Current status of the quantum metrology triangle*

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Abstract
The quantum metrology triangle is a test of the consistency of three quantum electrical standards: the single-electron tunnelling current standard, the Josephson voltage standard and the quantum Hall resistance standard. This paper considers what is known about each of these effects separately in terms of (1) theory, (2) empirical tests of universality and (3) 'direct' tests involving fundamental constants. The current status of each of the three 'legs' of the triangle is quite different, with the single-electron leg being the weakest by far. This leads to the conclusion that a recent experimental result for the triangle should be interpreted primarily in terms of corrections to the quantum of charge transferred by single-electron devices.

1. Introduction

A recent proposal [1] to redefine the kilogram, ampere and other base units of the International System of Units (SI) by choosing exact values for several fundamental constants has offered a concrete vision of a future system of units that one could rightly call a 'Quantum SI'. From the standpoint of electrical measurements, the key feature of this proposal is that it defines exact values for both the Planck constant h and the elementary charge e, which means that both the Josephson constant $K_J = 2e/h$ and the von Klitzing constant $R_K = h/e^2$ have exactly defined values. Since practical standards of voltage and resistance based on these quantum effects have long surpassed the uncertainty of the best SI realizations of the ampere and of derived electrical units, the new SI will automatically simplify and improve the uncertainty of precision electrical measurements. This is especially true for measurements that involve both electrical and mechanical units, such as electrostatic force balances used as primary standards for forces below 10^{-4} N [2].

A key element of the proposed Quantum SI is the assumption that the relations for K_J and R_K are exact. There are three types of arguments in support of this assumption; each will be examined in detail below. Theoretically, there are no current predictions for any correction terms. Empirically, several experiments have shown that K_J and R_K are independent of device design, material, measurement

setup, etc. This demonstration of universality is consistent with the exactness of the relations, but does not prove it outright. Finally, experimental data on fundamental constants can be analysed *independent* of the assumptions $K_J = 2e/h$ and $R_K = h/e^2$ to test for discrepancies.

Given the important role that K_J and R_K are likely to play in any new SI (regardless of the details of its construction), it is important to pursue further tests of their relations to h and e. Of particular value are empirical tests that do not focus on the universality of the effects but look directly for corrections to the predicted relations. One such test, first proposed in 1985 [3] and discussed in detail more recently [4, 5], is the quantum metrology triangle (QMT), which combines the Josephson and quantum Hall effects with a third quantum electrical effect, single-electron tunnelling. The QMT, either the original form or a closely related test [4,5], is being pursued by at least four national measurement institutes (NIST in the USA, PTB in Germany, LNE in France and NPL in the UK).

The purpose of this paper is to review the current status of the QMT in detail, and in particular to examine each of its 'legs' separately. This provides a useful context in which to interpret recent and forthcoming QMT experiments. Section 2 presents the basic equations for the QMT, and section 3 considers the quantum Hall, Josephson and single-electron tunnelling effects separately. Section 4 applies the results of section 3 to the interpretation of a recent QMT result from NIST and section 5 contains conclusions and an outlook for the future.
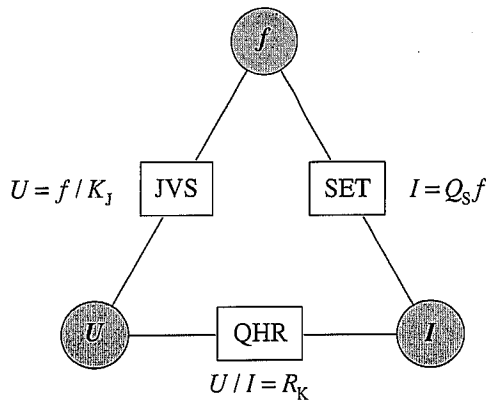
Figure 1. Original version of the QMT proposed in 1985 [3]. It links the Josephson effect (JVS), the quantum Hall effect (QHR) and the single-electron tunnelling effect (SET) by using Ohm's law.

2. The quantum metrology triangle

The QMT is illustrated in figure 1. It consists of quantum electrical standards for voltage, resistance and current linked by Ohm's law to form a triangle. (See [6] for a comprehensive review of the state of the art for each of these standards.) The defining relations for the three quantum standards are as follows.

