

# HIGH RESISTANCE SCALING FROM 10 kΩ AND QHR STANDARDS USING A CRYOGENIC CURRENT COMPARATOR

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## Abstract

We describe a cryogenic current comparator (CCC) bridge for resistance scaling that provides improved measurement uncertainty over a range of resistance values from 100 kΩ to 1 GΩ. This CCC is designed for high resistance scaling based directly on a quantized Hall resistance (QHR) standard as well as comparisons of resistance ratios of 1, 10 and 100. The QHR-to-decade-value winding ratio offset is chosen so as to approximately cancel the offset produced by resistive decade windings.

## Introduction

CCC bridges are an essential tool in modern electrical metrology, but in practice their applicability is limited by difficulties in the design, ease of use, and cost. This paper will describe the results of a project to design and build high-resistance CCC systems that provide the low uncertainty of this technique with reasonable cost. The two-terminal, single-source bridge design [1-3] reduces noise from the feedback electronics and other non-cryogenic components of the bridge. This technique allows high-value resistors to be compared directly against the QHR standard using two-terminal connections [4, 5] with uncertainties that approach those of four-terminal CCC resistance comparisons.

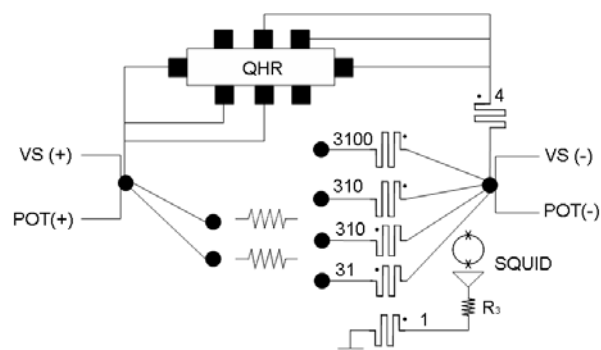
## CCC design and behavior

The two-terminal CCC electronics is based on a single ramping voltage source which enables comparisons of many different values of resistance, from 10 kΩ up to at least 1 GΩ. The voltage source is designed to produce a reversing output of about 0.5 V to 1 V for use with the QHR and 5 V to 10 V for use when only standard resistors are compared. Two superconducting junctions with external leads for voltage measurements provide a well-defined bridge voltage. The CCC bridge balance equations are straightforward to solve when this voltage, the number of turns in each

winding, and the bridge connection resistances are known.

This design derives its only bridge feedback current from a dc superconducting quantum interference device (dc SQUID). The feedback current is applied to a single-turn CCC winding to produce a constant dc magnetic flux, and is measured to determine the ampere-turns signal necessary to maintain the bridge balance condition.

The CCC windings are coupled to the SQUID by a 4 cm inner-diameter superconducting shield and flux transformer, with a current sensitivity of about  $1.3 \mu\text{A}\cdot\text{turn}/\Phi_0$ , where  $\Phi_0$  is the flux quantum. The ratios of the 4-turn QHR winding and 31-base-turn decade windings (31, 310, and 3100 turns) create an offset of about  $-246 \times 10^{-6}$  from the nominal resistance ratio, when decade-value resistors are compared to the QHR 12906.4035 Ω plateau. Canceling offsets are created by the winding resistance of the Phosphor-Bronze wire used for the windings and cryogenic leads. This tends to reduce the uncertainty contributions of the feedback current measurement and voltage source stability for low value resistors. In this implementation we compare the QHR to 1 MΩ (or 10 MΩ) using winding resistances of nominally 246 Ω (2460 Ω). All ratio windings except the one used with the QHR are made using various gauges of resistive wire, with enough resistance to minimize noise amplification in the SQUID detector near high-frequency resonances.



