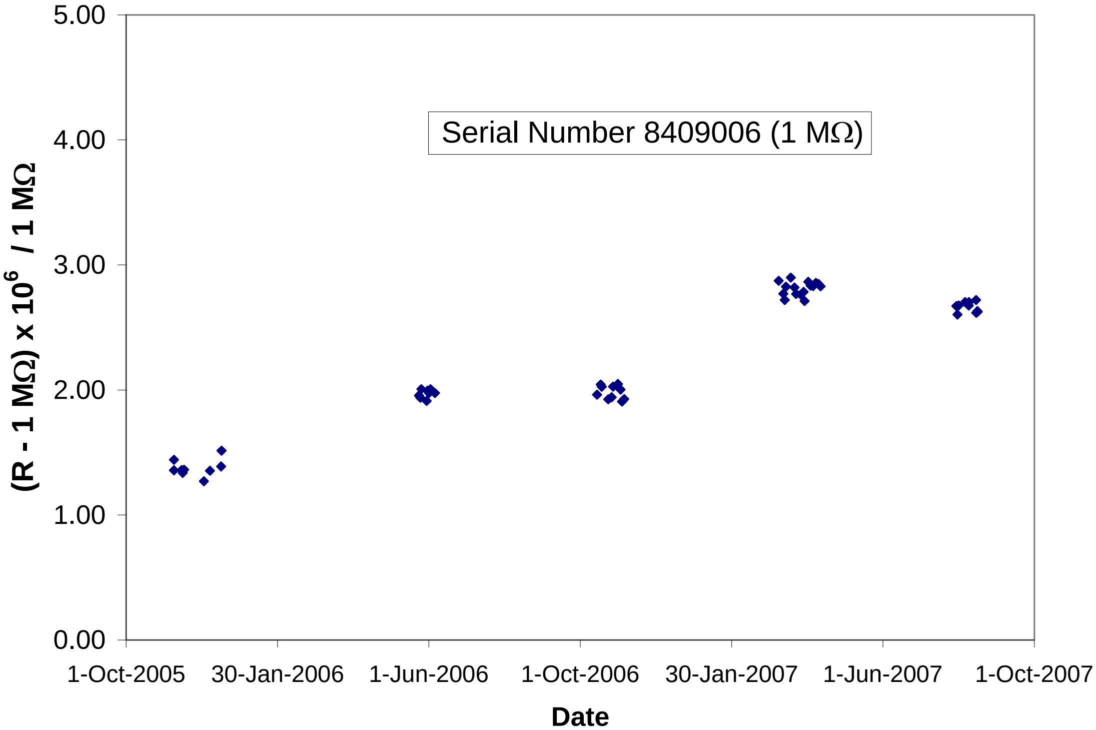
5. Pilot laboratory measurement results

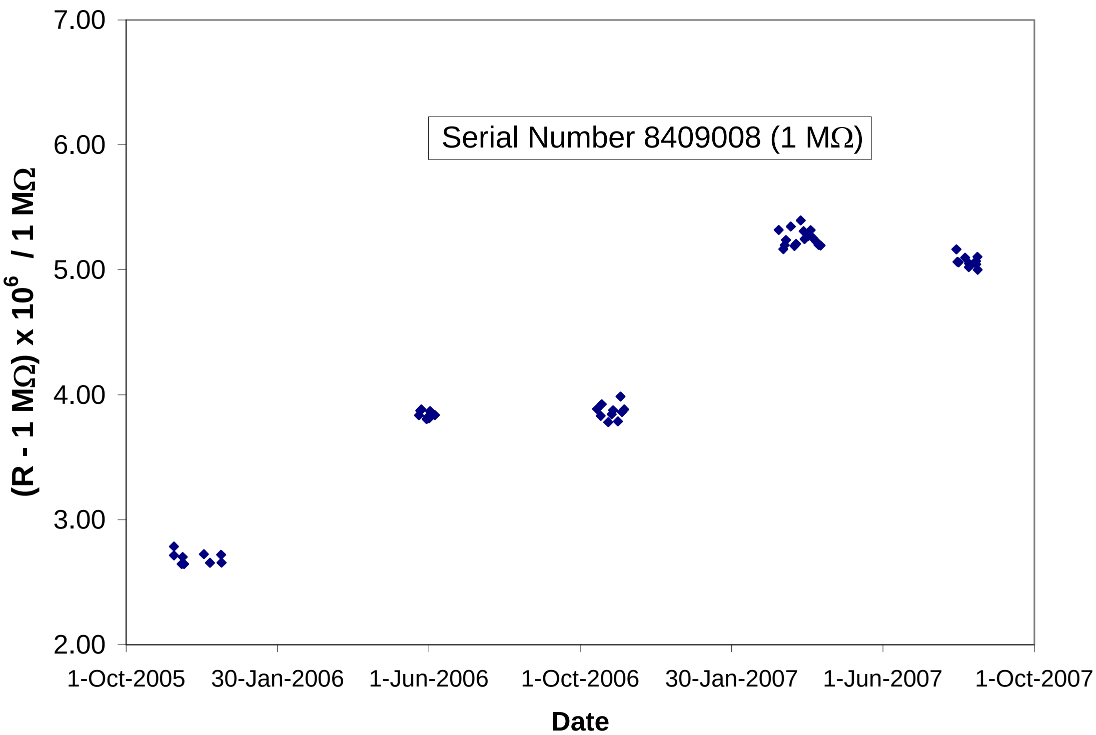
5.1 Pilot results at 1 Ω



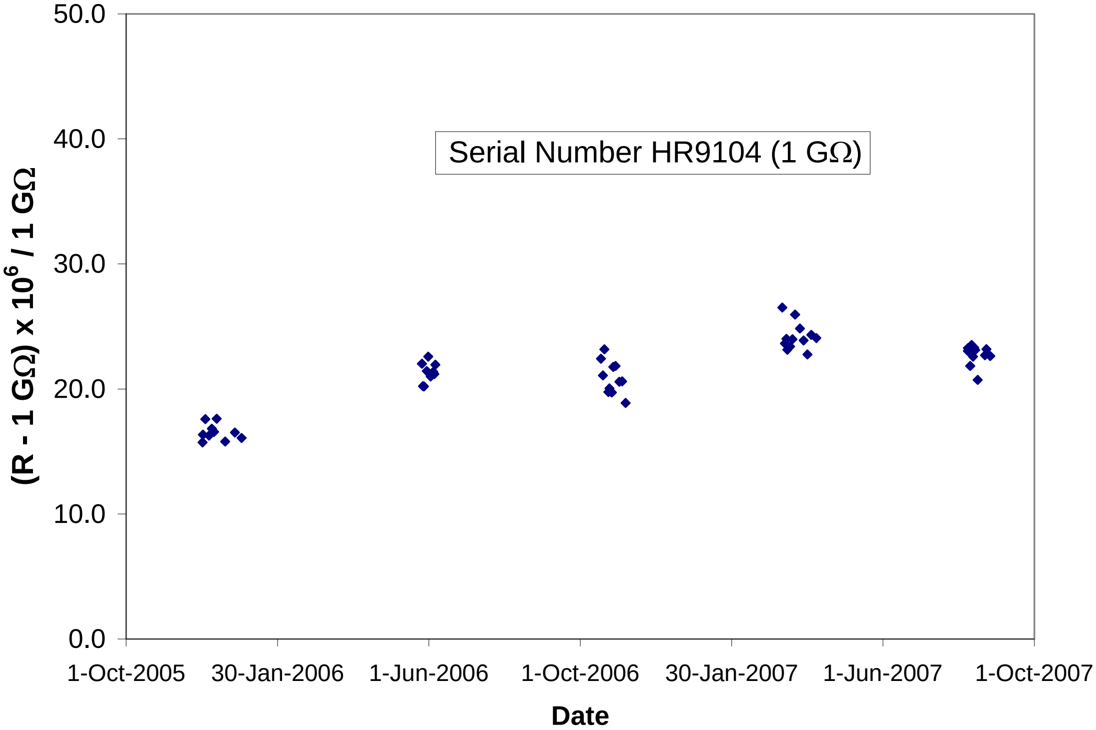


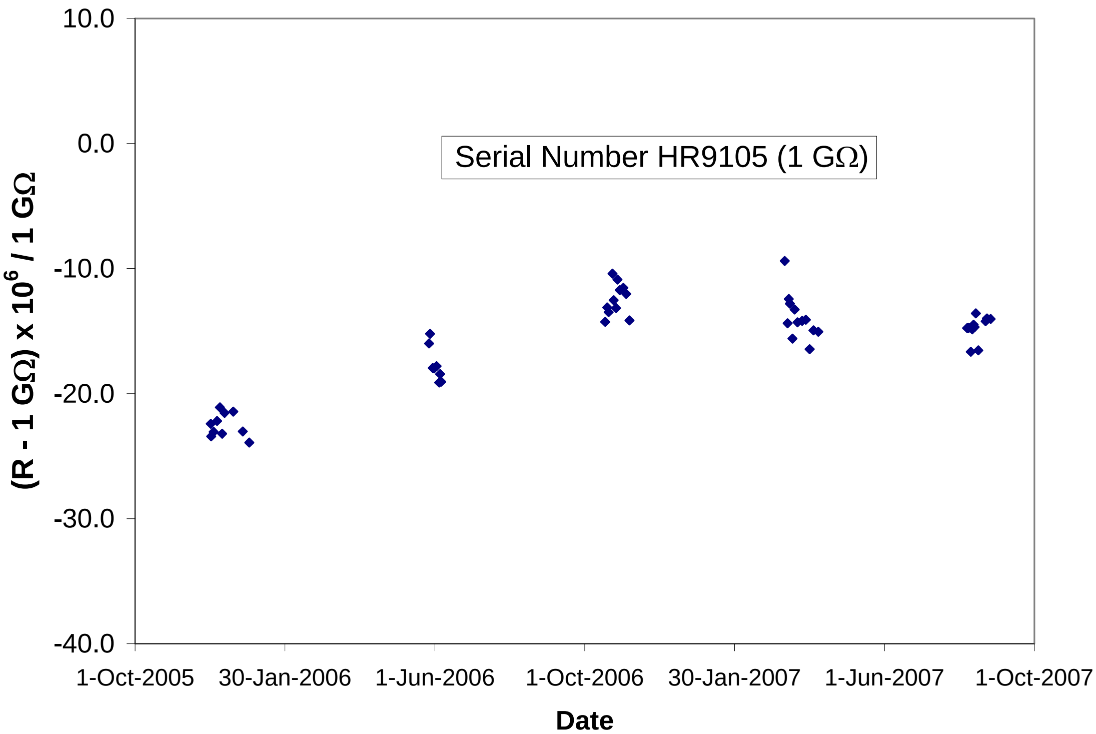
5.2 Pilot results at 1 MΩ





5.3 Pilot results at 1 GΩ



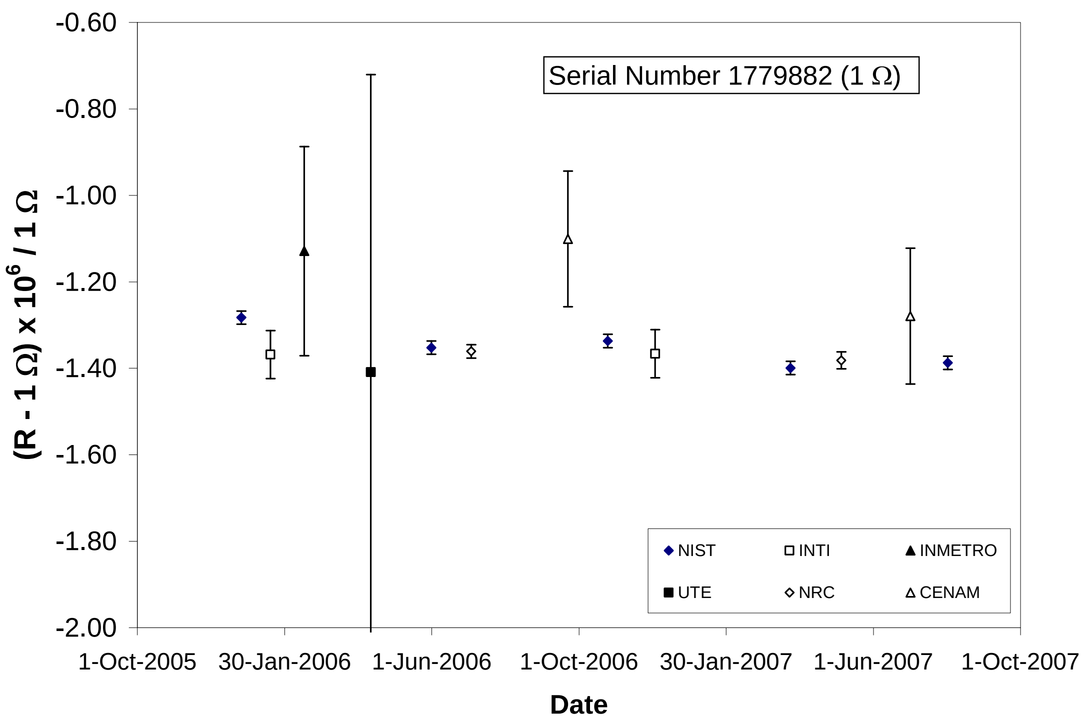


6. Reported results of comparisons

6.1 Results at 1 Ω for Serial Number 1779882

Laboratory	Mean date of measurements	Reported Resistance	Type A Standard Uncertainty (k=1)	Type B Standard Uncertainty (k=1)
NIST	26-Dec-2005	-1.283	0.0067	0.0137
INTI	19-Jan-2006	-1.368	0.0110	0.0544
INMETRO	16-Feb-2006	-1.129	0.0220	0.2410
UTE	12-Apr-2006	-1.409	0.2680	0.6340
NIST	1-Jun-2006	-1.352	0.0025	0.0137
NRC	4-Jul-2006	-1.361	0.0019	0.0154
CENAM	22-Sep-2006	-1.101	0.0022	0.1570
NIST	25-Oct-2006	-1.337	0.0032	0.0137
INTI	3-Dec-2006	-1.366	0.0110	0.0575
INMETRO	Laboratory did not participate in this round of the comparison			
UTE	Laboratory did not participate in this round of the comparison			
NIST	25-Mar-2007	-1.399	0.0024	0.0137
NRC	6-May-2007	-1.382	0.0144	0.0134
CENAM	2-Jul-2007	-1.279	0.0026	0.1570
NIST	2-Aug-2007	-1.387	0.0027	0.0137

Reported Resistance is the resistance correction from nominal, $X_l = \dfrac{(R_l - 1\Omega)\times 10^6}{1\Omega}$

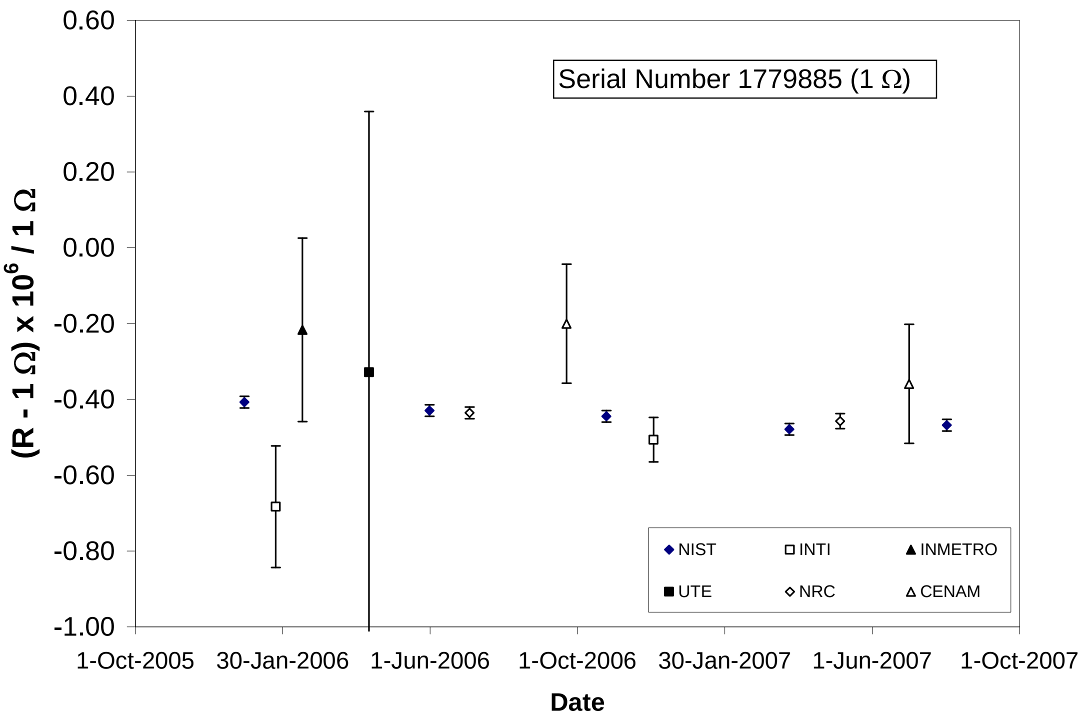


Reported resistance and combined standard uncertainty (k = 1)

