On the Use of Dipole Models to Correlate Emission Limits Between EMC Test Facilities

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Abstract— **Simple formulas are developed that enable emission limits from one EMC facility type to be translated to other EMC facility types. Facility types considered are free space, a half space as represented by an open area test site or a semi-anechoic chamber, TEM cells, and reverberation chambers. The formulas are based on simple dipole models. The formulas can provide valuable guidance for standards and product committees in setting product emission limits.**

I. Introduction

Emission and immunity tests can be made at a variety of EMC test facilities. These include open area test sites (OATS), semi-anechoic chambers (SAC), fully anechoic chambers (FAC), transverse electromagnetic (TEM) cells, and reverberation chambers (RC). Ideally each of these facilities would yield the same test result for a given test object, that is, a product that passes at one facility would pass at the others and a product that fails at one facility would fail at the others. This ideal case could be met if emission and immunity data could be exactly correlated between EMC facilities. However, because most test objects are quite complex and because present EMC facility test methods sample different subsets of the full range of possible emission and immunity test variables [1], exact correlation of test data is typically not possible.

An alternative to correlating complicated emission and immunity test data is to correlate test limits based on simple dipole models of the test object. Dipole models are a good starting point for setting "equivalent" test limits. These limits can then be adopted or modified by product committees as appropriate. Immunity limits are usually expressed in terms of electric field strength (V/m) at the test location in the absence of a test object. The field strength at the test location in a particular EMC test facility is then verified by field measurements using an isotropic receiving probe. In this way the interaction between the source and the test facility is normalized out. This makes setting equivalent immunity limits between EMC facilities straightforward. Emission limits are less straightforward. The equivalent to the immunity case would be to use a small isotropic source probe that could generate a variable field strength over the frequency range of interest (the reciprocal of a field probe that receives a variable field strength over the frequency range of interest). The field probe could then be used as a transfer standard to generate equivalent limits between EMC facilities. The difficulty is that a small, variable, isotropic source is not readily achievable. Thus, in the emission case analytical models are used to correlate the source and facility interactions. Dipole models are the simplest approach, and various dipole-based correlation algorithms have already been developed and have appeared in the literature [2-5]. This paper reviews the basics of these using a common terminology and notation. A dipole in free space (FS) is considered first and serves as a model for an ideal FAC. A dipole over a half space (HS) is considered second and serves as a model for an ideal OATS or SAC. A dipole in a transmission line (TL) models a TEM device. Finally, a dipole in an ideal cavity models a RC. The results of these models are then collected and can be used to derive equivalent test limits.

II. Dipole in Free Space

An ideal free space environment is the simplest case to analyze. Only the case of an electric dipole is considered. The results can be extended in a straightforward manner to a magnetic dipole, or combined electric and magnetic dipoles, as is discussed later. For a short electric dipole (length $d\ell$, peak current I_0) located at the origin and aligned with the z-axis, the far-field radiation is given by

$$E_\theta = \frac{j\omega\mu I_0 d\ell}{4\pi r}\sin\theta e^{-jkr} \text{ (V/m) and}$$
$$H_\varphi = \frac{jkI_0 d\ell}{4\pi r}\sin\theta e^{-jkr} \quad \text{(A/m)}, \tag{1}$$

where ω is the angular frequency, μ is the permeability of the medium in which the dipole is located (e.g., air), $k = 2\pi/\lambda$, where λ is the wavelength, (θ,φ,r) represents the usual spherical coordinate system, and a $e^{j\omega t}$ time convention is suppressed. The total power P_0 radiated by the electric dipole is found by integrating the Poynting vector over a sphere enclosing the dipole, resulting in

$$P_0 = \frac{2}{3}\eta\pi I_0{}^2\left(\frac{d\ell}{\lambda}\right)^2 \text{ (W)}, \tag{2}$$

where η is the intrinsic impedance of the medium (= 120π Ω for air). EMC measurements made in a FAC typically measure the electric field. For an emission measurement, the maximum electric field magnitude $E_{\max}$ would be sought over some scan geometry. For the above electric dipole geometry, the maximum in (1) occurs when $\theta = \pi/2$, where

$$E_{\max}^2 = \frac{\omega^2 \mu^2 (I_0 d\ell)^2}{(4\pi r)^2} \; . \tag{3}$$

Using (2), the above expression may be rewritten as

$$E_{\max}^2 = \frac{3}{2} \frac{\eta}{4\pi r^2} P_0 \; . \tag{4}$$

This expression is the electric dipole case of the more general expression given in [2],