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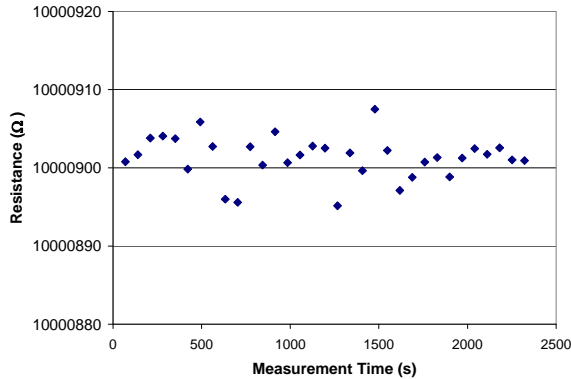
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**Figure 1.** Schematic diagram of the bridge, showing connections for room-temperature resistors and the voltage source (VS).

As shown in Fig. 1, three windings with 4 turns, 31 turns, and 310 turns are in opposition to a second 310 turn winding and the 3100 turn winding. All five CCC ratio windings are connected to the low side of the bridge and ground, and appropriate pairs of these windings can be used to compare resistance ratios of the QHR to 1 M $\Omega$  and 10 M $\Omega$ , as well as 1-to-1, 10-to-1, and 100-to-1 ratios. Sample measurements obtained with the bridge are shown in Fig. 2.



**Figure 2.** Typical comparison of a 10 M $\Omega$  Hamon-network resistor against the QHR  $i = 2$  plateau, measured at  $\pm 0.9$  V, with 10 s integration time for each point in each current direction. The relative standard deviation of the mean is  $0.05 \times 10^{-6}$ .

In an estimate of the fundamental noise current for a resistance  $R$  at temperature  $T$ ,  $I_n = (4k_B T/R)^{1/2}$  where  $k_B$  is the Boltzman constant, and is  $1.25 \times 10^{-10}$  A/Hz $^{1/2}$  for a 10 M $\Omega$  resistor at 293 C. When in series with the 3100 turn winding, this current noise level is equivalent to a flux noise of  $9.6 \times 10^{-5}$   $\Phi_0$ /Hz $^{1/2}$ , which is significantly larger than the typical dc SQUID-system noise and thus is likely to be one limiting factor in such a measurement. At a bridge voltage of 1 V, this 10 M $\Omega$  noise current is equivalent to  $0.4 \times 10^{-6}$ /Hz $^{1/2}$ , in terms of the relative uncertainty contribution.

### Decade Resistance Comparisons

The many components of uncertainty that influence our CCC comparisons for decade resistor values of 10 k $\Omega$  to 1 G $\Omega$  are described in a separate paper [6]. Below 1 M $\Omega$ , the winding-resistance and lead resistance corrections are the largest predicted component of the total combined uncertainty. At higher resistance values, Johnson noise in the resistors and the measurement of the feedback current become dominant, and leakage effects can be more critical. Many different types of film and wire-wound resistance elements exist in commercial standards for values above 1 M $\Omega$ , and we have investigated dielectric settling-time effects [7] using source reversal delays of 4 s to 64 s. At present, using a  $\pm 10$  V bridge

voltage, standard deviations of CCC bridge measurements with 20 s total integration time are approximately as shown in Table 1.

**Table 1.**

Secondary y ( $\Omega$ )	Primary ( $\Omega$ )	Standard Deviation ( $\mu\Omega/\Omega$ )
100 k $\Omega$	1 M $\Omega$	0.002
100 k $\Omega$	10 M $\Omega$	0.040
1 M $\Omega$	10 M $\Omega$	0.040
1 M $\Omega$	100 M $\Omega$	0.40
10 M $\Omega$	100 M $\Omega$	0.40
10 M $\Omega$	1 G $\Omega$	4.0

### Conclusion

We have built and tested two-terminal CCC bridges that allow improved combiner uncertainties for high-resistance scaling. Measurements of guarded Hamon devices [8] show good agreement in scaling, especially where film-type resistor elements make up the resistance networks. We have detected or verified settling time constants in these and certain other standards. Comparisons between the new systems and other CCC bridges designed for high-resistance scaling from the QHR will be presented at the conference. Comparisons with scaling based on 10 k $\Omega$  four-terminal CCC measurements will also be described.

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