6.2 Results at 1 Ω for Serial Number 1779885

Laboratory	Mean date of measurements	Reported Resistance	Type A Standard Uncertainty (k=1)	Type B Standard Uncertainty (k=1)
NIST	26-Dec-2005	-0.4071	0.0046	0.0137
INTI	19-Jan-2006	-0.6830	0.0110	0.1600
INMETRO	16-Feb-2006	-0.2163	0.0220	0.2410
UTE	12-Apr-2006	-0.3283	0.2730	0.6340
NIST	1-Jun-2006	-0.4293	0.0024	0.0137
NRC	4-Jul-2006	-0.4355	0.0018	0.0154
CENAM	22-Sep-2006	-0.2000	0.0031	0.1570
NIST	25-Oct-2006	-0.4447	0.0028	0.0137
INTI	3-Dec-2006	-0.5064	0.0110	0.0578
INMETRO	Laboratory did not participate in this round of the comparison			
UTE	Laboratory did not participate in this round of the comparison			
NIST	25-Mar-2007	-0.4788	0.0025	0.0137
NRC	6-May-2007	-0.4574	0.0142	0.0134
CENAM	2-Jul-2007	-0.3590	0.0022	0.1570
NIST	2-Aug-2007	-0.4679	0.0027	0.0137

Reported Resistance is the resistance correction from nominal, $X_l = \dfrac{(R_l - 1\Omega)\times 10^6}{1\Omega}$

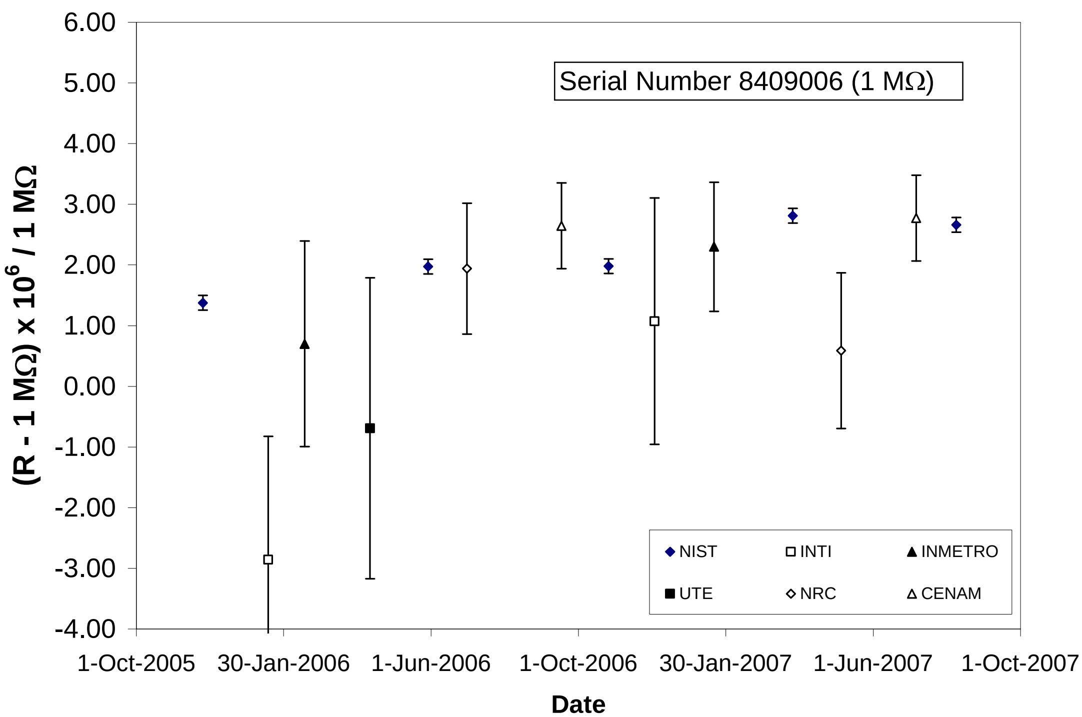


Reported resistance and combined standard uncertainty (k = 1)

6.3 Results at 1 MΩ for Serial Number 8409006

Laboratory	Mean date of measurements	Reported Resistance	Type A Standard Uncertainty (k=1)	Type B Standard Uncertainty (k=1)
NIST	25-Nov-2005	1.38	0.028	0.117
INTI	18-Jan-2006	-2.86	0.393	1.970
INMETRO	17-Feb-2006	0.70	0.100	1.690
UTE	12-Apr-2006	-0.69	0.047	2.480
NIST	30-May-2006	1.97	0.034	0.117
NRC	1-Jul-2006	1.94	0.172	1.063
CENAM	17-Sep-2006	2.65	0.011	0.709
NIST	26-Oct-2006	1.98	0.035	0.117
INTI	3-Dec-2006	1.07	0.169	1.908
INMETRO	21-Jan-2007	2.30	0.094	1.060
UTE	Laboratory did not participate in this round of the comparison			
NIST	27-Mar-2007	2.81	0.046	0.117
NRC	6-May-2007	0.590	0.219	1.264
CENAM	7-Jul-2007	2.77	0.015	0.705
NIST	9-Aug-2007	2.66	0.031	0.117

Reported Resistance is the resistance correction from nominal, $X_l = \dfrac{(R_l - 1M\Omega)\times 10^6}{1M\Omega}$

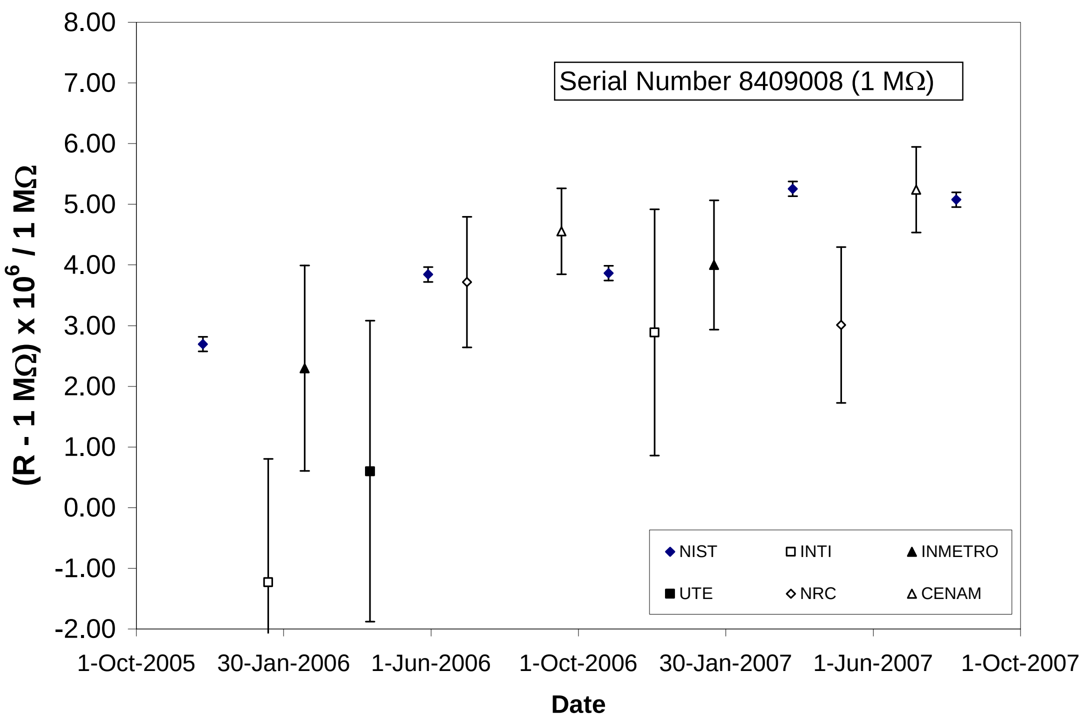


Reported resistance and combined standard uncertainty (k = 1)

6.4 Results at 1 MΩ for Serial Number 8409008

Laboratory	Mean date of measurements	Reported Resistance	Type A Standard Uncertainty (k=1)	Type B Standard Uncertainty (k=1)
NIST	25-Nov-2005	2.70	0.061	0.117
INTI	18-Jan-2006	-1.23	0.601	1.970
INMETRO	17-Feb-2006	2.30	0.071	1.690
UTE	12-Apr-2006	0.60	0.047	2.480
NIST	30-May-2006	3.84	0.045	0.117
NRC	1-Jul-2006	3.72	0.172	1.063
CENAM	17-Sep-2006	4.56	0.011	0.709
NIST	26-Oct-2006	3.87	0.026	0.117
INTI	3-Dec-2006	2.89	0.179	1.908
INMETRO	21-Jan-2007	4.00	0.097	1.060
UTE	Laboratory did not participate in this round of the comparison			
NIST	27-Mar-2007	5.26	0.025	0.117
NRC	6-May-2007	3.01	0.202	1.264
CENAM	7-Jul-2007	5.24	0.030	0.705
NIST	9-Aug-2007	5.076	0.040	0.117

Reported Resistance is the resistance correction from nominal, $X_l = \dfrac{(R_l - 1M\Omega)\times 10^6}{1M\Omega}$

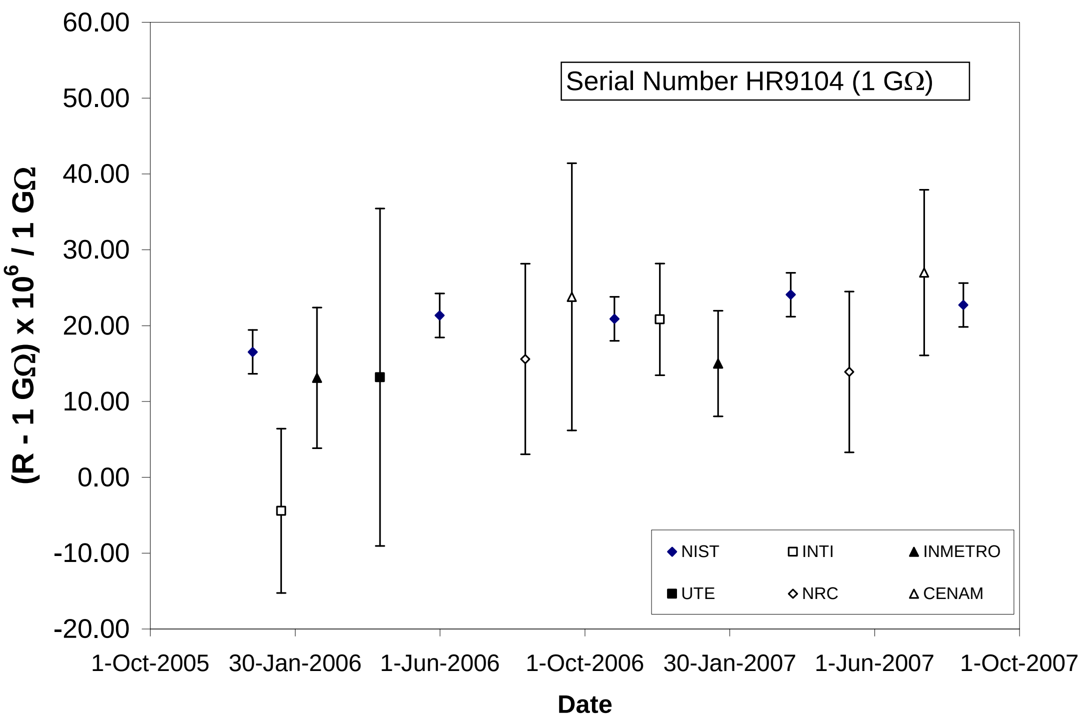


Reported resistance and combined standard uncertainty (k = 1)

6.5 Results at 1 GΩ for Serial Number HR9104

Laboratory	Mean date of measurements	Reported Resistance	Type A Standard Uncertainty (k=1)	Type B Standard Uncertainty (k=1)
NIST	26-Dec-2005	16.53	0.86	2.69
INTI	19-Jan-2006	-4.42	8.00	7.32
INMETRO	18-Feb-2006	13.10	7.00	6.09
UTE	12-Apr-2006	13.20	2.32	22.12
NIST	1-Jun-2006	21.34	0.88	2.69
NRC	12-Aug-2006	15.60	1.33	12.50
CENAM	20-Sep-2006	23.80	1.00	17.58
NIST	26-Oct-2006	20.89	1.35	2.69
INTI	3-Dec-2006	20.83	0.60	7.35
INMETRO	21-Jan-2007	15.00	3.31	6.12
UTE	Laboratory did not participate in this round of the comparison			
NIST	23-Mar-2007	24.08	1.12	2.69
NRC	11-May-2007	13.90	0.43	10.58
CENAM	13-Jul-2007	27.00	0.78	10.09
NIST	15-Aug-2007	22.72	0.92	2.69

Reported Resistance is the resistance correction from nominal, $X_l = \dfrac{(R_l - 1G\Omega)\times 10^6}{1G\Omega}$

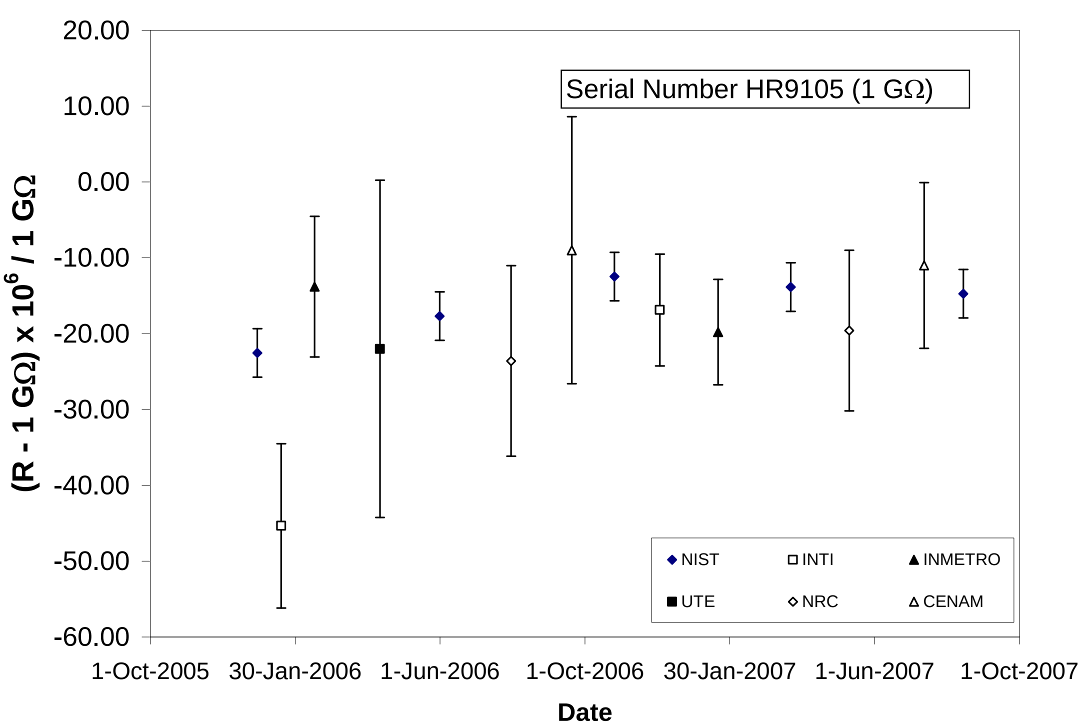


Reported resistance and combined standard uncertainty (k = 1)

6.6 Results at 1 GΩ for Serial Number HR9105

Laboratory	Mean date of measurements	Reported Resistance	Type A Standard Uncertainty (k=1)	Type B Standard Uncertainty (k=1)
NIST	26-Dec-2005	-22.53	1.39	2.69
INTI	19-Jan-2006	-45.35	8.00	9.41
INMETRO	18-Feb-2006	-13.80	6.80	6.98
UTE	12-Apr-2006	-22.00	1.47	22.12
NIST	1-Jun-2006	-17.69	1.65	2.69
NRC	12-Aug-2006	-23.60	1.58	12.58
CENAM	20-Sep-2006	-9.00	2.00	23.00
NIST	26-Oct-2006	-12.48	2.11	2.69
INTI	3-Dec-2006	-16.88	0.80	9.39
INMETRO	21-Jan-2007	-19.80	3.89	6.51
UTE	Laboratory did not participate in this round of the comparison			
NIST	23-Mar-2007	-13.86	2.11	2.69
NRC	11-May-2007	-19.60	0.75	10.58
CENAM	13-Jul-2007	-11.00	1.40	10.17
NIST	15-Aug-2007	-14.73	1.31	2.69

Reported Resistance is the resistance correction from nominal, $X_l = \dfrac{(R_l - 1G\Omega)\times 10^6}{1G\Omega}$



Reported resistance and combined standard uncertainty (k = 1)

7. References

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Appendix A. Analysis procedure

Resistance Interlaboratory Comparisons with Linear Trend for SIM Laboratories
Weiping Zhang, Nien Fan Zhang, Hung-kung Liu

1. Models and parameter estimation

It is well known that for a standard of resistance, the measurements typically show a trend in time, which we assume can be modeled as a linear trend. For the measurements of the SIM.EM-K1-K2-S6 comparisons, the linear trends were obvious. As in [6] and [7], we assume that the measurements of any particular laboratory have a linear trend in time and the slopes of the linear trends for the laboratories are the same, while we allow for different intercepts for different laboratories. In addition, since two traveling standards were used for each SIM comparison, the procedure proposed in [7] was considered. However, differing from the case in CCEM-K2, in the SIM.EM-K1-K2-S6 comparisons most non-pilot laboratories made measurements in two separate periods and for several of these laboratories, the Type B uncertainties assigned for the two resulting measurement data sets are not the same. Thus, a statistical analysis procedure related to [7] was developed to deal with this kind of data, which is described below. Based on this procedure, the results were calculated and listed in Appendix B.