$$E_{\max}^2 = D_{\max} \frac{\eta}{4\pi r^2} P_0 \; , \tag{5}$$

for the maximum electric field from an emitter with maximum directivity $D_{\max}$ (= 3/2 for an electric or magnetic dipole) and total radiated power P_0. $E_{\max}$ is actually measured as a voltage $V_{\max}$ at the output of an antenna with an antenna factor AF, where $E_{\max} = \mathrm{AF} V_{\max}$. Collecting results gives

$$V_{\max}^2 = \mathrm{AF}^{-2} D_{\max} \frac{\eta}{4\pi r^2} P_0 \; . \tag{6}$$

This expression can be further rewritten by defining a propagation-loss term $PL_{\mathrm{FS}} = 1/(4\pi r^2)$ giving

$$V_{\max,\mathrm{FS}}^2 = \eta \left(\mathrm{AF}_{\mathrm{FS}}^{-2} D_{\max,\mathrm{FS}} PL_{\mathrm{FS}} \right) P_0 \; . \tag{7}$$

Subscripts (FS) have been added to differentiate between the cases presented in the following sections. In particular, $\mathrm{AF}_{\mathrm{FS}}$ is not an inherent property of free space; rather, it simply denotes the AF of whatever antenna is used to make a free space measurement.

III. Dipole in Half Space

A half space is formed by introducing an ideal ground plane (a perfect conductor of infinite extent) into the free-space case. An electric dipole is here located at a height h above the ground plane and oriented either vertically or horizontally. Other orientations may be analyzed as the superposition of these two cases. The half-space case may be analyzed by introducing an image dipole. We introduce a rectangular coordinate system with the ground plane in the x-y plane, the dipole at $z = +h$, and the image dipole at $z = -h$. The distance from the source dipole to the measurement point is designated r_1, from the image dipole to the measurement point r_2, from the origin to the measurement point r, and the radial distance from the z-axis to the measurement point is designated ρ. In the far-field, the maximum electric field is given by [3, 5]

$$E_{\max}^2 = D_{\max} \frac{\eta}{4\pi r^2} g_{\max}^2 P_0 \; , \tag{8}$$

where $D_{\max}$ is again 3/2 for a single electric dipole. The geometry factor $g_{\max}$ is defined by

$$g_{\max} = \begin{cases} \left| \dfrac{r}{r_1} e^{-jkr_1} - \dfrac{r}{r_2} e^{-jkr_2} \right|_{\max} & \textit{for horiz.} \\ \left| \dfrac{\rho^2}{r_1^2} \dfrac{r}{r_1} e^{-jkr_1} + \dfrac{\rho^2}{r_2^2} \dfrac{r}{r_2} e^{-jkr_2} \right|_{\max} & \textit{for vert.,} \end{cases} \tag{9}$$

where the subscript max denotes the maximum value found over some scan geometry (e.g., a 1 to 4 m vertical height scan at some horizontal offset). We have introduced a normalizing factor r into (9) (versus the expressions found in [3, 5]) to give (8) a form similar to that of (5). If the radial distance ρ is significantly larger than the height above the ground plane h, such that $\rho / r_1 \approx 1$, $\rho / r_2 \approx 1$, $r / r_1 \approx 1$, and $r / r_2 \approx 1$, then $g_{\max}$ reduces to

$$g_{\max} = \begin{cases} 2 \sin k(r_1 - r_2)_{\max} & \textit{horizontal and} \\ 2 \cos k(r_1 - r_2)_{\max} & \textit{vertical.} \end{cases} \tag{10}$$

Under these assumptions, $g_{\max} = 2$ in both cases, assuming the measurement scan is such that, at some point, $k(r_1 - r_2) = \pi/2$ or π respectively (or some suitable multiple). Physically, this simply means that the ground plane doubles the maximum electric field through constructive interference due to the reflected path. Fig. 3 in [5] shows that $g_{\max} = 2$ is a good approximation above 200 MHz for a typical 3 m EMC emission test (3 m horizontal separation, 1 m source height, 1 to 4 m measurement height scan) and is a good approximation above 30 MHz for a typical 10 m EMC emission test (10 m horizontal separation, 1 m source height, 1 to 4 m measurement height scan). Thus, a value of $g_{\max}$ equal to 2 should be sufficient for setting equivalent EMC test limits and will be used here for the half-space case. Thus, the maximum measured voltage for an electric dipole above a perfect ground plane can be approximated as