(i) A Josephson voltage standard (JVS) driven at a frequency f_J and operating on the nth step produces a voltage

$$U_{JVS} = nf_J/K_J \qquad \text{with } K_J = \frac{2e}{h}(1 + \varepsilon_J). \qquad (1)$$

(ii) A quantum Hall resistance (QHR) standard quantized on the ith plateau has a resistance

$$R_{QHR} = R_K/i \qquad \text{with } R_K = \frac{h}{e^2}(1 + \varepsilon_K). \qquad (2)$$

(iii) A single-electron tunnelling (SET) current standard driven at a frequency f_S produces a current

$$I_{SET} = Q_S f_S \qquad \text{with } Q_S = e(1 + \varepsilon_S). \qquad (3)$$

For each quantum standard, a possible deviation from the expected relation involving h and/or e is parametrized by ε. Combining equations (1), (2) and (3) using Ohm's law $U = RI$, and letting $\mathcal{A}_1$ represent all known scaling factors in a real QMT experiment (such as bridge ratios), we have

$$\frac{n_J f_J}{K_J} = \mathcal{A}_1 \frac{R_K}{i} Q_S f_S, \qquad (4)$$

$$\frac{n_J i}{\mathcal{A}_1} \frac{f_J}{f_S} = K_J R_K Q_S, \qquad (5)$$

$$\frac{1}{2} \frac{n_J i}{\mathcal{A}_1} \frac{f_J}{f_S} = 1 + \varepsilon_J + \varepsilon_K + \varepsilon_S, \qquad (6)$$

where the last expression relies on the fact that each ε term is much less than 1.

In practice, the terms on the left side of equation (6) are known with negligible uncertainty and can be chosen so that

the left side is equal to 1. Thus if there are no corrections to any of the three quantum electrical standards, the QMT amounts the relation $1 = 1$. The result of an experimental realization of the QMT can be written as

$$1 = 1 + \Delta_{expt} \pm u_{expt}, \qquad (7)$$

where Δ_{expt} is the measured deviation from the expected relation $1 = 1$ and u_{expt} is the relative standard uncertainty of the result. If Δ_{expt} is less than u_{expt} the result 'closes' the QMT and provides evidence against corrections to the three quantum standards larger than u_{expt} (neglecting for the moment the possibility of cancellation between ε terms of opposite sign). If an experiment were to show that the QMT did not close, i.e. Δ_{expt} was larger than u_{expt}, it would indicate that one of the three quantum electrical standards does have a significant correction term, but it would not indicate which one.

3. Status of the individual legs of the QMT

The discussion thus far considers the QMT to be a test with binary outcomes of 'pass' or 'fail', and represents the conventional view of the QMT found in the literature and elsewhere. The uncertainty at which closing the QMT will improve confidence in the quantum electrical standards is generally said to be below about 1 part in 10^7. However, this view ignores the fact that the current situation for each of the three legs is quite different.

For each leg I will consider what is known about possible corrections in three areas: theory, empirical tests of universality and direct tests of the quantum relation for each leg. The third area requires some explanation. A direct test means one in which the expected relation between the quantum electrical standard and h and/or e is *not* assumed. For example, a JVS could be compared with a device that produces an SI volt to obtain a measurement of K_J in SI units, and this could then be compared with an SI value of the quantity $2e/h$. Although this is conceptually simple, two difficulties arise in practice. First, realizing an SI volt (or ohm or ampere) with the required uncertainty (below 1×10^{-6}) generally requires both a clever idea and a rather heroic effort. (An excellent example is the realization of the SI volt described in [7].) Second, the recommended values of fundamental constants are the result of a least-squares adjustment that *assumes the quantum relations are valid* [8] (for K_J and R_K only; the relation for Q_S has not entered the observational equations for any adjustments to date). Thus one cannot simply use the recommended value of $2e/h$ in a test of $K_J = 2e/h$ because this value is affected by experiments involving the Josephson effect. What can be done is to perform the least-squares adjustment with the assumptions $K_J = 2e/h$ and $R_K = h/e^2$ relaxed, and with adjustable correction factors ε_J and ε_K inserted into the relevant observational equations. The adjustment then provides the best values for ε_J and ε_K consistent with a wide variety of experiments. Such an analysis is described in appendix F of the 2002 CODATA report on the adjustment of fundamental constants [8]. An updated version based on the 2006 values of the fundamental constants [9] has been performed recently [10] and these results are summarized below.

3.1. Quantum Hall leg

Soon after the discovery of the quantum Hall effect in 1980 [11], Laughlin presented a topological argument for $R_K = h/e^2$ in an idealized two-dimensional electron gas (2DEG) [12]. As described by Jeckelmann and Jeanneret [13], various theories based on more realistic models of QHR devices also support the ideal relation. It must be noted, however, that real QHR devices show deviations of the Hall resistance from the expected value due to a host of effects such as temperature, measurement current and contact resistance [13]. There is no quantitative theory that includes all of these effects and the use of the QHR as a fundamental resistance standard requires adherence to a lengthy set of guidelines [14]. Thus it is only the value of the QHR *extrapolated* to zero longitudinal resistance (i.e. zero dissipation) that is found to be exactly quantized, and even with this restriction the fact that $\varepsilon_K = 0$ is generally viewed as 'a continuing surprise' [8].