Our model also assumes

- K traveling artifacts
- For all artifacts the i th laboratory ($i = 1,...,P$) makes k_i measurements with $k_i \geq 1$. For the l th artifact, the j th measurement (or the average of the measurements) made at laboratory i, $X_{ij}(l)$ is measured at the time t_{ij} ($j = 1,...k_i$). Without loss of generality, we assume that the pilot laboratory is the first one among all P laboratories with $k_1 > 1$.

For a fixed artifact, say l ($l = 1,\cdots,K$), we assume that a simple linear regression holds for the measurements, i.e.,

$$X_{ij}(l) = \alpha_i(l) + \beta(l)t_{ij} + e_{ij}(l), \quad (1)$$

for $j = 1,\cdots,k_i$, $i = 1,\cdots,p$, and $l = 1,\cdots,K$, where the random components $e_{ij}(l)$'s are statistically independent of each other and have zero mean and standard uncertainty of $\sigma_{ij}(l)$, which is the combination of the Type A and Type B evaluations of uncertainty. Specifically,

$$\sigma_{ij}(l) = \sqrt{\sigma_{ij,A}^2(l) + \sigma_{ij,B}^2(l)} \quad (2)$$

where $\sigma_{ij,A}(l)$ and $\sigma_{ij,B}(l)$ are the Type A and Type B uncertainties for the l th artifact measured at the j th time period by the i th lab, respectively. This indicates that the measurements of any artifact (whether by the same or by different laboratories) at different time periods are statistically independent.

Using matrix notations, the model in (1) is given by

$$X(l) = Z\theta(l) + \varepsilon(l), \tag{3}$$

where $X(l) = (X_{11}(l), \cdots, X_{1k_1}(l); \cdots, X_{P1}(l), \cdots, X_{Pk_P}(l))'$, $\theta = (\alpha_1(l), \cdots, \alpha_P(l), \beta(l))'$,

$$Z = \begin{pmatrix} 1 & 0 & \cdots & 0 & t_{11} \\ & & & & \vdots \\ 1 & 0 & \cdots & 0 & t_{1k_1} \\ 0 & 1 & \cdots & 0 & t_{21} \\ & & \vdots & & \vdots \\ 0 & 1 & \cdots & 0 & t_{2k_2} \\ & & \vdots & & \\ 0 & \cdots & 0 & 1 & t_{p1} \\ & & & & \vdots \\ 0 & \cdots & 0 & 1 & t_{pk_p} \end{pmatrix}.$$

Z is a $(k_1 + k_2 + \cdots + k_P)$ by $p+1$ matrix and $\varepsilon = (e_{11}(l), \cdots, e_{1k_1}(l); \cdots; e_{P1}(l), \cdots, e_{Pk_P}(l))'$ with mean $E[\varepsilon(l)] = 0$ and the covariance matrix $Cov(\varepsilon(l)) = \Sigma(l) = diag\{\sigma_{11}^2(l), \cdots, \sigma_{1k_1}^2(l), \cdots, \sigma_{P1}^2(l), \cdots, \sigma_{Pk_p}^2(l)\}$. We use ξ' to denote the transpose of a vector ξ. The matrix $diag\{c_1, \cdots, c_n\}$ is a diagonal matrix with elements $c_1, \cdots, c_n$.

It is well known [10] that the best linear unbiased estimator of $\theta(l)$ in model (3) is the generalized least square estimator, i.e.,

$$\hat{\theta}(l) = (Z'\Sigma^{-1}(l)Z)^{-1} Z'\Sigma^{-1}(l)X(l). \tag{4}$$

After laborious but straightforward mathematical operations not detailed here, it can be shown that the estimators of $\alpha_i(l)(i = 1, \cdots, p)$ and $\beta(l)$ can be written as

$$\hat{\alpha}_i(l) = X_i(l) - \hat{\beta}(l)t_i(l), i = 1, \cdots, p; \tag{5}$$

$$\hat{\beta}(l)=\frac{\sum_{i=1}^{P}\sum_{j=1}^{k_i}\frac{1}{\sigma_{ij}^2(l)}(t_{ij}-t_i(l))(X_{ij}(l)-X_i(l))}{\sum_{i=1}^{P}\sum_{j=1}^{k_i}\frac{1}{\sigma_{ij}^2(l)}(t_{ij}-t_i(l))^2}. \tag{6}$$

where

$$t_i(l)=\sum_{j=1}^{k_i}w_{ij}(l)t_{ij}, \qquad X_i(l)=\sum_{j=1}^{k_i}w_{ij}(l)X_{ij}(l) \tag{7}$$

with $w_{ij}(l)=\dfrac{1/\sigma_{ij}^2(l)}{\sum_{j=1}^{k_i}1/\sigma_{ij}^2(l)}$. Thus, $t_i(l)$ is a weighted mean of t_{ij} for the lth artifact and $X_i(l)$ is a weighted mean of $X_{ij}(l)$ ($j=1,...,k_i$). The corresponding uncertainty for $X_i(l)$, $u_i(l)$, for the lth artifact in the ith laboratory is given by

$$u_i^2(l)=\frac{1}{\sum_{j=1}^{k_i}1/\sigma_{ij}^2(l)}. \tag{8}$$

The corresponding uncertainties for the estimators (5) and (6) are given by

$$\begin{aligned} u_{\hat{\beta}(l)}^2 &= 1/[\sum_{i=1}^{P}\sum_{j=1}^{k_i}\frac{1}{\sigma_{ij}^2(l)}(t_{ij}-t_i(l))^2], \\ u_{\hat{\alpha}_i(l)}^2 &= u_i^2(l)+u_{\hat{\beta}(l)}^2 t_i^2(l), \end{aligned} \tag{9}$$

In addition, the following equations regarding the covariances hold.

$$\begin{aligned} &Cov(X_i(l),\hat{\beta}(l))=0, \\ &Cov(\hat{\alpha}_i(l),\hat{\beta}(l))=\frac{-t_i(l)}{\sum_{i=1}^{P}\sum_{j=1}^{k_i}\frac{1}{\sigma_{ij}^2(l)}(t_{ij}-t_i(l))^2}, \\ &Cov(\hat{\alpha}_i(l),\hat{\alpha}_j(l))=\frac{t_i(l)t_j(l)}{\sum_{i=1}^{P}\sum_{j=1}^{k_i}\frac{1}{\sigma_{ij}^2(l)}(t_{ij}-t_i(l))^2}, i\neq j. \end{aligned} \tag{10}$$

Hence the predicted regression line for the measurement from the ith laboratory made with the lth artifact at time t is given by

$$L_{il}(t)=\hat{\alpha}_i(l)+\hat{\beta}(l)t, \tag{11}$$

and its corresponding uncertainty is given by

$$u^2_{\hat{\alpha}_i(l)+\hat{\beta}(l)t} = u_i^2(l) + u^2_{\hat{\beta}(l)}(t_i(l)-t)^2. \tag{12}$$

2. Comparison reference value

For the comparison reference value (CRV) at any time t (denoted by CRV_t), we use a weighted mean of $\hat{\alpha}+\hat{\beta}t$ over all the laboratories $i=1,\cdots,p$ and all the artifacts $l=1,\cdots,K$. i.e.,

$$CRV_t(\omega,v) = \sum_{i=1}^{P}\omega_i\left(\sum_{l=1}^{K} v_{il}L_{il}(t)\right), \tag{13}$$

where $L_{il}(t)$ is given by (11), which is the prediction for the value of the lth artifact (based on the lth regression line) for the ith laboratory at time t. The time t is allowed to be different for different artifacts. Namely, the time is a vector denoted by $\vec{t} = (t(1),...,t(K))$. In this case,

$$\begin{aligned} CRV_{\vec{t}}(\omega,v) &= \sum_{i=1}^{P}\omega_i\left(\sum_{l=1}^{K} v_{il}L_{il}(t(l))\right) \\ &= \sum_{i=1}^{P}\omega_i\left(\sum_{l=1}^{K} v_{il}[X_i(l)+\hat{\beta}(l)(t(l)-t_i(l))]\right) \end{aligned} \tag{14}$$

The weights $\omega = (\omega_1,\cdots,\omega_p)$ represent the effects of all the participating laboratories and are scaled so that they sum to 1, and v_{il} are essentially the weights for the l traveling artifacts. In practice, the weights are determined by the residual variances of the fitted regressions corresponding to the artifacts. In this case, similar to (16) in [6], the variance of CRV at time $\vec{t}$ is

$$\begin{aligned} &Var[CRV_{\vec{t}}(\omega,v)] \\ &= \sum_{i=1}^{P}\omega_i^2\sum_{l=1}^{K}v_{il}^2u_i^2(l) + \sum_{l=1}^{K}\left[\sum_{i=1}^{P}\omega_i v_{il}(t(l)-t_i(l))\right]^2 u^2_{\hat{\beta}(l)}. \end{aligned} \tag{15}$$

In metrology, it is commonly assumed that the weights v_{il} do not depend on the laboratory. Namely, $v_{il} = v_l$ for $i=1,\cdots.P$ as in CCEM-K2 [3]. In this case the second term on the right hand side of (15) is

$$\sum_{l=1}^{K}v_l^2\left[\sum_{i=1}^{P}\omega_i(t(l)-t_i(l))\right]^2 u^2_{\hat{\beta}(l)}.$$

Because the weights, $\omega_1, \cdots, \omega_p$, are scaled to sum to 1, this term will vanish when choosing $t(l)$ for the l th artifact to be

$$t(l) = \sum_{i=1}^{P} \omega_i t_i(l) \tag{16}$$

for $l = 1, ..., K$. With this choice of $t(l)$, the corresponding standard uncertainty of $CRV_{\vec{t}}(\omega, v)$ in (15) is

$$u^2_{CRV_{\vec{t}}}(\omega, v) = \sum_{i=1}^{P} \omega_i^2 \sum_{l=1}^{K} v_l^2 u_i^2(l). \tag{17}$$

For a fixed set of v_l, $u^2_{CRV_{\vec{t}}}(\omega, v)$ is minimized when the weights $\{\omega_i\}$ are given by

$$\omega_i^* = \frac{1/\sum_{l=1}^{K} v_l^2 u_i^2(l)}{\sum_{i=1}^{P} [1/\sum_{l=1}^{K} v_l^2 u_i^2(l)]}. \tag{18}$$

With the weights of $\{\omega_i^*\}$, from (16) the corresponding

$$t^*(l) = \sum_{i=1}^{p} \omega_i^* t_i(l). \tag{19}$$

Denote $\vec{t}^* = (t^*(1), ..., t^*(K))$. The corresponding CRV in (14) is given by

$$CRV_{\vec{t}*}(\omega^*, v) = \sum_{i=1}^{P} \omega_i^* \sum_{l=1}^{K} v_l X_i(l) \tag{20}$$

The standard uncertainty of CRV is given by

$$u_{CRV_{\vec{t}^*}}(\omega^*, v) = \sqrt{\frac{1}{\sum_{i=1}^{p} [1/\sum_{l=1}^{K} v_l^2 u_i^2(l)]}}.$$

(21)

In practice, a choice of v_l can be formed by the `mean-squared residual' for the l th regression line for the pilot libratory as in CCEM-K2, i.e.,

$$v_l = \frac{1/\rho^2(l)}{\sum_{l=1}^{K} 1/\rho^2(l)} \tag{22}$$

where

$$\rho^2(l) = \frac{\sum_{j=1}^{k_1}(X_{1j}(l) - \hat{\alpha}_1(l) - \hat{\beta}(l)t_{1j})^2}{k_1 - 2}. \tag{23}$$

Thus, from the given data of $\{X_{ij}(l)\}$ and their corresponding standard uncertainties and (8), (9), (23), (22), and (18)-(21), the CRV and its standard uncertainty can be calculated.

3. Degrees of equivalence

3.1. Degrees of equivalence between national measurement standards and the CRV

For the degree of equivalence between the national measurement standard from the i th laboratory and the CRV, we only consider the case when $v_{il} = v_l, \omega_i = \omega_i^*$ and $\vec{t} = \vec{t}^*$ as given by (18) and (19). This degree of equivalence is defined as the difference

$$D_{i,CRV_{\vec{t}^*}}(\omega,v) = \sum_{l=1}^{K} v_l(\hat{\alpha}_i(l) + \hat{\beta}(l)t^*(l)) - CRV_{\vec{t}^*}(\omega^*, v). \tag{24}$$

From (12), (10), and (21) the corresponding standard uncertainty isgiven by,

$$u[D_{i,CRV_{\vec{t}^*}}(\omega,v)] = \sqrt{(1-2\omega_i^*)\sum_{l=1}^{K} v_l^2 u_i^2(l) + \sum_{l=1}^{K} \frac{v_l^2(t_i(l) - t^*(l))^2}{\sum_{i=1}^{P}\sum_{j=1}^{k_i} \frac{1}{\sigma_{ij}^2(l)}(t_{ij} - t_i(l))^2} + \frac{1}{\sum_{i=1}^{P}[1/\sum_{l=1}^{K} v_l^2 u_i^2(l)]}}$$

3.2. Degrees of equivalence between pairs of national measurement standards

The degree of equivalence between two national measurement standards at time $\vec{t}$ is defined as in [6], i.e.,

$$D_{i,j} = \sum_{l=1}^{K} v_l[(\hat{\alpha}_i(l) + \hat{\beta}(l)t(l)) - \sum_{l=1}^{K} v_l[(\hat{\alpha}_j(l) + \hat{\beta}(l)t(l))] = \sum_{l=1}^{K} v_l[\hat{\alpha}_i(l) - \hat{\alpha}_j(l)] \tag{26}$$

when $i \neq j$. Thus the quantity is independent of $\vec{t}$. Since $\hat{\alpha}_i(l)$ are independent for different l, by (10) the corresponding standard uncertainty is given by

$$u_{D_{i,j}} = \sqrt{\sum_{l=1}^{K} v_l^2 [u_i^2(l) + u_j^2(l)] + \sum_{l=1}^{K} v_l^2 \left[\frac{(t_i(l) - t_j(l))^2}{\sum_{i=1}^{P} \sum_{j=1}^{k_i} \frac{1}{\sigma_{ij}^2(l)} (t_{ij} - t_i(l))^2} \right]}. \tag{27}$$

In this analysis, we assume that the simple linear regression models hold and the uncertainties $\sigma_{ij}(l)$ are known. However, in practice, the uncertainties are usually unknown. In this case, discretions need to be used regarding the calculations of the uncertainties, in particular the uncertainty associated with the weighted mean. See [11].

Appendix B. Analysis results

The results were calculated and listed below.

1. 1 Ω

The regression estimates are: $\hat{\beta} =$ (-0.057796583, -0.040525765) and the intercepts for NIST are $\hat{\alpha}_1 =$ (114.6339529, 80.8816171) for the 2 regression lines based on NIST data.

The CRV is calculated at $\vec{t}* = (2006.83, 2006.82)$. The corresponding CRV = -0.5962 from (20). The standard uncertainty of the CRV is u_{CRV} = 0.0047 from (21).