$$V_{\max,HS}^2 = \eta \left(\mathrm{AF}_{\mathrm{HS}}^2 D_{\max,\mathrm{HS}} PL_{\mathrm{HS}} \right) P_0 \; , \tag{11}$$

where $PL_{\mathrm{HS}} = 4/(\pi r^2)$ for the half-space case. However, if the ground-plane geometry needs to be accounted for more accurately, we can use $PL_{\mathrm{HS}} = g_{\max}{}^2/(4\pi r^2)$.

IV. Dipole in a TEM Transmission Line

A TEM transmission line (stripline, TEM cell) seeks to approximate a linearly polarized plane-wave field over some test volume. A dipole placed in a TEM line will couple to the TEM mode and to higher-order modes, if present, and produce a voltage at the measurement port. This measured voltage combined with suitable rotations of the dipole (similar in purpose to the above receive antenna scans) can be used to determine the dipole moment [1] or the total radiated power from the dipole [1, 4]. For example, in IEC 61000-4-20 [6], P_0 for an electric dipole is given as

$$P_0 = \frac{2\eta}{3\pi} \frac{k^2}{e_{0y}{}^2 Z_0} S_V{}^2 \; , \tag{12}$$

where Z_0 is the characteristic impedance of the TL (typically 50 Ω), e_{0y} is a normalized field factor, $e_{0y}^2 \approx Z_0 / d^2$, where d is the plate separation at the dipole location, and S_V represents the sum of the measured output port voltage over a set of dipole rotations. These expressions ignore contributions from higher-order modes. If the dipole is oriented for maximum coupling, as in the previous sections, then the voltage at the measurement port will be maximized: $S_V^2 = V_{\max}^2$ (no rotations needed), and

$$V_{\max}^2 = \frac{3}{2}\frac{\pi}{\eta}\frac{Z_0^2}{k^2 d^2} P_0 \ , \tag{13}$$

where a substitution for e_{0y} has been made. Replacing 3/2 with $D_{\max}$, this may be rewritten as

$$V_{\max}^2 = \left(\frac{Z_0}{\eta}\frac{r}{d}\lambda\right)^2 D_{\max} \frac{\eta}{4\pi r^2} P_0 \ . \tag{14}$$

Comparing this result to (6), we see that (14) defines an equivalent antenna factor for a TEM cell (the term in parenthesis). The distance r is interpreted as the radial distance to the test volume. In a uniform cross section TEM line (e.g., a standard two-port TEM cell tapered at each end) r is the radial distance from the measurement port along the cell taper to the location of the dipole projected back to the plane beginning the uniform section of the transmission line. In a constant flare TEM line (e.g., GHz TEM cell) r is simply the radial distance from the measurement port to the dipole location. The equivalent antenna factor is then given by

$$AF_{\mathrm{TL}} = \frac{\eta}{Z_0}\frac{d}{r}\frac{1}{\lambda} \ . \tag{15}$$

This expression is consistent with a similar equivalent gain factor for TEM lines that can be derived using power expressions. Using this definition, we again have

$$V_{\max,\mathrm{TL}}^2 = \eta\left(\mathrm{AF}_{\mathrm{TL}}^{-2} D_{\max,\mathrm{TL}} PL_{\mathrm{TL}}\right) P_0 \ , \tag{16}$$

where PL_{TL} is defined the same as PL_{FS} but using r as defined in this section.

V. Dipole in a Reverberation Chamber

A reverberation chamber is an over-moded cavity that seeks to statistically approximate a uniform set of plane waves over some test volume. The ideal set of plane waves would include all directions and polarizations. A good reverberation chamber approaches this ideal. One can show that the average power $\langle P_r \rangle$ (averaged over multiple modal distributions) received by a matched, lossless reference antenna due to a source in the cavity is given by [7]

$$\langle P_r \rangle = \frac{\lambda^3 Q}{16\pi^2 V} P_0 \ , \tag{17}$$

where Q is the quality factor of the chamber, V is the volume of the chamber, and as before, P_0 is the total radiated power from the source, which is an electric dipole in this case. The difficulties with (17) are that Q is often not well characterized and we need to correct for antenna-loss effects. To avoid these difficulties, the more usual method of determining P_0 in a reverberation chamber is to make a comparative measurement with a reference source of known power P_{ref}, while keeping the chamber conditions the same: then

$$P_0 = \frac{P_{ref}}{\langle P_{r,ref} \rangle} \langle P_r \rangle \ . \tag{18}$$