Empirically, the extrapolated value of the QHR has been shown to be independent of the particular device, the host material for the 2DEG (GaAs heterostructure or Si MOSFET), the growth technique for GaAs 2DEGs (MBE or MOCVD), the plateau index i and the mobility of the 2DEG within a relative standard uncertainty of 3×10^{-10} or less [13, 15]. A new method involving four devices on the same chip in a bridge configuration yields the deviation of one device from the three others and promises a large improvement in sensitivity [16]. This work reports a relative deviation of 3×10^{-10} and a statistical uncertainty of 8×10^{-11}, but other uncertainties that may explain the deviation have not yet been quantified. As mentioned in the Introduction, these results do not prove $\varepsilon_K = 0$, but they do put tight constraints on any correction mechanism.

The direct test of $R_K = h/e^2$ based on the 2006 values of the constants [10] found

$$\varepsilon_K = (20 \pm 18) \times 10^{-9}. \tag{8}$$

Thus the best estimate of ε_K when the assumptions $K_J = 2e/h$ and $R_K = h/e^2$ are relaxed is consistent with zero. It turns out that in the case of R_K one can arrive at essentially the same result without the full least-squares adjustment, and it is instructive to present this argument as a check on the more rigorous but less intuitive analysis. Precise comparisons of QHR standards with the SI ohm have been done with a Thompson–Lampard calculable capacitor, a special resistor having a calculable ac/dc difference, and a lengthy chain of ac, quadrature and dc bridges (the example with the smallest uncertainty is [17]). Concise summaries of these experiments are given in [8, 18]. The weighted mean of five such experiments gives the following value for R_K in terms of the SI ohm [8],

$$R_K = 25\,812.808\,18(47)\ \Omega\ [1.8 \times 10^{-8}]. \tag{9}$$

(Here the number in parentheses is the standard uncertainty referred to the last digits of the quoted value and the number in square brackets is the relative standard uncertainty.) As for the SI value of h/e^2, it can be expressed in terms of the fine structure constant α, the speed of light c and the magnetic constant μ_0 as

$$\frac{h}{e^2} = \frac{\mu_0 c}{2\alpha} = 25\,812.807\,557(18)\ \Omega\ [6.8 \times 10^{-10}], \tag{10}$$

where the numerical value is the 2006 recommended value [9]. Since c and μ_0 are defined constants in the current SI, h/e^2 depends only on α. Although the experiments involving calculable capacitors do give values of α that affect the numerical value of h/e^2 slightly, two other types of experiments (based on the electron magnetic moment anomaly in one case and photon recoil of atoms in the other) have uncertainties so much smaller that the final value of α is nearly independent of experiments involving the QHR. Thus a fairly good test of $R_K = h/e^2$ can be done by comparing the numerical values in equations (9) and (10). This gives

$$\varepsilon_K = (24 \pm 18) \times 10^{-9}, \tag{11}$$

which is in good agreement with the more rigorous value in equation (8).

The status of the QHR leg of the QMT can be summarized as follows. (1) There is no theoretical prediction that ε_K is not zero, provided that the QHR value is extrapolated to zero longitudinal resistance. However, the general topological arguments for exact quantization apply only to ideal systems and an explanation for the exactness observed in real devices remains elusive. (2) There is considerable empirical evidence that the QHR value, again extrapolated to zero longitudinal resistance, is universal at the level of a few parts in 10^{10}. (3) A direct test of $R_K = h/e^2$ indicates that ε_K is smaller than a few parts in 10^8. Although this is widely viewed as good enough to allow the proposed redefinition of the SI to proceed, there is considerable room for improvement since the experimental uncertainty of modern QHR standards is a few parts in 10^9 [6].

3.2. Josephson leg

Several arguments for the exactness of $K_J = 2e/h$ were given around 1970 [19–22], all based on very general properties such as gauge invariance and the requirement that the wavefunction of the superconducting condensate be single valued. Unlike the case of the quantum Hall effect, real JVS devices are believed to satisfy quite well the conditions assumed in the theory. As shown by Fulton [20], this fact can be seen as a consequence of the exactness of flux conservation in superconductors of closed geometry. Thus these theoretical arguments are generally viewed as providing a solid reason to expect $\varepsilon_J = 0$.

Empirically, the universality of the voltage from a JVS has been established by numerous experiments. Quoting relative standard uncertainties in all cases, arrays of the same type of junction agreed within 2×10^{-17} [23], arrays of different types of junctions agreed within a few parts in 10^{10} [24,25] and an array of high-temperature superconductor junctions agreed with a conventional array within 2×10^{-8} [26]. Also, a pair of single junctions agreed within 3×10^{-19} [27].