The degrees of equivalence between the national measurement standards and the CRV and their corresponding standard uncertainties from (24) and (25) are:
:

(× 10^{-6})	NIST	INTI	INMETRO	UTE	NRC	CENAM
$D_{i,CRV_{t*}}$	0.0003	-0.0732	0.1995	0.0663	-0.0001	0.1791
$u_{D_{i,CRV}}$	0.0025	0.0464	0.2060	0.5875	0.0092	0.0944

The degrees of equivalence between pairs of national measurement standards and their corresponding standard uncertainties (in the parenthses underneath) from (26) and (27) are:

(× 10^{-6})	NIST	INTI	INMETRO	UTE	NRC	CENAM
NIST		0.0735 (0.0469)	-0.1992 (0.2061)	-0.0660 (0.5875)	0.0004 (0.0116)	-0.1788 (0.0947)
INTI	-0.0735 (0.0469)		-0.2727 (0.2113)	-0.1395 (0.5894)	-0.0731 (0.0477)	-0.2523 (0.1054)
INMETRO	0.1992 (0.2061)	0.2727 (0.2113)		0.1332 (0.6226)	0.1996 (0.2063)	0.0203 (0.2268)
UTE	-0.0660 (0.5875)	0.1395 (0.5894)	-0.1332 (0.6226)		0.0664 (0.5876)	-0.1129 (0.5951)
NRC	-0.0004 (0.0116)	0.0731 (0.0477)	-0.1996 (0.2063)	-0.0664 (0.5876)		-0.1792 (0.0951)
CENAM	0.1788 (0.0947)	0.2523 (0.1054)	-0.0203 (0.2268)	0.1129 (0.5951)	0.1792 (0.0951)	

2. 1 MΩ

The regression estimates are: $\hat{\beta} = (0.804982076, 1.471552860)$ and the intercepts for NIST are $\hat{\alpha}_1 =$ (-1613.2753494, -2948.9471028) for the 2 regression lines based on NIST data.

The CRV is calculated at $\vec{t}^* = (2006.788, 2006.825)$. The corrsponding CRV = 2.6871. The standard uncertainty of the CRV is u_{CRV} = 0.0423.

The degrees of equivalence between the national measurement standards and the CRV and their corresponding standard uncertainties are:

($\times 10^{-6}$)	NIST	INTI	INMETRO	UTE	NRC	CENAM
$D_{i,CRV_{t*}}$	0.0069	-2.7316	-0.3930	-2.5238	-0.8096	0.2993
$u_{D_{i,CRV}}$	0.0063	1.0847	0.7031	1.9392	0.6434	0.3893

The degrees of equivalence of pairs of national measurement standards and their corresponding standard uncertainties (in the parenthses underneath) from (26) and (27) are::

($\times 10^{-6}$)	NIST	INTI	INMETRO	UTE	NRC	CENAM
NIST		2.7385 (1.0864)	0.3999 (0.7057)	2.5307 (1.9401)	0.8165 (0.6462)	-0.2924 (0.3939)
INTI	-2.7385 (1.0864)		-2.3385 (1.2941)	-0.2077 (2.2224)	-1.9220 (1.2627)	-3.0309 (1.1544)
INMETRO	-0.3999 (0.7057)	2.3385 (1.2941)		2.1308 (2.0636)	0.4166 (0.9549)	-0.6924 (0.8059)
UTE	-2.5307 (1.9401)	0.2077 (2.2224)	-2.1308 (2.0636)		-1.7143 (2.0441)	-2.8232 (1.9792)
NRC	-0.8165 (0.6462)	1.9220 (1.2627)	-0.4166 (0.9549)	1.7143 (2.0441)		-1.1089 (0.7543)
CENAM	0.2924 (0.3939)	3.0309 (1.1544)	0.6924 (0.8059)	2.8232 (1.9792)	1.1089 (0.7543)	

3. 1 GΩ

The regression estimates are: $\hat{\beta} = (4.058579309, 4.803570868)$ and the intercepts for NIST are $\hat{\alpha}_1 = (-8123.6671700, -9656.3000061)$ for the 2 regression lines based on NIST data.

The CRV is calculated at $\vec{t}* = (2006.800, 2006.798)$. The corrsponding CRV = 10.2401. The standard uncertainty of the CRV is u_{CRV} = 0.9477.

The degrees of equivalence between the national measurement standards and the CRV and their corresponding standard uncertainties are:

$(\times 10^{-6})$	NIST	INTI	INMETRO	UTE	NRC	CENAM
$D_{i,CRV_{t*}}$	0.6539	-7.5813	-4.2259	-4.3737	-6.2296	3.6206
$u_{D_{i,CRV}}$	0.3652	4.8011	4.2672	17.2829	6.2380	6.8733

The degrees of equivalence between pairs of national measurement standards and their corresponding standard uncertainties (in the parenthses underneath) from (26) and (27) are:

$(\times 10^{-6})$	NIST	INTI	INMETRO	UTE	NRC	CENAM
NIST		8.2352 (4.9977)	4.8798 (4.4874)	5.0276 (17.3383)	6.8835 (6.3912)	-2.9666 (7.0131)
INTI	-8.2352 (4.9977)		-3.3554 (6.5565)	-3.2076 (17.9748)	-1.3517 (7.9961)	-11.2019 (8.5182)
INMETRO	-4.8798 (4.4874)	3.3554 (6.5565)		0.1479 (17.8462)	2.0037 (7.6813)	-7.8464 (8.2142)
UTE	-5.0276 (17.3383)	3.2076 (17.9748)	-0.1479 (17.8462)		1.8558 (18.4385)	-7.9943 (18.6881)
NRC	-6.8835 (6.3912)	1.3517 (7.9961)	-2.0037 (7.6813)	-1.8558 (18.4385)		-9.8501 (9.3468)
CENAM	2.9666 (7.0131)	11.2019 (8.5182)	7.8464 (8.2142)	7.9943 (18.6881)	9.8501 (9.3468)	

Appendix C. Uncertainty budgets for 1 ohm

1. Detailed uncertainty budget, CENAM

Serial No. of item(s): 1779882, 1779885	Standard uncertainty	Distribution /method of evaluation	Sensitivity coefficient	Uncertainty contribution	Degrees of freedom
Influence factor y_i	$u(y_i)$	*Method/(A, B)*	c_i	$u(R_i)$ (Ω)	ν_i
Scaling / traceability	1.30E-07	Normal/B	1 Ω	1.30E-07	∞
Reference standard(s)	5.00E-08	Normal/B	1 Ω	5.00E-08	∞
Measuring apparatus	5.77E-08	Rectangular/B	1 Ω	5.77E-08	∞
Ambient conditions:					
Temperature	2.00E-01 °C	Normal/B	2.18E-06 Ω/°C	4.36E-08	∞
Pressure	6.00E-02 kPa	Normal/B	5.75E-09 Ω/kPa	3.45E-10	∞
Standard deviation	2.58E-09	Normal/A	1 Ω	2.58E-09	19
Repeatability	5.00E-09	Normal/B	1 Ω	5.00E-09	19
Corrections applied					
Pressure	6.00E-10 Ω/kPa	Normal/B	18.17 kPa	1.09E-08	∞
RSS of Type A standard uncertainties and effective degrees of freedom:				2.58E-09	19
RSS of Type B standard uncertainties and effective degrees of freedom:				1.57E-07	∞
Combined standard uncertainty and effective degrees of freedom:				1.57E-07	∞
Expanded uncertainty (95 % coverage factor):				3.13E-07 Ω	

Method: Traceability is derived from the CENAM quantum Hall effect standard used with potentiometric comparison to 10 kΩ. A commercial current comparator bridge was used with two Hamon transfer standards of 1 kΩ per step to determine the value of a 100 Ω standard resistor. Similarly, a commercial current comparator bridge was used with two Hamon transfer standards of 10 Ω per step to determine the value of a 1 Ω standard resistor. At each level, interchange of like-value standards was used with the current comparator bridge to reduce bridge ratio errors. Measurements were repeated on ten days in each round of the comparison.

Measurement temperature control: The reference standards and comparison standards were measured in a mineral oil bath maintained at nominal 25.00 °C.
Test current: Direct current with reversal, measured at 100 mA.
Pressure: Typical barometric pressure including oil above the resistors was 83.2 kPa.
Humidity: Relative humidity in the laboratory averaged 44 % in the first round, 56.5 % in the second round.

2. Detailed uncertainty budget, INMETRO

Serial No. of item(s): 1779882, 1779885	Standard uncertainty	Distribution /method of evaluation	Sensitivity coefficient	Uncertainty contribution	Degrees of freedom
Influence factor y_i	$u(y_i)$	*Method/(A, B)*	c_i	$u(R_i)$ (Ω)	ν_i
Scaling / traceability	1.70E-08	Normal/B	1 Ω	1.70E-08	∞
Reference standard(s)	1.60E-07	Normal/B	1 Ω	1.60E-07	7
Measuring apparatus	1.50E-07	Rectangular/B	1 Ω	1.50E-07	∞
Ambient conditions:					
Temperature (Rs)	2.00E-02 °C	Rectangular/B	4.30E-06 Ω/°C	8.60E-08	∞
Temperature (R*x*)	2.00E-02 °C	Rectangular/B	2.18E-06 Ω/°C	4.36E-08	∞
Pressure	4.60E-01 kPa	Normal/B	0.35E-09 Ω/kPa	1.60E-10	∞
Standard deviation	2.20E-08	Normal/A	1 Ω	2.20E-08	15
Repeatability	2.50E-08	Normal/B	1 Ω	2.50E-08	15
Corrections applied					
Pressure	6.00E-10 Ω/kPa	Normal/B	0.78 kPa	4.80E-10	∞
Power level	1.00E-06 Ω/W	Normal/B	0.0075 W	7.50E-09	∞
RSS of Type A standard uncertainties and effective degrees of freedom:				2.20E-08	15
RSS of Type B standard uncertainties and effective degrees of freedom:				2.41E-07	∞
Combined standard uncertainty and effective degrees of freedom:				2.42E-07	∞
Expanded uncertainty (95 % coverage factor):				4.85E-07 Ω	

Method: All measurements were carried out with an automated current comparator bridge. Traceability was obtained through a 1 Ω standard calibrated with respect to the BIPM quantum Hall effect standard. Measurements were repeated on eight days in the first round of the comparison. No final results were reported in the second round.

Measurement temperature control: The comparison standards were measured in a mineral oil bath maintained at nominal 25.00 °C. The INMETRO 1 Ω standards were maintained at 23.00 °C.
Test current: Direct current with reversal, measured at 50 mA.
Pressure: Typical barometric pressure including oil above the resistors was 102.1 kPa.
Humidity: Relative humidity in the laboratory averaged 55 % in the first round.

3. Detailed uncertainty budget, INTI

Serial No. of item(s): 1779882, 1779885 Influence factor y_i	Standard uncertainty $u(y_i)$	Distribution /method of evaluation *Method/(A, B)*	Sensitivity coefficient c_i	Uncertainty contribution $u(R_i)$ (Ω)	Degrees of freedom ν_i
Scaling / traceability	1.00E-08	Rectangular/B	1 Ω	1.00E-08	50
Reference standard(s)	3.00E-08	Normal/B	1 Ω	3.00E-08	50
Measuring apparatus	6.00E-09	Rectangular/B	1 Ω	6.00E-09	50
Ambient conditions:					
Temperature	3.00E-03 °C	Normal/B	7.54E-06 Ω/°C	2.26E-08	50
Pressure	3.30E-01 kPa	Normal/B	5.75E-09 Ω/kPa	1.90E-09	50
Standard deviation	1.10E-08	Normal/A	1 Ω	1.10E-08	19
Repeatability (1779882)	4.30E-09	Normal/B	1 Ω	4.30E-09	19
Repeatability (1779885)	1.50E-07	Normal/B	1 Ω	1.50E-07	19
Corrections applied					
Temperature	6.40E-9 Ω/°C	Normal/B	4.974 °C	3.18E-08	∞
Pressure	6.00E-10 Ω/kPa	Normal/B	2.18 kPa	1.31E-09	∞
Power level	3.60E-06 Ω/W	Normal/B	0.0075 W	2.70E-08	∞
RSS of Type A standard uncertainties and effective degrees of freedom:				1.10E-08	19
RSS of Type B standard uncertainties and effective degrees of freedom:				5.75E-08	∞
Combined standard uncertainty and effective degrees of freedom:				5.76E-08	∞
Expanded uncertainty (95 % coverage factor):				1.15E-07 Ω	

Method: All measurements were carried out with an automated current comparator bridge. Traceability was obtained through a 1 Ω standard calibrated with respect to the PTB quantum Hall effect standard. Six INTI 1 Ω standard resistors were used in this comparison, with interchange of like-value standards in the current comparator bridge to reduce bridge ratio errors. Measurements were repeated on ten days in each round of the comparison. In the first round, one comparison standard (SN 1779885) was apparently recovering from transportation effects, and only the last five measurement results were used in the analysis. A larger value of the Repeatability uncertainty was assigned to this first round data for SN 1779885.

Measurement temperature control: The reference standards and comparison standards were measured in a silicone oil bath maintained at nominal 20.02 °C.
Test current: Direct current with reversal, measured at 50 mA.
Pressure: Typical barometric pressure including oil above the resistors was 103.5 kPa.
Humidity: Relative humidity in the laboratory averaged 55 % in the first round, 51 % in the second round.

4. Detailed uncertainty budget, NIST

Serial No. of item(s): 1779882, 1779885 Influence factor y_i	Standard uncertainty $u(y_i)$	Distribution /method of evaluation *Method/(A, B)*	Sensitivity coefficient c_i	Uncertainty contribution $u(R_i)$ (Ω)	Degrees of freedom ν_i
Scaling / traceability	1.00E-08	Normal/B	1 Ω	1.00E-08	100
Reference standard(s)	6.50E-09	Normal/B	1 Ω	6.50E-09	∞
Measuring apparatus	4.00E-09	Rectangular/B	1 Ω	4.00E-09	∞
Ambient conditions:					
Temperature	2.00E-03 °C	Normal/B	2.10E-06 Ω/°C	4.20E-09	∞
Pressure	1.00E-01 kPa	Normal/B	5.75E-09 Ω/kPa	5.75E-10	∞
Standard deviation	2.70E-09	Normal/A	1 Ω	2.70E-09	29
Repeatability	3.50E-09	Normal/B	1 Ω	3.50E-09	29
Corrections applied					
Pressure	6.00E-10 Ω/kPa	Normal/B	0.19 kPa	1.20E-10	∞
RSS of Type A standard uncertainties and effective degrees of freedom:				2.70E-09	29
RSS of Type B standard uncertainties and effective degrees of freedom:				1.37E-08	∞
Combined standard uncertainty and effective degrees of freedom:				1.38E-08	∞
Expanded uncertainty (95 % coverage factor):				2.76E-08 Ω	

Method: The NIST quantum Hall effect standard was compared twice each year to five 100 Ω standards using a cryogenic current comparator bridge. Four times each year, these five standards were compared to two or three 1 Ω standards of the CSIRO design, using a second cryogenic current comparator. These CSIRO-type standards were then used as transfer standards to maintain the mean reference value of a group of five Thomas-type 1 Ω standards in an automated potentiometer measurement system based on a commercial current comparator bridge. The 1 Ω comparison standards were compared to this reference group using the same potentiometric system. Measurements were repeated on twelve to seventeen days in each pilot laboratory segment of the comparison. The first three measurement results for both standards in NIST segment 2 (25-May-2006 through 27-May-2006) were eliminated from the analysis because of apparent transportation effects.

Measurement temperature control: The reference standards and comparison standards were measured in a mineral oil bath maintained at nominal 25.000 °C.
Test current: Direct current with reversal, measured at 100 mA.
Pressure: Typical barometric pressure including oil above the resistors was 101.51 kPa.
Humidity: Relative humidity in the laboratory averaged 32 % in the first segment, 34 % in the second segment, 32 % in the third segment, 36 % in the fourth segment, and 42 % in the fifth segment.