Solving for $\langle P_r \rangle$, and rewriting this in terms of the average received voltage yields

$$\langle V^2 \rangle = Z_c \frac{\langle P_{r,ref} \rangle}{P_{ref}} P_0 \ , \tag{19}$$

where $P_r = V^2/Z_c$ (Z_c is the impedance at the antenna measurement port, typically 50 Ω). In this case there is no max subscript since there is no need to scan the receive antenna or rotate the test object. There is also no directivity term as this is negated by averaging over incidence angles and polarization. For consistency with previous expressions, define the following for the reverberation chamber case: $D_{\max,\mathrm{RC}} = 1$ (no directivity), $V_{\max,RC}^2 \equiv \langle V^2 \rangle$ (the receive antenna scan in previous methods is replaced by averaging over the modal distribution variations), $AF_{RC}^2 = \eta / s^2 Z_c$ (s = 1 m is included to give consistent units), and

$$PL_{RC} = \frac{1}{s^2}\frac{\langle P_{r,ref} \rangle}{P_{ref}} \ . \tag{20}$$

This yields

$$V_{\max,RC}^2 = \eta\left(\mathrm{AF}_{\mathrm{RC}}^{-2} D_{\max,RC} PL_{RC}\right) P_0 \ . \tag{21}$$

Note that $V_{\max,\mathrm{RC}}$ does not denote the maximum voltage measured over the multiple modal distributions; it denotes the average, as defined in (20) above.

VI. Limit Correlation

The above yields expressions (7, 11, 16, 21) for the received voltage from an electric dipole located in four different test environments: free space (FS), half space (HS), a TEM line (TL), and a reverberation chamber (RC). Assuming the total radiated power from the dipole is the same in each case, one can form the ratios of these expressions to correlate between EMC test facilities,

$$\frac{V_{\max,A}^2}{V_{\max,B}^2} = \frac{AF_A^{-2}}{AF_B^{-2}} \frac{D_{\max,A}}{D_{\max,B}} \frac{PL_A}{PL_B} \tag{22}$$

where A and B can be any combination of FS, HS, TL and RC.

The above expression (22) is based on consideration of an electric dipole. However, the identical form could be used for a magnetic dipole ($D_{\max} = 3/2$), a combination of an electric and a magnetic dipole ($D_{\max} = 3$), or, in a general sense, for test objects having directivities other than these dipole values. If one is not correlating to a reverberation chamber, then the directivity ratio is unity regardless of the test object. For the correlation to a reverberation chamber where $D_{\max,RC} = 1$, $D_{\max}$ for a test object in the other facility must be either known (typically not the case) or must be estimated, as was done in [8], based on the electrical size of the test object. The emission correlation parameters are summarized in Table 1. The terms used in Table I may be found in the appropriate sections of the paper. Emission limits may be correlated by substituting $E_{\max}$ for $V_{\max}$AF

TABLE I
SUMMARY OF THE EMISSION CORRELATION PARAMETERS

EMC Test Facility	AF	$D_{\max}$	PL
FS (FAC)	receiving antenna	3/2 for dipole or see [8]	$1/(4\pi r^2)$
HS (SAC, OATS)	receiving antenna	3/2 for dipole or see [8]	$4/(4\pi r^2)$ or $g_{\max}^2/(4\pi r^2)$ (9)
TL (TEM Cell, GTEM Cell, stripline)	$\frac{\eta}{Z_0}\frac{d}{r}\frac{1}{\lambda}$	3/2 for dipole or see [8]	$1/(4\pi r^2)$
RC (reverb. chamber)	$(\frac{\eta}{Z_c})^{1/2}\frac{1}{s}$	1 for all emitters	$\frac{1}{s^2}\frac{\langle P_{r,ref}\rangle}{P_{ref}}$

VII. CONCLUSIONS

Formulas have been presented that enable emission limits from one EMC facility type to be translated to other EMC facility types. Facility types considered are free space, a half space as represented by an open area test site or semi-anechoic chamber, TEM cells, and reverberation chambers. The formulas are based on simple dipole models. The formulas can provide valuable guidance for standards and product committees in setting limits that equivalently test a product .

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