The direct test of $K_J = 2e/h$ based on the 2006 values of the constants [10] found

$$\varepsilon_J = (-77 \pm 80) \times 10^{-9}. \tag{12}$$

Thus the best estimate of ε_J when the assumptions $K_J = 2e/h$ and $R_K = h/e^2$ are relaxed is consistent with zero. However, further analysis has revealed that this result is not as robust as originally thought. It turns out that the value of ε_J is determined predominately by two different routes, i.e. two different types of observational equations and input data. Mohr *et al* [10] have performed the adjustment with certain input data deleted to reveal the contribution of each route and found

$$\text{Route 1: } \varepsilon_J = (-281 \pm 95) \times 10^{-9},$$
$$\text{Route 2: } \varepsilon_J = (407 \pm 143) \times 10^{-9}. \tag{13}$$

Thus the routes individually give values of ε_J that *differ significantly from zero but have opposite sign*. The result in equation (12) must therefore be seen as fortuitous, and confidence in the direct test of $K_J = 2e/h$ cannot be said to extend below a few parts in 10^7. The origins of the discrepancy between the two routes can be traced to inconsistencies among the input data that were already discussed in the 2002 CODATA report. To find a result free of these inconsistencies, Mohr *et al* also performed the adjustment with both sets of discrepant data deleted [10] and found

$$\varepsilon_J = (238 \pm 720) \times 10^{-9}. \tag{14}$$

Thus using only nondiscrepant data gives a result that is consistent with zero but with an uncertainty of about 7 parts in 10^7. For all of the alternative tests just described, the value and uncertainty of ε_K are essentially unchanged from the result of equation (8).

Unlike the case described above for $R_K = h/e^2$, a simplified version of the direct test of $K_J = 2e/h$ is not possible. This is not due to a lack of precise measurements of K_J in terms of the SI volt. A watt balance combined with the SI ohm from a calculable capacitor yields a value of K_J, and the best result of this type to date has an uncertainty[1] of about 2×10^{-8}. The difficulty is rather that there is no independent value of $2e/h$ with comparable precision. Because the CODATA adjustment of fundamental constants assumes $K_J = 2e/h$ exactly, the 2006 recommended value of $2e/h$ is dominated by this same watt balance result. The best value of $2e/h$ that does not assume $K_J = 2e/h$ comes from a measurement of the molar volume of Si. (This somewhat unintuitive link can be seen by writing $2e/h = (8\alpha/\mu_0 ch)^{1/2}$ and taking h from equation (144) of [8].) Using the 2006 recommended values [9], this value of $2e/h$ has an uncertainty of about 1 part in 10^7, but it must be noted that the discrepancy between the molar volume of Si result and other results, discussed at length in [8], remains unresolved in the 2006 recommended values.

The status of the JVS leg of the QMT can be summarized as follows. (1) There is no theoretical prediction that ε_J is not zero, and the general arguments for exact quantization are viewed as applicable to real devices. (2) There is considerable evidence that the voltage produced by a JVS is universal at

the level of 10^{-10}, and possibly much lower. (3) A direct test of $K_J = 2e/h$ is complicated by discrepancies among the input data for various fundamental constants. A conservative test excluding discrepant input data indicates that ε_J is smaller than about 7 parts in 10^7, while other tests raise questions about possible corrections at 3 or 4 parts in 10^7. As with the QHR leg, this is well above the experimental uncertainty of modern JVS systems, which is a few parts in 10^{10} [6].

3.3. Single-electron tunnelling leg

In remarkable contrast to the cases of the QHR and the JVS, little attention has been paid to the question of whether the relation $Q_S = e$ is exact, either theoretically or otherwise. This is probably due to the fact that SET standards are not currently used for calibrations and intercomparisons in electrical metrology, so there has been little practical reason to worry about a correction in this case. In the area of current, using resistance and voltage artefacts traceable to the QHR and JVS is adequate for existing calibration needs and in fact this may continue to be the preferred path even after adoption of an SI in which h and e are defined constants. In the area of capacitance, a prototype standard based on counting electrons has been demonstrated [30] but it seems unlikely to rival the performance of calculable capacitors in either uncertainty or ease of use. Nevertheless, investigation of a correction to $Q_S = e$ remains an important fundamental question and it is possible that a need for calibrations directly involving SET standards of current or charge will arise in the future. This seems particularly likely at the extremes of small currents or charges, small voltages and high resistances. From the perspective of the QMT, it is necessary to consider the status of all three legs in order to interpret any experimental results.