5. Detailed uncertainty budget, NRC

Serial No. of item(s): 1779882, 1779885 Influence factor y_i	Standard uncertainty $u(y_i)$	Distribution /method of evaluation *Method/(A, B)*	Sensitivity coefficient c_i	Uncertainty contribution $u(R_i)$ (Ω)	Degrees of freedom ν_i
Scaling / traceability			(included below)		
Reference standard(s)	1.00E-08	Normal/B	1 Ω	1.00E-08	4.9
Measuring apparatus	2.50E-09	Rectangular/B	1 Ω	2.50E-09	4899
Ambient conditions:					
Temperature	1.80E-03 °C	Normal/B	2.10E-06 Ω/°C	3.70E-09	39
Pressure	1.01E-02 kPa	Normal/B	5.75E-09 Ω/kPa	1.00E-10	39
Standard deviation	1.42E-08	Normal/A	1 Ω	1.42E-08	34
Repeatability	0	–	1 Ω	–	–
Corrections applied					
Pressure	6.00E-10 Ω/kPa	Normal/B	1.60 kPa	9.60E-10	∞
Power level	1.00E-06 Ω/W	Normal/B	0.0075 W	7.50E-09	∞
RSS of Type A standard uncertainties and effective degrees of freedom:				1.42E-08	34
RSS of Type B standard uncertainties and effective degrees of freedom:				1.34E-08	32
Combined standard uncertainty and effective degrees of freedom:				1.96E-08	32
Expanded uncertainty (95 % coverage factor):				3.92E-08 Ω	

Method: The measurements were made using an automated current comparator bridge, using two CSIRO-type 1 Ω resistors as working standards. The working standards have been maintained with respect to the NRC quantum Hall effect standard since 1991. Their values are established regularly, using a cryogenic current comparator in a two-step process via a 100 Ω standard maintained in its own oil bath. Each comparison resistor was measured in turn, with interchange of like-value standards in the current comparator bridge to reduce any error due to bridge asymmetry. Measurements were repeated on twenty separate occasions in the first round and on 35 separate occasions in the second round of the comparison.

Measurement temperature control: The reference standards and comparison standards were measured in a mineral oil bath maintained at nominal 25.000 °C.
Test current: Direct current with reversal, measured at 100 mA.
Pressure: Typical barometric pressure including oil above the resistors was 102.9 kPa.
Humidity: Relative humidity in the laboratory averaged 37 % in the first round, 25 % in the second round.

6. Detailed uncertainty budget, UTE

Serial No. of item(s): 1779882, 1779885	Standard uncertainty	Distribution /method of evaluation	Sensitivity coefficient	Uncertainty contribution	Degrees of freedom
Influence factor y_i	$u(y_i)$	*Method/(A, B)*	c_i	$u(R_i)$ (Ω)	ν_i
Scaling / traceability	5.00E-07	Normal/B	1 Ω	5.00E-07	∞
Reference standard(s)	2.66E-07	Normal/B	1 Ω	2.66E-07	4
Measuring apparatus	1.75E-07	Rectangular/B	1 Ω	1.75E-07	∞
Ambient conditions:					
Temperature	1.00E-01 °C	Normal/B	1.20E-06 Ω/°C	1.20E-07	∞
Pressure	2.00E-01 kPa	Normal/B	6.00E-09 Ω/kPa	1.20E-09	∞
Standard deviation	2.70E-07	Normal/A	1 Ω	2.70E-07	11
Repeatability	1.90E-07	Normal/B	1 Ω	1.90E-07	11
Corrections applied					
Temperature	6.40E-9 Ω/°C	Normal/B	2.80 °C	1.79E-08	∞
Pressure	6.00E-10 Ω/kPa	Normal/B	0.32 kPa	1.90E-10	∞
RSS of Type A standard uncertainties and effective degrees of freedom:				2.70E-07	11
RSS of Type B standard uncertainties and effective degrees of freedom:				6.34E-07	∞
Combined standard uncertainty and effective degrees of freedom:				6.89E-07	∞
Expanded uncertainty (95 % coverage factor):				1.38E-06 Ω	

Method: All measurements were carried out with a stable DC current applied to the resistors (under test and reference) connected in series, and the voltage drop across each resistor measured with a high-resolution voltmeter. Traceability was obtained through a 1 Ω standard calibrated with respect to the PTB quantum Hall effect standard in 1998, and calibrated by INTI in 2001 and 2004. Measurements were repeated on six days in the first round of the comparison. The UTE did not participate in the second round.

Measurement temperature control: The reference standards and comparison standards were measured in a mineral oil bath maintained at room temperature. The temperature uncertainty reflects the correlation between the temperatures of the resistors.
Test current: Direct current with reversal, measured at 100 mA.
Pressure: Typical barometric pressure including oil above the resistors was 100.7 kPa.
Humidity: Relative humidity in the laboratory averaged 48 % in the first round.

Appendix D. Uncertainty budgets for 1 megohm

7. Detailed uncertainty budget, CENAM

Serial No. of item(s): 8409006, 8409008	Standard uncertainty	Distribution /method of evaluation	Sensitivity coefficient	Uncertainty contribution	Degrees of freedom
Influence factor y_i	$u(y_i)$	*Method/(A, B)*	c_i	$u(R_i)$ (MΩ)	ν_i
Scaling / traceability	3.20E-07	Normal/B	1 MΩ	3.20E-07	∞
Reference standard(s)	3.75E-07	Normal/B	1 MΩ	3.75E-07	∞
Measuring apparatus	5.70E-08	Rectangular/B	1 MΩ	5.70E-08	∞
Leakage effects	5.00E-07	Normal/B	1 MΩ	5.00E-07	∞
Ambient conditions:					
Temperature	2.00E-01 °C	Normal/B	0.02 Ω/°C	4.00E-09	∞
Standard deviation	1.52E-09	Normal/A	1 MΩ	1.52E-09	19
Repeatability	9.00E-09	Normal/B	1 MΩ	9.00E-09	19
Corrections applied:					
Temperature	6.00E-03 Ω/°C	Normal/B	0.4 °C	2.40E-09	∞
RSS of Type A standard uncertainties and effective degrees of freedom:				1.10E-08	19
RSS of Type B standard uncertainties and effective degrees of freedom:				7.05E-07	∞
Combined standard uncertainty and effective degrees of freedom:				7.05E-07	∞
Expanded uncertainty (95 % coverage factor):				1.41E-06 MΩ	

Method: The system used for the measurement of the 1 MΩ resistors was a programmable automatic high resistance bridge, operating on the principle of a potentiometer based on a binary voltage divider. Traceability is derived from the CENAM quantum Hall effect standard used with potentiometric comparison to 10 kΩ. The automatic high resistance bridge was used with two Hamon transfer standards of 100 kΩ per step to determine the value of a CENAM 1 MΩ standard resistor. Interchange of the like-value 1 MΩ standards was used to reduce bridge ratio errors. Measurements were repeated on ten days in each round of the comparison.

Measurement temperature control: The reference standard was measured in a mineral oil bath maintained at nominal 25.00 °C with a standard uncertainty of 0.02 °C; no observable variations in the oil bath temperature was detected during the measurements. The comparison standards were measured in laboratory air at an average temperature of 23.4 °C.
Test voltage: Direct current with reversal, measured at 25 V.
Pressure: Typical barometric pressure including oil above the resistors was 81.3 kPa.
Humidity: Relative humidity in the laboratory averaged 45 % in the first round, 54 % in the second round.

8. Detailed uncertainty budget, INMETRO

Serial No. of item(s): 8409006, 8409008	Standard uncertainty	Distribution /method of evaluation	Sensitivity coefficient	Uncertainty contribution	Degrees of freedom
Influence factor y_i	$u(y_i)$	*Method/(A, B)*	c_i	$u(R_i)$ (MΩ)	ν_i
Scaling / traceability	6.00E-07	Rectangular/B	1 MΩ	6.00E-07	∞
Reference standard(s)	7.10E-07	Normal/B	1 MΩ	7.10E-07	6
Measuring apparatus	5.00E-07	Rectangular/B	1 MΩ	5.00E-07	∞
Leakage effects	0	Rectangular/B	0	0	∞
Ambient conditions:					
Temperature (Rs)	1.00E-02 °C	Rectangular/B	7.00 Ω/°C	7.00E-08	∞
Temperature (R*x*)	1.10E-01 °C	Rectangular/B	0.02 Ω/°C	2.20E-09	∞
Standard deviation	9.35E-08	Normal/A	1 MΩ	9.35E-08	35
Repeatability	7.08E-08	Normal/B	1 MΩ	7.08E-08	35
Corrections applied:					
Temperature	6.00E-03 Ω/°C	Normal/B	0	0	∞
RSS of Type A standard uncertainties and effective degrees of freedom:				9.35E-08	35
RSS of Type B standard uncertainties and effective degrees of freedom:				1.06E-06	∞
Combined standard uncertainty and effective degrees of freedom:				1.06E-06	∞
Expanded uncertainty (95 % coverage factor):				2.13E-06 MΩ	

Method: All measurements of the 1 MΩ resistors were carried out using the four terminal configuration of an automatic high resistance bridge, operating on the principle of a potentiometer based on a binary voltage divider with 10 V maximum supply voltage. Traceability was obtained through 1 Ω and 10 kΩ standards calibrated with respect to the BIPM quantum Hall effect standard. The results refer to the mean value of 21 series of 10 measurements in the first round of the comparison and 18 series of 10 measurements in the second round.

Measurement temperature control: The INMETRO 100 kΩ standard was measured in a mineral oil bath maintained at nominal 23.00 °C with a standard uncertainty of 0.01 °C. The comparison standards were maintained in an air bath at 23.00 °C with a standard uncertainty of 0.11 °C.
Test voltage: The voltage divider works with the voltage assuming several values between 0 and 10 V. The test voltage is assumed to be a rectangular distribution with 5 V average and 5 V half-width.
Humidity: Relative humidity in the laboratory averaged 55 % in both rounds.

9. Detailed uncertainty budget, INTI

Serial No. of item(s): 8409006, 8409008	Standard uncertainty	Distribution /method of evaluation	Sensitivity coefficient	Uncertainty contribution	Degrees of freedom
Influence factor y_i	$u(y_i)$	*Method/(A, B)*	c_i	$u(R_i)$ (MΩ)	ν_i
Scaling / traceability	1.89E-06	Normal/B	1 MΩ	1.89E-06	100
Reference standard(s)	2.00E-07	Normal/B	1 MΩ	2.00E-07	100
Measuring apparatus	Included in Scaling /traceability				
Leakage effects	Negligible in this method				
Ambient conditions:					
Temperature (22.75 °C)	2.50E-01 °C	Rectangular/B	0.02 Ω/°C	5.00E-09	100
Standard deviation	1.74E-07	Normal/A	1 MΩ	1.74E-07	21
Repeatability	5.20E-07	Normal/B	1 MΩ	5.20E-07	21
Corrections applied:					
Temperature	6.00E-03 Ω/°C	Normal/B	0.25 °C	1.50E-09	∞
RSS of Type A standard uncertainties and effective degrees of freedom:				1.79E-07	21
RSS of Type B standard uncertainties and effective degrees of freedom:				1.97E-06	∞
Combined standard uncertainty and effective degrees of freedom:				1.98E-06	∞
Expanded uncertainty (95 % coverage factor):			3.96E-06 MΩ		

Method: Both resistors were measured with an automated guarded dual voltage source bridge. The bridge balance condition is calculated from the mean of two sets of zero readings and two sets of readings near the bridge balance point. The measurement is repeated at the opposite polarity of applied voltage. The resistance ratio is calculated after corrections are applied to the measured voltages, and is the average for both polarities.

Traceability was obtained through a 10 kΩ standard calibrated with respect to the PTB quantum Hall effect standard and verified against INTI's quantum Hall effect standard. A Hamon transfer standard of 100 kΩ per step was used, first in the parallel configuration against the 10 kΩ standard. Then the comparison resistors were measured against this recently calibrated 100 kΩ Hamon resistor. Finally, the two 1 MΩ comparison standards were compared against each other. A matrix equation was solved to find the value of each unknown. Measurements were repeated on nine days in the first round of the comparison, and eleven days in the second.

Measurement temperature control: The reference and comparison standards were measured in laboratory air at an average temperature of 23.09 °C in the first round, and 22.75 °C in the second round.
Test voltage: The test voltage for the resistors was 10 V.
Humidity: Relative humidity in the laboratory averaged 46 % in the first round, 45 % in the second round.

10. Detailed uncertainty budget, NIST

Serial No. of item(s): 8409006, 8409008	Standard uncertainty	Distribution /method of evaluation	Sensitivity coefficient	Uncertainty	Degrees of freedom
Influence factor y_i	$u(y_i)$	*Method/(A, B)*	c_i	$u(R_i)$ (MΩ)	ν_i
Scaling / traceability	5.00E-08	Normal/B	1 MΩ	5.00E-08	∞
Reference standard(s)	1.00E-07	Normal/B	1 MΩ	1.00E-07	32
Measuring apparatus	1.00E-08	Rectangular/B	1 MΩ	1.00E-08	∞
Leakage effects	3.00E-08	Normal/B	0	3.00E-08	∞
Ambient conditions:					
Temperature (23.50 °C)	1.00E-01 °C	Rectangular/B	0.02 Ω/°C	2.00E-09	∞
Standard deviation	3.50E-08	Normal/A	1 MΩ	3.50E-08	119
Repeatability	9.00E-09	Normal/B	1 MΩ	9.00E-09	119
Corrections applied:					
Temperature (to 23 °C)	6.00E-03 Ω/°C	Normal/B	0.50 °C	3.00E-09	∞
RSS of Type A standard uncertainties and effective degrees of freedom:				3.50E-08	119
RSS of Type B standard uncertainties and effective degrees of freedom:				1.17E-07	∞
Combined standard uncertainty and effective degrees of freedom:				1.21E-07	∞
Expanded uncertainty (95 % coverage factor):				2.41E-07 MΩ	

Method: The NIST quantum Hall effect standard was compared twice each year to a group of 1 MΩ standards using two cryogenic current comparator bridges. These bridges are two-terminal resistance bridges that make use of six contacts of the quantum Hall device to reduce lead resistance effects. A digital multimeter was used to make lead resistance measurements which are subtractive corrections to the 1 MΩ resistance values. An automatic guarded bridge of the Warshawsky design was used to make four-terminal resistance comparisons between these 1 MΩ standards and the comparison resistors. Measurements were repeated on nine to sixteen days in each pilot laboratory segment of the comparison.

Measurement temperature control: The reference and comparison standards were measured in laboratory air at an average temperature of 23.47 °C in the first segment, 23.50 °C in the second segment, 23.50 °C in the third segment, 23.38 °C in the fourth segment, and 23.10 °C in the fifth segment.
Test voltage: Direct current with reversal, measured at 33.1 V.
Humidity: Relative humidity in the laboratory averaged 32 % in the first segment, 34 % in the second segment, 32 % in the third segment, 36 % in the fourth segment, and 42 % in the fifth segment.

11. Detailed uncertainty budget, NRC

Serial No. of item(s): 8409006, 8409008	Standard uncertainty	Distribution /method of evaluation	Sensitivity coefficient	Uncertainty	Degrees of freedom
Influence factor y_i	$u(y_i)$	*Method/(A, B)*	c_i	$u(R_i)$ (MΩ)	ν_i
Scaling / traceability	(included below)				
Reference standard(s)	1.06E-06	Normal/B	1 MΩ	1.06E-06	503
Measuring apparatus	7.99E-08	Normal/B	1 MΩ	7.99E-08	2799
Leakage effects	1.00E-08	Normal/B	1 MΩ	1.00E-08	4.9
Ambient conditions:					
Temperature (23.54 °C)	1.48E-01 °C	Normal/B	0.02 Ω/°C	2.96E-09	1463
6) Standard deviation	2.19E-07	Normal/A	1 MΩ	2.19E-07	125
7) Repeatability	–	–	1 MΩ	–	–
8) Corrections applied:					
Temperature (to 23 °C)	6.00E-03 Ω/°C	Normal/B	0.54 °C	3.24E-09	∞
RSS of Type A standard uncertainties and effective degrees of freedom:				2.19E-07	125
RSS of Type B standard uncertainties and effective degrees of freedom:				1.06E-06	∞
Combined standard uncertainty and effective degrees of freedom:				1.08E-06	∞
Expanded uncertainty (95 % coverage factor):			2.15E-06 MΩ		

Method: Measurements were made with an automated Wheatstone bridge circuit. The resistors each were compared with two well-known 1 MΩ working standards in turn, using the bridge scanner to switch the resistors. The working standards were also compared to each other, as were the two working standards. During each comparison the two resistors were interchanged to minimize bridge error. Before and after each measurement period, the same bridge and a pair of 100 kΩ standards were used to relate the 1 MΩ working standards to a 10 kΩ working standard located in the same bath. The 10 kΩ working standard is compared regularly with the NRC quantized Hall effect standard using the NRC cryogenic current comparator. Each comparison resistor was measured on six separate occasions in the first round, and three separate occasions in the second round.

Measurement temperature control: The reference and comparison standards were measured in laboratory air at an average temperature of 23.71 °C in the first round, and 23.54 °C in the second round.
Test voltage: The test voltage for the resistors was 10 V.
Humidity: Relative humidity in the laboratory averaged 38 % in the first round, 25 % in the second round.

12. Detailed uncertainty budget, UTE

Serial No. of item(s): 8409006, 8409008	Standard uncertainty	Distribution /method of evaluation	Sensitivity coefficient	Uncertainty	Degrees of freedom
Influence factor y_i	$u(y_i)$	*Method/(A, B)*	c_i	$u(R_i)$ (MΩ)	ν_i
Scaling / traceability	1.93E-06	Normal/B	1 MΩ	1.93E-06	∞
Reference standard(s)	1.06E-06	Normal/B	1 MΩ	1.06E-06	4
Measuring apparatus	1.01E-06	Rectangular/B	1 MΩ	1.01E-06	∞
Leakage effects	0	Normal/B	0	0	∞
Ambient conditions:					
Temperature (22.20 °C)	1.00E-01 °C	Rectangular/B	3.50 Ω/°C	3.50E-07	∞
6) Typ. standard deviation	4.66E-08	Normal/A	1 MΩ	4.66E-08	11
7) Repeatability	3.70E-07	Normal/B	1 MΩ	3.70E-07	11
8) Corrections applied:					
Temperature (to 23 °C)	9.80E-03 Ω/°C	Normal/B	0.80 °C	7.90E-09	∞
RSS of Type A standard uncertainties and effective degrees of freedom:				4.66E-08	11
RSS of Type B standard uncertainties and effective degrees of freedom:				2.48E-06	∞
Combined standard uncertainty and effective degrees of freedom:				2.48E-06	∞
Expanded uncertainty (95 % coverage factor):				4.95E-06 MΩ	

Method: A stable DC voltage was applied to both resistors connected in series, and the voltage drop across each resistor measured with a Kelvin-Varley divider and a high-resolution voltmeter. The reference resistor was a 100 kΩ standard, with scaling from 1 Ω provided using the same comparison method. Traceability was obtained through a 1 Ω standard calibrated with respect to the PTB quantum Hall effect standard in 1998, and calibrated by INTI in 2001 and 2004. Measurements were repeated on six days in the first round of the comparison. The UTE did not participate in the second round.

Measurement temperature control: The scaling and reference standards were measured in a mineral oil bath at room temperature. The comparison standards were maintained in an air enclosure at room temperature. The temperature uncertainty reflects the correlation between the temperatures of the resistors.
Test voltage: Direct current with reversal, measured at 30 V.
Humidity: Relative humidity in the laboratory averaged 46 % in the first round.

Appendix E. Uncertainty budgets 1 for gigaohm

13. Detailed uncertainty budget, CENAM

Serial No. of item(s): HR9104, HR9105	Standard uncertainty	Distribution /method of evaluation	Sensitivity coefficient	Uncertainty contribution	Degrees of freedom
Influence factor y_i	$u(y_i)$	*Method/(A, B)*	c_i	$u(R_i)$ (GΩ)	ν_i
Scaling / traceability	5.00E-06	Normal/B	1 GΩ	5.00E-06	∞
Reference standard(s)	5.00E-06	Normal/B	1 GΩ	5.00E-06	400
Measuring apparatus	5.00E-06	Normal/B	1 GΩ	5.00E-06	∞
Leakage effects	5.00E-06	Normal/B	1 GΩ	5.00E-06	∞
Ambient conditions:					
Temperature (Rs)	5.00E-02 °C	Normal/B	2.20E-05 GΩ/°C	1.10E-06	∞
Temperature (Rs)	2.00E-01 °C	Normal/B	5.00E-06 GΩ/°C	1.00E-06	∞
Standard deviation	7.79E-07	Normal/A	1 GΩ	7.79E-07	19
Repeatability	4.50E-07	Normal/B	1 GΩ	4.50E-07	19
Corrections applied:					
Temperature	1.30E-06 GΩ/°C	Normal/B	0.5 °C	6.50E-07	∞
RSS of Type A standard uncertainties and effective degrees of freedom:				4.50E-07	379
RSS of Type B standard uncertainties and effective degrees of freedom:				1.01E-05	∞
Combined standard uncertainty and effective degrees of freedom:				1.02E-05	∞
Expanded uncertainty (95 % coverage factor):				2.03E-05 GΩ	

Method: The system used for the measurement of the 1 GΩ resistors was a manual active-arm bridge which is formed by substituting two of the resistive arms of a Wheatstone bridge circuit with low impedance voltage calibrator sources, which are traceable to the Josephson voltage standard maintained at CENAM. The current flowing in the unknown resistor R_X is changed by altering the voltage of the supply connected across this resistor. The balance voltage V_X is subsequently calculated from the mean of the two sets of zero readings and three sets of readings near the bridge balance point. The measurement is repeated at the opposite polarity and the final ratio is the average of V_X / V_S obtained for both polarities.

Resistance standards traceability is derived from the CENAM quantum Hall effect standard used with potentiometric comparison to 10 kΩ. An automatic high resistance bridge was used with two Hamon transfer standards of 100 kΩ per step to determine the value of a CENAM 1 MΩ standard resistor. The resistance values of the 10 MΩ and 100 MΩ resistors were determined with 10:1 ratio measurements using the automatic high resistance bridge. Measurements were repeated ten times over five days in the first round of the comparison, and ten times over ten days in the second round.

Measurement temperature control: The reference and comparison standards were measured in laboratory air at an average temperature of 23.06 °C in the first round, and 22.88 °C in the second round. The mean temperature detected by the thermistors in the resistors under test was 23.35 °C in the first round, and 22.86 °C in the second round.
Test voltage: The test voltage for the resistors was 100 V.
Humidity: Relative humidity in the laboratory averaged 51 % in the first round, 52 % in the second round.

14. Detailed uncertainty budget, INMETRO

Serial No. of item(s): HR9104, HR9105	Standard uncertainty	Distribution /method of evaluation	Sensitivity coefficient	Uncertainty contribution	Degrees of freedom
Influence factor y_i	$u(y_i)$	*Method/(A, B)*	c_i	$u(R_i)$ (GΩ)	ν_i
Scaling / traceability	3.10E-06	Rectangular/B	1 GΩ	3.10E-06	∞
Reference standard(s)	3.97E-06	Normal/B	1 GΩ	3.97E-06	6
Measuring apparatus	5.00E-07	Rectangular/B	1 GΩ	5.00E-07	∞
Ambient conditions:					
Temperature	1.10E-01 °C	Rectangular/B	2.20E-05 GΩ/°C	2.42E-06	∞
Voltage	2.89E-00 V	Rectangular/B	3.80E-08 GΩ/V	1.10E-07	∞
Standard deviation	3.31E-06	Normal/A	1 GΩ	3.31E-06	38
Repeatability	2.40E-06	Normal/B	1 GΩ	2.40E-06	38
Corrections applied:					
Voltage	4.00E-09 GΩ/V	Normal/B	95 V	3.80E-07	∞
RSS of Type A standard uncertainties and effective degrees of freedom:				3.31E-06	38
RSS of Type B standard uncertainties and effective degrees of freedom:				6.12E-06	∞
Combined standard uncertainty and effective degrees of freedom:				6.95E-06	∞
Expanded uncertainty (95 % coverage factor):				1.39E-05 GΩ	

Method: All measurements of the 1 GΩ resistors were carried out using an automatic high resistance bridge, operating on the principle of a potentiometer based on a binary voltage divider with 10 V maximum supply voltage. Ratios of 10:1 were used step up from 10 kΩ to 1 GΩ. Traceability was obtained through 1 **Ω** and 10 k**Ω** standards calibrated with respect to the BIPM quantum Hall effect standard. The results refer to the mean value of 21 series of 10 measurements in the first round of the comparison and 12 series of 10 measurements in the second round.

Measurement temperature control: The INMETRO standards and the comparison standards were maintained in an air bath at 23.00 °C with a standard? uncertainty of 0.2 °C in the first round, reduced to a standard uncertainty of 0.11 °C in the second round. The mean temperature detected by the thermistors in the resistors under test was 23.17 °C in the first round, and 23.92 °C in the second round. Because of the difference in the second round between the reported air bath and thermistor temperatures, and after discussion with the pilot laboratory, INMETRO elected to base the results on the air bath temperature in the second round.
Test voltage: The voltage divider works with the voltage assuming several values between 0 and 10 V. The test voltage is assumed to be a rectangular distribution with 5 V average and 5 V half-width.
Humidity: Relative humidity in the laboratory averaged 55 % in both rounds.

15. Detailed uncertainty budget, INTI

Serial No. of item(s): HR9104, HR9105	Standard uncertainty	Distribution /method of evaluation	Sensitivity coefficient	Uncertainty contribution	Degrees of freedom
Influence factor y_i	$u(y_i)$	*Method/(A, B)*	c_i	$u(R_i)$ (GΩ)	ν_i
Scaling / traceability	1.97E-06	Normal/B	1 GΩ	1.97E-06	100
Reference standard(s)	2.00E-07	Normal/B	1 GΩ	2.00E-07	100
Measuring apparatus	(included in Scaling / traceability)				
Ambient conditions:					
Temperature	2.89E-01 °C	Rectangular/B	2.26E-05 GΩ/°C	6.53E-06	100
Standard deviation	8.00E-07	Normal/A	1 GΩ	8.00E-07	23
Repeatability	2.70E-06	Normal/B	1 GΩ	2.70E-06	23
Corrections applied:					
Temperature	1.30E-06 GΩ/°C	Normal/B	0.37 °C	4.80E-07	∞
RSS of Type A standard uncertainties and effective degrees of freedom:				8.00E-07	23
RSS of Type B standard uncertainties and effective degrees of freedom:				7.35E-06	∞
Combined standard uncertainty and effective degrees of freedom:				7.40E-06	∞
Expanded uncertainty (95 % coverage factor):				1.48E-05 GΩ	

Method: Both resistors were measured with an automated guarded active-arm bridge that is formed by substituting two of the resistive arms of a Wheatstone bridge circuit with low impedance voltage calibrator sources. The bridge balance condition was calculated from the mean of two sets of detector readings at zero voltage and two sets of readings near the bridge balance point. The measurement was repeated at the opposite polarity of applied voltage. The resistance ratio was calculated after corrections were applied to the measured voltages, and is the average for both polarities.

Traceability was obtained through a 10 kΩ standard calibrated with respect to the PTB quantum Hall effect standard and verified against INTI's PTB quantum Hall effect standard. A Hamon transfer standard of 100 kΩ per step was used, first in the parallel configuration against the 10 kΩ standard. A second Hamon standard of 10 MΩ per step was compared to this transfer standard, then the comparison resistors were measured using this Hamon standard in the 100 MΩ series configuration. A matrix equation was solved to find the value of each unknown. Measurements were repeated on eight days in the first round of the comparison, and 12 days in the second round.

Measurement temperature control: The reference and comparison standards were measured in laboratory air at an average temperature of 23.10 °C in the first round, and 22.63 °C in the second round.
Test voltage: The test voltage for the resistors was 100 V.
Humidity: Relative humidity in the laboratory averaged 48 % in the first round, 45 % in the second round.

16. Detailed uncertainty budget, NIST

Serial No. of item(s): HR9104, HR9105	Standard uncertainty	Distribution /method of evaluation	Sensitivity coefficient	Uncertainty contribution	Degrees of freedom
Influence factor y_i	$u(y_i)$	*Method/(A, B)*	c_i	$u(R_i)$ (GΩ)	ν_i
Scaling / traceability	5.00E-07	Normal/B	1 GΩ	5.00E-07	∞
Reference standard(s)	2.00E-06	Normal/B	1 GΩ	2.00E-06	400
Measuring apparatus	1.12E-06	Rectangular/B	1 GΩ	1.12E-06	∞
Leakage effects	1.00E-07	Rectangular/B	1 GΩ	1.00E-07	∞
Ambient conditions:					
Temperature	5.00E-02 °C	Rectangular/B	2.20E-05 GΩ/°C	1.10E-06	∞
Voltage	2.00E-01 V	Rectangular/B	3.80E-08 GΩ/V	7.60E-09	∞
Standard deviation	9.20E-07	Normal/A	1 GΩ	9.20E-07	89
Repeatability	2.20E-07	Normal/B	1 GΩ	2.20E-07	89
Corrections applied:					
Temperature	1.30E-06 GΩ/°C	Normal/B	0.02 °C	2.60E-08	∞
RSS of Type A standard uncertainties and effective degrees of freedom:				9.20E-07	89
RSS of Type B standard uncertainties and effective degrees of freedom:				2.69E-06	∞
Combined standard uncertainty and effective degrees of freedom:				2.81E-06	∞
Expanded uncertainty (95 % coverage factor):				5.62E-06 GΩ	

Method: The NIST quantum Hall effect standard was compared twice each year to a group of 1 MΩ standards using two cryogenic current comparator bridges. Also twice a year, two or three of these 1 MΩ standards were compared to Hamon standards using an automatic guarded active-arm bridge. The scaling procedure used intermediate Hamon standards, one of value 1 MΩ per step and two of 10 MΩ per step, to assign values to one 100 MΩ per step Hamon standard. This Hamon standard is used in the 1 GΩ configuration as the reference standard in active-arm bridge comparisons with the 1 GΩ comparison standards. Measurements were repeated on nine to twelve days in each pilot laboratory segment of the comparison.

Measurement temperature control: The reference and comparison standards were measured in laboratory air at an average temperature of 23.47 °C in the first segment, 23.50 °C in the second segment, 23.50 °C in the third segment, 23.38 °C in the fourth segment, and 23.10 °C in the fifth segment.
Test voltage: Direct current with reversal, measured at 100 V.
Humidity: Relative humidity in the laboratory averaged 32 % in the first segment, 34 % in the second segment, 32 % in the third segment, 36 % in the fourth segment, and 42 % in the fifth segment.

17. Detailed uncertainty budget, NRC

Serial No. of item(s): HR9104, HR9105	Standard uncertainty	Distribution /method of evaluation	Sensitivity coefficient	Uncertainty contribution	Degrees of freedom
Influence factor y_i	$u(y_i)$	*Method/(A, B)*	c_i	$u(R_i)$ (GΩ)	ν_i
Scaling / traceability	(included below)				
Reference standard(s)	2.42E-06	Normal/B	1 GΩ	2.42E-06	251
Measuring apparatus	7.33E-07	Normal/B	1 GΩ	7.33E-07	251
Leakage Effects	1.00E-05	Normal/B	1 GΩ	1.00E-05	4.9
Ambient conditions:					
Temperature	1.00E-01 °C	Normal/B	2.10E-05 GΩ/°C	2.10E-06	243
Voltage	6.00E-02 V	Normal/B	3.80E-08 GΩ/V	3.80E-09	1599
Standard deviation	7.47E-07	Normal/A	1 GΩ	1.90E-06	125
Repeatability	(data not supplied)				
Corrections applied:					
Temperature	1.30E-06 GΩ/°C	Normal/B	0.75 °C	9.80E-07	∞
Voltage	4.00E-09 GΩ/V	Normal/B	9.09 V	3.60E-08	∞
RSS of Type A standard uncertainties and effective degrees of freedom:				1.90E-06	125
RSS of Type B standard uncertainties and effective degrees of freedom:				1.05E-05	∞
Combined standard uncertainty and effective degrees of freedom:				1.06E-05	∞
Expanded uncertainty (95 % coverage factor):				2.16E-05 GΩ	

Method: Measurements were made with an automated Wheatstone bridge circuit. The resistors each were compared with two well-known 100 MΩ working standards in turn, using the bridge scanner to switch the resistors. Throughout the measurement period, the working standards were also compared to each other. Before and after the measurement period, the same bridge and a set of decade value standards were used to relate the 100 MΩ working standards to a 10 kΩ working standard located in the same bath. The 10 kΩ working standard is compared regularly with the NRC quantized Hall effect standard using the NRC cryogenic current comparator. Because the values of the two test resistors appeared to vary in a somewhat unexpected way with time, results were derived from the last two to five days' results in the first round of the comparison. During the second round, all the data collected were used to determine average values for the resistors.

Measurement temperature control: The reference and comparison standards were measured in laboratory air at an average temperature of 23.67 °C in the first round, and 23.75 °C in the second round.
Test voltage: The test voltage for the resistors was 90.91 V.
Humidity: Relative humidity in the laboratory averaged 38 % in the first round, 25 % in the second round.