What are the theoretical constraints on the charge transferred through the tunnel junctions in an SET device? Although some general properties of fractional charge have been described [31, 32], it appears there has been no detailed analysis of this question. Perhaps the following discussion will show why there should be. Within a single conductor, the transfer of charge between two points is not quantized at all. One can imagine displacing the electron gas relative to the lattice by an arbitrarily small amount, which will result in an arbitrary charge on the surfaces. At the other extreme, charge transfer between two completely isolated conductors is also well understood. Since the charge on any isolated object is an integer multiple of e, this is the smallest unit of charge that can be transferred between objects (various aspects of charge quantization and the neutrality of matter are reviewed in [33]). The regime of SET is precisely at the crossover between the two extremes just described: the very nature of a tunnel junction is to provide *partial* isolation between the metal electrodes on either side. Thus one is compelled to wonder whether it is possible that Q_S might be slightly different from e in this crossover regime.

One aspect of the middle ground occupied by SET is understood, and may be a fruitful starting point for a detailed analysis of possible corrections to $Q_S = e$. As illustrated in figure 2(a), the conditions under which charge transfer

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[1] The chain from the calculable capacitor to a resistance of $100\,\Omega$ has an uncertainty of 2.4×10^{-8} [28], and the best watt balance result to date has an uncertainty of 3.6×10^{-8} [29]. Combining these yields an uncertainty for the SI watt of about 4×10^{-8}, and half this uncertainty for the SI value of K_J since power is proportional to voltage squared.
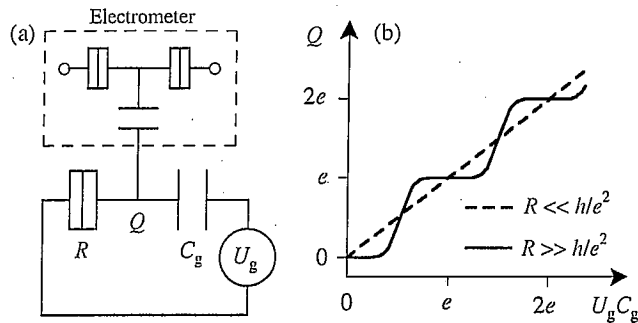
Figure 2. The transition between discrete and continuous charge transfer through a tunnel junction. (*a*) Schematic of an 'electron box' measurement [34]. (*b*) Island charge Q versus charge induced by the gate voltage U_g.

through a tunnel junction becomes *discrete* are twofold. The charge Q of an island isolated by a tunnel junction, and coupled to a voltage source and an electrometer by capacitors [34], will show discrete steps when two sources of fluctuations are suppressed. First, the single-electron charging energy of the island, $E_C = e^2/2C_{tot}$ where C_{tot} is the total island capacitance, must be larger than the energy of thermal fluctuations at temperature T, $E_{th} = kT$, where k is the Boltzmann constant. Second, the junction must be opaque enough that quantum fluctuations are suppressed, which can be quantified as follows. The lifetime of the island charge set by tunnelling is $\tau_{isl} \sim RC_{tot}$. The timescale for quantum fluctuations of the island charge is $\Delta E \Delta t \sim h/2\pi$ with $\Delta E = E_C$. The island charge will be well defined, and thus observable as discrete multiples of e, when $\tau_{isl} \gg \Delta t$, which occurs when $R \gg h/e^2 \approx 26\,k\Omega$. Thus the charge measured by the electrometer as a function of the voltage U_g will vary smoothly for $R \ll h/e^2$ and show discrete steps for $R \gg h/e^2$, as illustrated in figure 2(*b*). (A rigorous treatment of the conditions for observing discrete charge transfer in tunnel junctions can be found in [35].)

Given this picture of the transition between continuous and discrete charge transfer through a tunnel junction, one must then ask whether it is *necessarily* the case that the amount of charge transferred becomes exactly e at the same point where it becomes discrete. In other words, is it possible that for some regime of junction resistance (and/or temperature) SET devices could transfer charge in discrete units that were slightly different from e? It is tempting to say 'No' and take the view that any deviation from a quantized island charge simply reflects fluctuations in the *number* of quanta and not in their *value*, but it would be reassuring to have a rigorous analysis to support this intuitive view.

There have been no empirical tests of the universality of the current or charge from an SET standard. This is clearly an area that needs more attention [36], but to date progress has been limited by the fact that few groups have been able to operate SET current standards with the required accuracy.

Since there are currently no observational equations in the CODATA analysis that involve Q_S, a direct test similar to those done for the QHR and JVS legs has not been done. There are two experiments that have measured the current produced by an SET device with a calibrated commercial

ammeter [37, 38]. Both found agreement with the expected value of $I = ef$ within a relative standard uncertainty of about 1×10^{-4}. However, these experiments could not distinguish between two possible effects: failure of the SET device to transfer any charge during some cycles (transfer errors) and a deviation from $Q_S = e$ for cycles that did transfer charge [36]. A conclusive test requires that transfer errors be measured independently (a method for doing this is described in [39]).

The status of the SET leg of the QMT can be summarized as follows. (1) There is no theoretical prediction that ε_S is not zero, but the question of possible corrections has received little attention. Furthermore, it is worrisome that SET devices operate precisely in a regime where one might expect to find subtle corrections. (2) The universality of SET standards has not been tested at all. (3) The best direct tests of $Q_S = e$ have large uncertainty and do not rule out confounding effects. For comparison, the measured error rate per cycle in a 7-junction SET pump, which measures mistakes in transferring charge quanta but not the value of the quanta, is of order 1 part in 10^8 [39, 40].

Given all this, it is remarkable that discussions of the QMT have generally focused on what it can reveal about the JVS and QHR legs when the SET leg is by far the weakest!

3.4. Implications for QMT experiments

In light of the very different status of the three legs of the QMT, how should one interpret the results of an actual QMT experiment? The current state of knowledge leads to various thresholds in uncertainty at which different conclusions can be drawn. A result at about 1 part in 10^6 or above should be interpreted primarily in terms of the SET leg, since there is little question about the QHR and JVS legs in this regime. A result between about 7 parts in 10^7 and 3 parts in 10^8 would bear on both the SET and JVS legs, but do little in terms of additional confidence in the QHR leg. Absent a better understanding of the SET leg, the possibility of offsetting corrections would be important in this regime. A result below about 3 parts in 10^8 would bear on all three legs, and again offsetting corrections would have to be considered.

4. Recent NIST result for the QMT

A first result for the QMT, with a relative standard uncertainty slightly less than 1 part in 10^6, has recently been completed at NIST. I will briefly describe the experiment and then use the discussion above to draw conclusions from the result. The experiment involves NIST's first-generation Electron Counting Capacitance Standard, ECCS-1, described in [30]. Although ECCS-1 was first demonstrated in 1998, it is only recently that a full uncertainty budget for the comparison with a calculable capacitor has been completed. This is due in large part to recent progress in determining the frequency dependence of the cryogenic capacitor used in the ECCS [41]. Reference [42] has a complete description of the uncertainty budget and operational details of the ECCS.

The essential idea of the ECCS circuit is illustrated in figure 3. An SET pump operated for N cycles transfers a
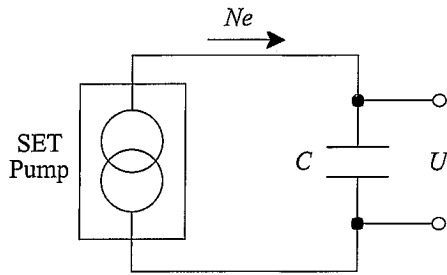
Figure 3. Schematic diagram of the electron counting capacitance standard.

charge NQ_S onto a capacitor C, causing a voltage change ΔU, and from the definition of capacitance we have

$$C = \frac{NQ_S}{\Delta U}. \tag{15}$$

The capacitor is measured in terms of the SI farad using a capacitance bridge traceable to NIST's calculable capacitor, and we denote this value as C_0. The voltage change is measured with a voltmeter calibrated using a JVS and the value of K_J adopted in 1990, $K_{J\text{-}90} \equiv 483\,597.9\,\text{GHz V}^{-1}$ [43]. Thus ΔU is measured in terms of a '1990 volt' V_{90}, and the conversion to SI volts is made via the defining relation

$$\frac{V_{90}}{V} \equiv \frac{K_{J\text{-}90}}{K_J}. \tag{16}$$

Using the notation that a quantity X is the product of its numerical value and its unit, $X = \{X\}_Y\, Y = \{X\}_{Y'}\, Y'$, we can write ΔU as

$$\Delta U = \{\Delta U\}_{SI}\, V = \{\Delta U\}_{90}\, V_{90} = \{\Delta U\}_{90} \frac{K_{J\text{-}90}}{K_J}\, V. \tag{17}$$

Equation (15) then becomes

$$C_0 = \frac{NQ_S K_J}{\{\Delta U\}_{90} K_{J\text{-}90}\, V} = C_{ECCS}(1 + \varepsilon_S)(1 + \varepsilon_J), \tag{18}$$

where C_{ECCS} is the value of $NQ_S/\Delta U$ assuming $\varepsilon_S = 0$ and $\varepsilon_J = 0$. The comparison of the ECCS with a calculable capacitor can then be expressed as a measurement of the ratio

$$\frac{C_0}{C_{ECCS}} = (1 + \varepsilon_S)(1 + \varepsilon_J) \approx 1 + \varepsilon_S + \varepsilon_J. \tag{19}$$