18. Detailed uncertainty budget, UTE

Serial No. of item(s): HR9104, HR9105	Standard uncertainty	Distribution /method of evaluation	Sensitivity coefficient	Uncertainty contribution	Degrees of freedom
Influence factor y_i	$u(y_i)$	*Method/(A, B)*	c_i	$u(R_i)$ (GΩ)	ν_i
Scaling / traceability	2.44E-06	Normal/B	1 GΩ	2.44E-06	∞
Reference standard(s)	5.66E-07	Normal/B	1 GΩ	5.66E-07	4
Measuring apparatus	2.18E-05	Rectangular/B	1 GΩ	2.18E-05	∞
Ambient conditions:					
Temperature	1.00E-01 °C	Rectangular/B	2.25E-05 GΩ/°C	2.25E-06	∞
Voltage	1.00E-01 V	Rectangular/B	5.00E-08 GΩ/V	5.00E-09	∞
Standard deviation	1.90E-06	Normal/A	1 GΩ	1.90E-06	11
Repeatability	9.50E-07	Normal/B	1 GΩ	9.50E-07	11
Corrections applied:					
Temperature	1.30E-06 GΩ/°C	Normal/B	0.32 °C	4.16E-07	∞
RSS of Type A standard uncertainties and effective degrees of freedom:				1.90E-06	11
RSS of Type B standard uncertainties and effective degrees of freedom:				2.21E-05	∞
Combined standard uncertainty and effective degrees of freedom:				2.22E-05	∞
Expanded uncertainty (95 % coverage factor):				4.44E-05 GΩ	

Method: A stable DC voltage was applied to both resistors connected in series, and the voltage drop across each resistor measured with a high-resolution voltmeter. The reference resistor was a 1 MΩ standard, with scaling from 1 Ω provided using the voltage divider method with a Kelvin-Varley divider. Traceability was obtained through a 1 Ω standard calibrated with respect to the PTB quantum Hall effect standard in 1998, and calibrated by INTI in 2001 and 2004. Measurements were repeated on six days in the first round of the comparison. The UTE did not participate in the second round.

Measurement temperature control: The scaling and reference standards were measured in a mineral oil bath at room temperature. The comparison standards were maintained in an air enclosure at room temperature. The temperature uncertainty reflects the correlation between the temperatures of the resistors.
Test voltage: Direct current with reversal, measured at 100 V.
Humidity: Relative humidity in the laboratory averaged 48 % in the first round.

Appendix F. Summary results for RMO comparisons

Table F.1. Matrix of Equivalence for RMO Comparison SIM.EM-K1 at 1 Ω.

RMO Comparison SIM.EM-K1

MEASURAND: Resistance **NOMINAL VALUE: 1 Ω**

The comparison reference value (CRV) for this comparison is obtained from the weighted average of the participants. The comparison reference value is X_{CRV} = -0.5962 x 10^{-6} and the expanded relative uncertainty (k = 2) is U_{CRV} = 0.0094 x 10^{-6} for the 1 Ω resistance level. See equation (21) of Appendix A.

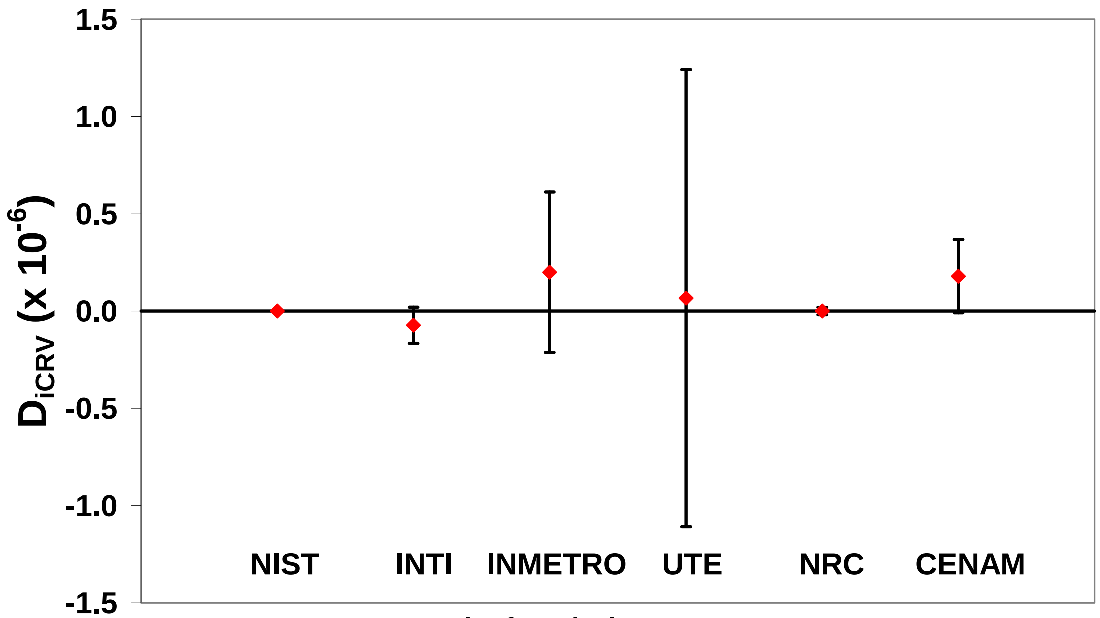
The degree of equivalence of each laboratory with respect to the reference value is given by a pair of numbers: $D_{i\,CRV} = D_{i\,COMB} - X_{CRV}$ and the expanded relative uncertainty $U_{i\,CRV} = (U_{i\,COMB}^2 - U_{CRV}^2)^{1/2}$ where $D_{i\,COMB}$ is the weighted mean of the two 1 Ω traveling standards' differences from a least-squares linear regression of the participants' values for each traveling standard. The calculations are described in detail in equations (24) and (25) of Appendix A.

The degree of equivalence between two laboratories is given by a pair of numbers: $D_{ij} = D_{i\,CRV} - D_{j\,CRV}$ and the expanded relative uncertainty U_{ij}; these calculations are described in detail in equations (26) and (27) of Appendix A.

			Lab *j* →					
			NIST		INTI		INMETRO	
Lab *i* ↓	D_{iCRV}	U_{iCRV}	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}
	(x 10^{-6})		(x 10^{-6})		(x 10^{-6})		(x 10^{-6})	
NIST	0.0003	0.0050			0.0735	0.0938	-0.1992	0.4122
INTI	-0.0732	0.0928	-0.0735	0.0938			-0.2727	0.4226
INMETRO	0.1995	0.4120	0.1992	0.4122	0.2727	0.4226		
UTE	0.0663	1.1750	-0.0660	1.1750	0.1395	1.1788	-0.1332	1.2452
NRC	-0.0001	0.0184	-0.0004	0.0232	0.0731	0.0954	-0.1996	0.4126
CENAM	0.1791	0.1888	0.1788	0.1894	0.2523	0.2108	-0.0203	0.4536

			Lab *j* →					
			UTE		NRC		CENAM	
Lab *i* ↓	D_{iCRV}	U_{iCRV}	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}
	(x 10^{-6})		(x 10^{-6})		(x 10^{-6})		(x 10^{-6})	
NIST	0.0003	0.0050	-0.0660	1.1750	0.0004	0.0232	-0.1788	0.1894
INTI	-0.0732	0.0928	-0.1395	1.1788	-0.0731	0.0954	-0.2523	0.2108
INMETRO	0.1995	0.4120	0.1332	1.2452	0.1996	0.4126	0.0203	0.4536
UTE	0.0663	1.1750			0.0664	1.1752	-0.1129	1.1902
NRC	-0.0001	0.0184	-0.0664	1.1752			-0.1792	0.1902
CENAM	0.1791	0.1888	0.1129	1.1902	0.1792	0.1902		

Graph of equivalence for 1 Ω data from Table F.1.



Graph of Equivalence: SIM.EM-K1
Differences from the Comparison Reference Value at 1 Ω, (*k = 2*)

Table F.2. Matrix of Equivalence for RMO Comparison SIM.EM-S6 at 1 MΩ (k = 2).

RMO Comparison SIM.EM-S6

MEASURAND: Resistance — NOMINAL VALUE: 1 MΩ

The comparison reference value (CRV) for this comparison is obtained from the weighted average of the participants. The comparison reference value is X_{CRV} = 2.6871 x 10^{-6} and the expanded relative uncertainty (k = 2) is U_{CRV} = 0.0846 x 10^{-6} for the 1 MΩ resistance level. See equation (21) of Appendix A.

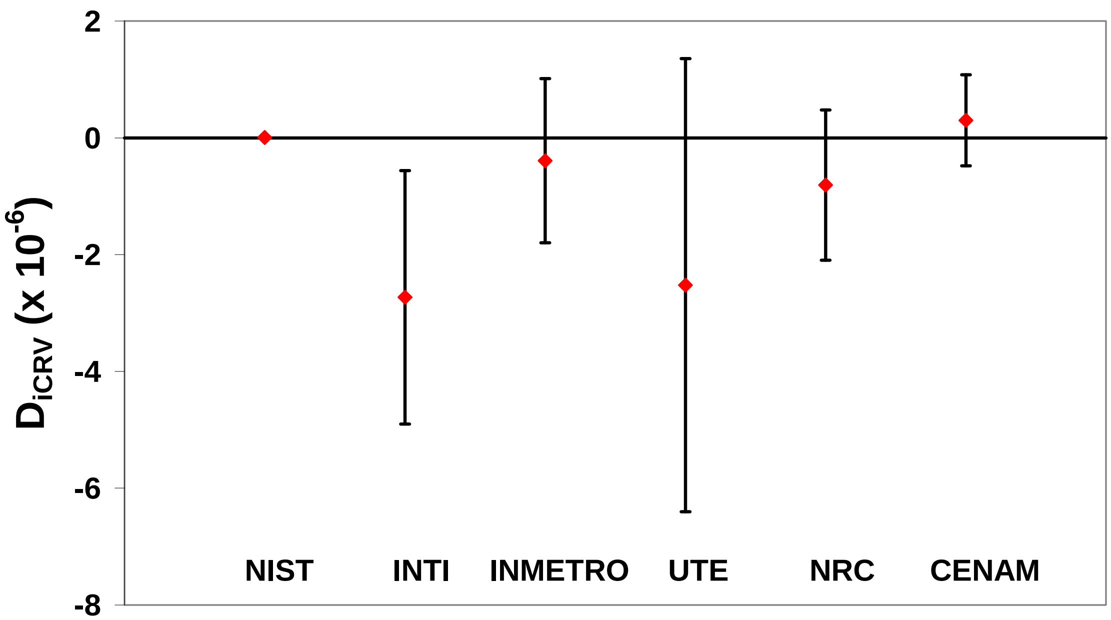
The degree of equivalence of each laboratory with respect to the reference value is given by a pair of numbers: $D_{i\,CRV} = D_{i\,COMB} - X_{CRV}$ and the expanded relative uncertainty $U_{i\,CRV} = (U_{i\,COMB}^2 - U_{CRV}^2)^{1/2}$ where $D_{i\,COMB}$ is the weighted mean of the two 1 MΩ traveling standards' differences from a least-squares linear regression of the participants' values for each traveling standard. The calculations are described in detail in equations (24) and (25) of Appendix A.

The degree of equivalence between two laboratories is given by a pair of numbers: $D_{ij} = D_{i\,CRV} - D_{j\,CRV}$ and the expanded relative uncertainty U_{ij}; these calculations are described in detail in equations (26) and (27) of Appendix A.

Lab i ↓ / Lab j →	D_{iCRV} (x 10^{-6})	U_{iCRV} (x 10^{-6})	NIST D_{ij} (x 10^{-6})	NIST U_{ij}	INTI D_{ij} (x 10^{-6})	INTI U_{ij}	INMETRO D_{ij} (x 10^{-6})	INMETRO U_{ij}
NIST	0.0069	0.0126			2.7385	2.1728	0.3999	1.4114
INTI	-2.7316	2.1694	-2.7385	2.1728			-2.3385	2.5882
INMETRO	-0.3930	1.4062	-0.3999	1.4114	2.3385	2.5882		
UTE	-2.5238	3.8784	-2.5307	3.8802	0.2077	4.4448	-2.1308	4.1272
NRC	-0.8096	1.2868	-0.8165	1.2924	1.9220	2.5254	-0.4166	1.9098
CENAM	0.2993	0.7786	0.2924	0.7878	3.0309	2.3088	0.6924	1.6118

Lab i ↓ / Lab j →	D_{iCRV} (x 10^{-6})	U_{iCRV} (x 10^{-6})	UTE D_{ij} (x 10^{-6})	UTE U_{ij}	NRC D_{ij} (x 10^{-6})	NRC U_{ij}	CENAM D_{ij} (x 10^{-6})	CENAM U_{ij}
NIST	0.0069	0.0126	2.5307	3.8802	0.8165	1.2924	-0.2924	0.7878
INTI	-2.7316	2.1694	-0.2077	4.4448	-1.9220	2.5254	-3.0309	2.3088
INMETRO	-0.3930	1.4062	2.1308	4.1272	0.4166	1.9098	-0.6924	1.6118
UTE	-2.5238	3.8784			-1.7143	4.0882	-2.8232	3.9584
NRC	-0.8096	1.2868	1.7143	4.0882			-1.1089	1.5086
CENAM	0.2993	0.7786	2.8232	3.9584	1.1089	1.5086		

Graph of equivalence for 1 MΩ data from Table F.2 ($k = 2$).



Graph of Equivalence: SIM.EM-S6
Differences from the Comparison Reference Value at 1 MΩ, (*k = 2*)

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Table F.3. Matrix of Equivalence for RMO Comparison SIM.EM-K2 at 1 GΩ (k = 2).

RMO Comparison SIM.EM-K2

MEASURAND: Resistance | NOMINAL VALUE: 1 GΩ

The comparison reference value (CRV) for this comparison is obtained from the weighted average of the participants. The comparison reference value is X_{CRV} = 10.2401 x 10^{-6} and the expanded relative uncertainty (k = 2) is U_{CRV} = 1.8954 x 10^{-6} for the 1 GΩ resistance level. See equation (21) of Appendix A.

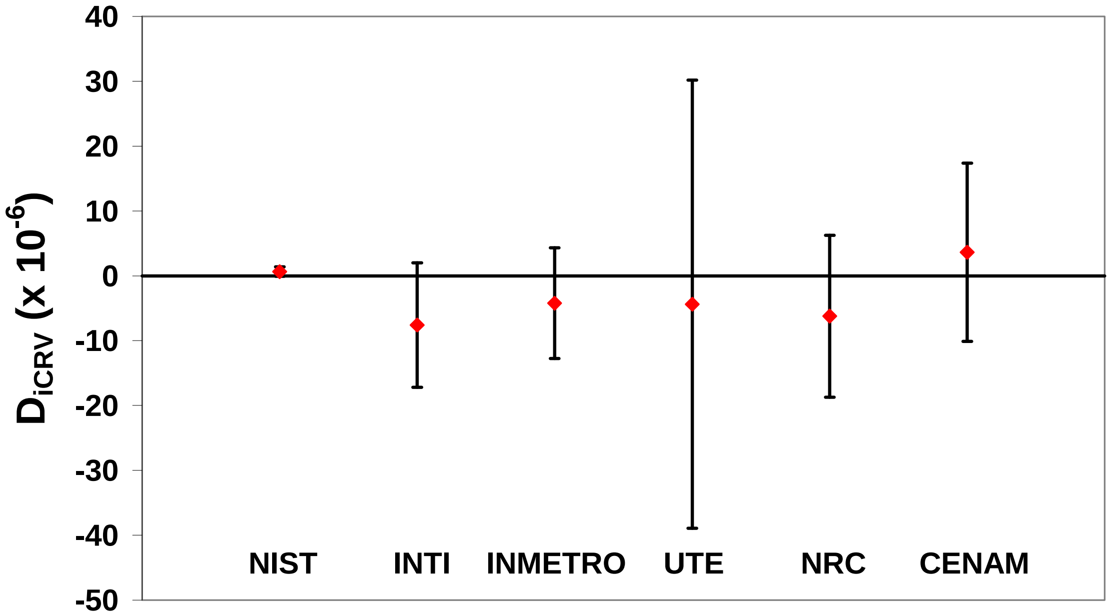
The degree of equivalence of each laboratory with respect to the reference value is given by a pair of numbers: $D_{iCRV} = D_{iCOMB} - X_{CRV}$ and the expanded relative uncertainty $U_{iCRV} = (U_{iCOMB}^2 - U_{CRV}^2)^{1/2}$ where D_{iCOMB} is the weighted mean of the two 1 GΩ traveling standards' differences from a least-squares linear regression of the participants' values for each traveling standard. The calculations are described in detail in equations (24) and (25) of Appendix A.

The degree of equivalence between two laboratories is given by a pair of numbers: $D_{ij} = D_{iCRV} - D_{jCRV}$ and the expanded relative uncertainty U_{ij}; these calculations are described in detail in equations (26) and (27) of Appendix A.