As described in section 3.1, calculable capacitors have been linked to the QHR with an uncertainty of about 2×10^{-8}. In these experiments, the balance of reactive and resistive impedances realized with a quadrature bridge gives

$$R_{QHR} = \frac{\mathcal{A}_2}{\omega C_0}, \tag{20}$$

where $\mathcal{A}_2$ represents various known factors such as bridge ratios. Using the defining expression for the QHR in equation (2), C_0 is then given by

$$C_0 = \frac{\mathcal{A}_2 i}{\omega} \frac{e^2/h}{1 + \varepsilon_K}. \tag{21}$$

Combining this with equation (19) yields an expression similar to that in equation (6), where again $\mathcal{A}_2$, i and ω can be chosen such that we have

$$1 = 1 + \varepsilon_J + \varepsilon_S + \varepsilon_K. \tag{22}$$

Thus the comparison of an ECCS with a calculable capacitor, combined with the link between the calculable capacitor[2] and the QHR, yields a QMT that provides the same test as the original version described in section 2.

The mean of the three values obtained using NIST's ECCS-1 is [42]

$$\frac{C_0}{C_{ECCS}} - 1 = (-0.10 \pm 0.92) \times 10^{-6}. \tag{23}$$

According to the conventional view of the QMT, this result closes the QMT with an uncertainty slightly below 1 part in 10^6, which is too large to provide any additional confidence in the quantum standards. However, the discussion above shows how we can say more than this. First, since the QHR leg is not essential to this result (see equation (19)), we can immediately narrow the discussion to ε_J and ε_S only. Furthermore, since even the largest values of ε_J indicated by the tests of Mohr *et al* (see equations (13) and (14)) are smaller than the uncertainty of the ECCS-1 result, it is reasonable to exclude ε_J also. Thus the primary conclusion should be that the SET leg has been tested at an uncertainty of 9 parts in 10^7 and found to have no correction, i.e. the experiment shows $\varepsilon_S < 9.2 \times 10^{-7}$. Given the lack of other experimental tests of ε_S to date, this is a significant result.

5. Conclusions

Considering theory, universality and direct tests involving fundamental constants, the current status of the individual legs of the QMT is quite different. On all three counts, the SET leg is currently much weaker than the other two. Further investigation of both the theory of discrete charge transfer through tunnel junctions and of the universality of SET devices is needed to increase confidence in quantum standards of current and charge. A particularly interesting study would be measurements of the value of Q_S for a range of junction resistance [36]. Any systematic dependence on resistance would provide a clue to possible correction mechanisms, and the absence of such a dependence would put a useful constraint on such mechanisms. While today it seems entirely likely that all three quantum electrical standards are indeed exact, searching for the limits of these effects may yet lead to a better fundamental understanding of them and possibly to entirely new discoveries.

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[2] Note that the calculable capacitor is not an *essential* element of this test, since the link between capacitance and resistance could be made using artifact standards only. Here the calculable capacitor simply plays the role of a stable reference that allows the ECCS to be linked to the QHR without repeating the quadrature bridge chain.

Acknowledgments

References

[1] Mills I M, Mohr P J, Quinn T J, Taylor B N and Williams E R 2006 Redefinition of the kilogram, ampere, kelvin and mole: a proposed approach to implementing CIPM recommendation 1 (CI-2005) *Metrologia* **43** 227–46

[2] Pratt J R, Kramar J A, Newell D B and Smith D T 2005 Review of SI traceable force metrology for instrumented indentation and atomic force microscopy *Meas. Sci. Technol.* **16** 2129–37

[3] Likharev K K and Zorin A B 1985 Theory of Bloch-wave oscillations in small Josephson junctions *J. Low Temp. Phys.* **59** 347–82

[4] Piquemal F and Genevès G 2000 Argument for a direct realization of the quantum metrological triangle *Metrologia* **37** 207–11

[5] Keller M W 2001 Standards of current and capacitance based on single-electron tunnelling devices *Recent Advances in Metrology and Fundamental Constants* ed T J Quinn *et al* *Proc. Int. School of Physics 'Enrico Fermi' (Amsterdam)* vol CXLVI (Amsterdam: IOS Press) pp 291–316

[6] Piquemal F, Bounouh A, Devoille L, Feltin N, Thevenot O and Trapon G 2004 Fundamental electrical standards and the quantum metrological triangle *C. R. Physique* **5** 857–79

[7] Clothier W K, Sloggett G J, Bairnsfather H, Currey M F and Benjamin J D 1989 A determination of the volt *Metrologia* **26** 9–46

[8] Mohr P J and Taylor B N 2005 CODATA recommended values of the fundamental physical constants: 2002 *Rev. Mod. Phys.* **77** 1–107