			Lab j →					
			NIST		INTI		INMETRO	
Lab i ↓	D_{iCRV}	U_{iCRV}	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}
	(x 10^{-6})		(x 10^{-6})		(x 10^{-6})		(x 10^{-6})	
NIST	0.6539	0.7304			8.2352	9.9954	4.8798	8.9748
INTI	-7.5813	9.6022	-8.2352	9.9954			-3.3554	13.1130
INMETRO	-4.2259	8.5344	-4.8798	8.9748	3.3554	13.1130		
UTE	-4.3737	34.5658	-5.0276	34.6766	3.2076	35.9496	-0.1479	35.6924
NRC	-6.2296	12.4760	-6.8835	12.7824	1.3517	15.9922	-2.0037	15.3626
CENAM	3.6206	13.7466	2.9666	14.0262	11.2019	17.0364	7.8464	16.4284

			Lab j →					
			UTE		NRC		CENAM	
Lab i ↓	D_{iCRV}	U_{iCRV}	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}
	(x 10^{-6})		(x 10^{-6})		(x 10^{-6})		(x 10^{-6})	
NIST	0.6539	0.7304	5.0276	34.6766	6.8835	12.7824	-2.9666	14.0262
INTI	-7.5813	9.6022	-3.2076	35.9496	-1.3517	15.9922	-11.2019	17.0364
INMETRO	-4.2259	8.5344	0.1479	35.6924	2.0037	15.3626	-7.8464	16.4284
UTE	-4.3737	34.5658			1.8558	36.8770	-7.9943	37.3762
NRC	-6.2296	12.4760	-1.8558	36.8770			-9.8501	18.6936
CENAM	3.6206	13.7466	7.9943	37.3762	9.8501	18.6936		

Graph of equivalence for 1 GΩ data from Table F.3 (k = 2).



Graph of Equivalence: SIM.EM-K2
Differences from the Comparison Reference Value at 1 GΩ, (*k = 2*)

Appendix G. Analysis and results of linking CCEM comparisons and SIM RMO comparisons

Linking CCEM-K2 and BIPM Bilateral to SIM.EM Comparisons

Nien Fan Zhang

1. Linkage between CCEM-K2 and SIM.EM-K2 comparisons

For the CCEM-K2 and SIM.EM comparisons, there are two linking labs: NIST and NRC. In general, we assume that there are *k* linking labs. Based on [1], a correction or a difference between the two comparisons is estimated from the degrees of equivalence between the KCRV and CRV for the linking labs in these two comparisons. Specifically, we denote the degree of equivalence between the n^{th} lab and the KCRV in CCEM-K2 by $D_{n,KCRV}$. The results can be found in the final report of CCEM-K2 [2]. Similarly, we denote the degree of equivalence between the m^{th} lab and the CRV in SIM.EM comparison by $D_{m,CRV}$. The results can be found in Appendix B of this report.

For the k^{th} linking laboratory, the difference between the two degrees of equivalence is

$$D_k = D_{k,KCRV} - D_{k,CRV} \tag{1}$$

for $k = 1,...,K$. From [1] the correction or the difference of the two comparisons is estimated by a weighted mean of $\{D_k\}$. Namely,

$$\hat{D} = \sum_{k=1}^{K} \psi_k D_k \tag{2}$$

where $\{\psi\}$ are the weights, e.g., $\psi_k = 1/k$, which leads to a simple average or

$$\psi_k = \frac{1/\mathrm{u}_{D_\mathrm{k}}^2}{\sum_{j=1}^{K} 1/\mathrm{u}_{D_\mathrm{j}}^2}, \tag{3}$$

where u_{D_j} is the uncertainty for the j^{th} lab including Type A and Type B uncertainties. The quantity $\hat{D}$ is used to estimate the differences between pairs of laboratories for which one laboratory only participated in the CCEM-K2 and the second laboratory only participated in the SIM.EM-K2 comparison.

Specifically, for the m^{th} laboratory, which participated only in the SIM.EM-K2 comparison the $D_{m,CRV}$ needs to have an adjustment to get an estimator of $D_{m,KCRV}$. The estimator is given by

$$D'_{m,KCRV} = D_{m,CRV} + \hat{D}, \tag{4}$$

where $\hat{D}$ is the estimated difference between the two comparisons. $D'_{m,KCRV}$ is the estimated degree of equivalence between the KCRV of the CCEM-K2 and the m^{th} laboratory that participated in SIM.EM-K2 **had this laboratory participated in the CCEM-K2.**

For the pair-wise comparisons - degrees of equivalence of pairs of national measurement standards, i.e., the degrees of equivalence for any pair of two different laboratories in the two comparisons there are three cases.

(1) For any two laboratories participating in the CCEM-K2 (no matter whether they participated in the SIM.EM-K2 comparisons or not), the degrees of equivalence and the corresponding uncertainties are based on the results from CCEM-K2.

(2) If two laboratories participated only in the SIM.EM-K2 comparison or one laboratory participated in both CCEM-K2 and SIM.EM-K2 comparison and the second one only participated in the SIM.EM-K2 comparison, then the corresponding degree of equivalence and the uncertainties are the corresponding quantities in the SIM.EM-K2 comparison.

(3) In the case that the n^{th} laboratory participated only in the CCEM-K2 and the m^{th} laboratory participated only in the SIM.EM-K2 comparison, their degree of equivalence is estimated by

$$\begin{aligned} D'_{nm} &= D_{n,KCRV} - D'_{m,KCRV} \\ &= D_{n,KCRV} - D_{m,CRV} - \hat{D} \end{aligned}. \tag{5}$$

In CCEM-K2 and SIM.EM-K2 comparisons, the measurands have drifts. The statistical analyses proposed in [3], [7], and [8] were used to treat the case of drifts and the degrees of equivalences and their corresponding uncertainties were obtained. The uncertainties for the degrees of equivalence for $D'_{m,KCRV}$ and D'_{nm} in (4) and (5) are calculated.

2. Linkage between BIPM bilateral and SIM.EM-K2 comparisons

We will rely on a 2007 bilateral comparison of 1 Ω resistance standards, BIPM.EM-K13.a, to link the SIM.EM-K1 resistance comparison to other CCEM results. The BIPM and NIST participated in BIPM KC BIPM.EM-K13.a specifically for the purpose of this linkage, because the much earlier CCEM-K1 comparison is considered to be provisional. The BIPM KC BIPM.EM-K13.a comparison involved three traveling standards. We denote the mean value of BIPM NIST's measurements on the three standards by X_1 and the mean value of BIPM's measurements by X_2. The reference value is defined as

$$BCRV = \frac{X_1 + X_2}{2}. \tag{6}$$

The degrees of equivalence between each of the two labs and the BCRV is given by

$$D_{i,BCRV} = X_i - BCRV \tag{7}$$

for $i = 1, 2$. When $i = 1$,

$$D_{NIST,BCRV} = X_1 - BCRV = \frac{X_1 - X_2}{2}. \tag{8}$$

When $i = 2$,

$$D_{BIPM,BCRV} = X_2 - BCRV = \frac{X_2 - X_1}{2} \tag{9}$$

When we link the BIPM bilateral comparison to the SIM.EM-K1 comparison, NIST is the only linking lab. The difference of the NIST measurements between the two comparisons is given by

$$\begin{aligned} D &= D_{1,CRV} - D_{NIST,BCRV} \\ &= D_{1,CRV} + \frac{X_2 - X_1}{2} \end{aligned} \tag{10}$$

The degree of equivalence between BIPM and. the CRV of the SIM.EM-K1 comparison is given by

$$\begin{aligned} D'_{BIPM,CRV} &= D_{BIPM,BCRV} + D \\ &= \frac{X_2 - X_1}{2} + D_{1,CRV} + \frac{X_2 - X_1}{2} \\ &= X_2 - X_1 + D_{1,CRV} \end{aligned} \tag{11}$$

Given $D_{1,CRV} = 0.0003$ μΩ/Ω and $X_1 - X_2 = -0.014$ μΩ/Ω, $D'_{BIPM,CRV} = -0.0137$ μΩ/Ω.

The standard uncertainty of this result is given by

$$\begin{aligned} u_{D'_{BIPM,CRV}} &= \sqrt{u^2_{X_2} + u^2_{X_1} + u^2_{D_{1,CRV}}} \\ &= \sqrt{u^2_c + u^2_{D_{1,CRV}}} \end{aligned} \tag{12}$$

where u_c is given in the report of BIPM comparison. Given $u_{D_{1,CRV}}$ = 0.0025 μΩ/Ω and $u_c = 0.021$ μΩ/Ω, then $u_{D'_{BIPM,CRV}}$ = 0.0212 μΩ/Ω.

From (11), the pair-wise degree of equivalence between the *j*th lab in the SIM comparison and BIPM is given by

$$\begin{aligned} D'_{j,BIPM} &= D_{j,CRV} - D'_{BIPM,CRV} \\ &= D_{j,CRV} - [X_2 - X_1 + D_{1,CRV}]. \\ &= D_{j,CRV} - D_{1,CRV} + X_1 - X_2 \end{aligned} \tag{13}$$

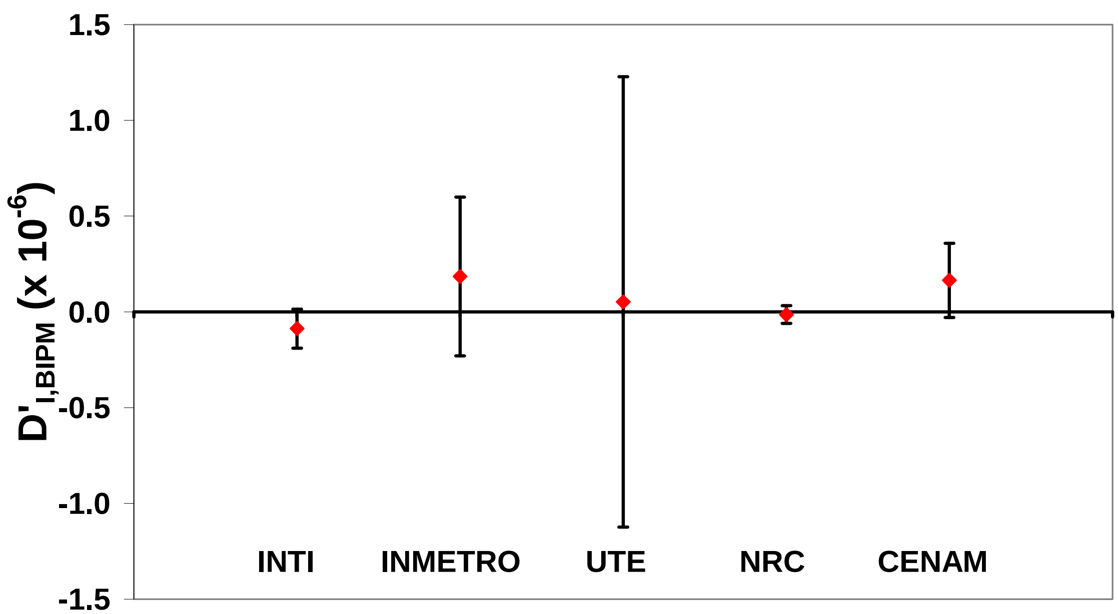
The corresponding standard uncertainties are calculated and given in Table G.1.

Table G.1. Linkage between BIPM.EM-K13a and SIM.EM-K1. The expanded uncertainties are computed using a coverage factor of $k = 2$.

Lab *i* ↓	$D'_{i,BIPM}$ (x 10^{-6})	U_{iCOMB} (x 10^{-6})
INTI	**-0.088**	**0.102**
INMETRO	**0.185**	**0.414**
UTE	**0.052**	**1.176**
NRC	**-0.014**	**0.046**
CENAM	**0.165**	**0.194**

Graph of equivalence for 1 Ω data from Table G.1.

With relative differences from the key comparison reference value of BIPM.EM-K13.a, for which the BIPM value is chosen



Relative differences from the key comparison reference value of BIPM.EM-K13a at 1 Ω, for which the BIPM value is chosen ($k = 2$).

Table G.2. Linkage between SIM.EM-K2 and CCEM-K2:
Pair-wise Matrix of Equivalence at 1 GΩ between non-linking SIM and CCEM-K2 labs with expanded uncertainties computed using a coverage factor of $k = 2$.

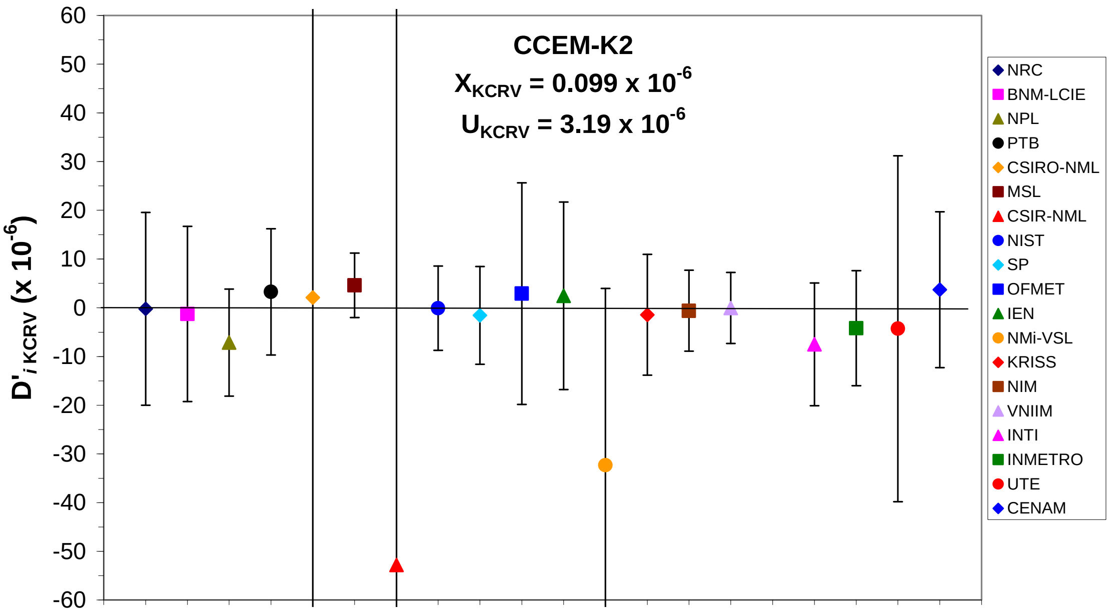
Lab j →			NIST		NRC		LCIE		NPL	
Lab i ↓	D_{iKCRV}	U_{iCOMB}	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}
	(x 10^{-6})		(x 10^{-6})		(x 10^{-6})		(x 10^{-6})		(x 10^{-6})	
INTI	-7.5	12.6	link	link	link	link	6.2	21.9	0.3	16.7
INMETRO	-4.2	11.8	link	link	link	link	2.9	21.5	-3.0	16.1
UTE	-4.3	35.5	link	link	link	link	3.0	39.8	-2.9	37.2
CENAM	3.7	16.0	link	link	link	link	-5.0	24.1	-10.9	19.4

Lab j →			PTB		CSIRO-NML		MSL		CSIR-NML	
Lab i ↓	D_{iKCRV}	U_{iCOMB}	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}
	(x 10^{-6})		(x 10^{-6})		(x 10^{-6})		(x 10^{-6})		(x 10^{-6})	
INTI	-7.5	12.6	10.8	18.1	9.6	68.0	12.1	14.2	-45.5	581.1
INMETRO	-4.2	11.8	7.5	17.5	6.3	67.8	8.8	13.5	-48.8	581.1
UTE	-4.3	35.5	7.6	37.8	6.4	75.6	8.9	36.1	-48.7	582.1
CENAM	3.7	16.0	-0.4	20.6	-1.6	68.7	0.9	17.3	-56.7	581.2

Lab j →			SP		OFMET		IEN		NMi	
Lab i ↓	D_{iKCRV}	U_{iCOMB}	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}
	(x 10^{-6})		(x 10^{-6})		(x 10^{-6})		(x 10^{-6})		(x 10^{-6})	
INTI	-7.5	12.6	5.9	16.1	10.4	26.0	10.0	23.0	-24.8	38.4
INMETRO	-4.2	11.8	2.6	15.4	7.1	25.7	6.7	22.6	-28.1	38.2
UTE	-4.3	35.5	2.7	36.9	7.2	42.2	6.8	40.4	-28.0	50.8
CENAM	3.7	16.0	-5.3	18.8	-0.8	27.8	-1.2	25.0	-36.0	39.7

Lab j →			KRISS		NIM		VNIIM	
Lab i ↓	D_{iKCRV}	U_{iCOMB}	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}
	(x 10^{-6})		(x 10^{-6})		(x 10^{-6})		(x 10^{-6})	
INTI	-7.5	12.6	6.0	17.7	6.9	15.1	7.5	14.5
INMETRO	-4.2	11.8	2.7	17.1	3.6	14.4	4.2	13.8
UTE	-4.3	35.5	2.8	37.6	3.7	36.5	4.3	36.2
CENAM	3.7	16.0	-5.2	20.2	-4.3	18.0	-3.7	17.5

Graph of equivalence for 1 GΩ data from Table G.2.



Graph of Equivalence: Linking SIM.EM-K2 to CCEM-K2
Differences from CCEM-K2 Key Comparison Reference Value at 1 GΩ (*k = 2*)