[9] The 2006 CODATA recommended values of the constants became available in March 2007 at http://physics.nist.gov/constants (Mohr P J, Taylor B N and Newell D B CODATA recommended values of the fundamental physical constants: 2006, arXiv:0801.0028)

[10] Mohr P J, Taylor B N and Newell D B 2007 Concise summary of tests of the exactness of the relations $K_J = 2e/h$ and $R_K = h/e^2$ from the 2006 CODATA adjustment of the values of the constants *Report Submitted to the 18th Meeting of the CCU (Sèvres, 11–13 June 2007)*

[11] von Klitzing K, Dorda G and Pepper M 1980 New method for high-accuracy determination of the fine structure constant based on quantized Hall resistance *Phys. Rev. Lett.* **45** 494–7

[12] Laughlin R B 1981 Quantized Hall conductivity in two dimensions *Phys. Rev.* B **23** 5632–3

[13] Jeckelmann B and Jeanneret B 2001 The quantum Hall effect as an electrical resistance standard *Rep. Prog. Phys.* **64** 1603–55

[14] Delahaye F and Jeckelmann B 2003 Revised technical guidelines for reliable dc measurements of the quantized Hall resistance *Metrologia* **40** 217–23

[15] Jeckelmann B, Jeanneret B and Inglis D 1997 High precision measurements of the quantized Hall resistance: experimental conditions for universality *Phys. Rev.* B **55** 13124–34

[16] Schopfer F and Poirier W 2007 Testing universality of the quantum Hall effect by means of the Wheatstone bridge *J. Appl. Phys.* **102** 54903

[17] Jeffery A, Elmquist R E, Shields J Q, Lee L H, Cage M E, Shields H S and Dziuba R F 1998 Determination of the von Klitzing constant and the fine-structure constant through a comparison of the quantized Hall resistance and the ohm derived from the NIST calculable capacitor *Metrologia* **35** 83–96

[18] Mohr P J and Taylor B N 2000 CODATA recommended values of the fundamental physical constants: 1998 *Rev. Mod. Phys.* **72** 351–495

[19] Bloch F 1970 Josephson effect in a superconducting ring *Phys. Rev.* B **2** 109–21

[20] Fulton T A 1973 Implications of solid-state corrections to the Josephson voltage-frequency relation *Phys. Rev.* B **7** 981–2

[21] Hartle J B, Scalapino D J and Sugar R L 1971 Absence of quantum-electrodynamic corrections to the charge of the electron as measured in Josephson junction experiments *Phys. Rev.* B **3** 1778–81

[22] Langenberg D N and Schrieffer J R 1973 Comments on quantum-electrodynamic corrections to the electron charge in metals *Phys. Rev.* B **3** 1776–8

[23] Krasnopolin I Y, Behr R and Niemeyer J 2002 Highly precise comparison of $Nb/Al/AlO_x/Al/AlO_x/Al/Nb$ Josephson junction arrays using a SQUID as a null detector *Supercond. Sci. Technol.* **15** 1034–6

[24] Behr R, Schulze H, Muller F, Kohlmann J and Niemeyer J 1999 Josephson arrays at 70 GHz for conventional and programmable voltage standards *IEEE Trans. Instrum. Meas.* **48** 270–3

[25] Jeanneret B, Rufenacht A and Burroughs C J 2001 High precision comparison between SNS and SIS Josephson voltage standards *IEEE Trans. Instrum. Meas.* **50** 188–91

[26] Klushin A M, Behr R, Numssen K, Siegel M and Niemeyer J 2002 Accurate measurements of quantum voltage steps on arrays of bicrystal Josephson junctions *Appl. Phys. Lett.* **80** 1972–4

[27] Jain A K, Lukens J E and Tsai J S 1987 Test for relativistic gravitational effects on charged particles *Phys. Rev. Lett.* **58** 1165–8

[28] Jeffery A, Elmquist R E, Shields J Q, Lee L H, Cage M E, Shields S H and Dziuba R F 1998 Determination of the von Klitzing constant and the fine-structure constant through a comparision of the quantized Hall resistance and the ohm derived from the NIST calculable capacitor *Metrologia* **35** 83–96

[29] Steiner R L, Williams E R, Liu R and Newell D B 2007 Uncertainty improvements of the NIST electronic kilogram *IEEE Trans. Instrum. Meas.* **56** 592–6

[30] Keller M W, Eichenberger A L, Martinis J M and Zimmerman N M 1999 A capacitance standard based on counting electrons *Science* **285** 1706–9

[31] Goldhaber A S 2003 Fractional charge definitions and conditions SLAC-PUB-10483 *J. Math Phys.* **44** 3607 (revised version)

[32] Goldhaber A S and Kivelson S A 1991 Local charge versus Aharonov–Bohm charge *Phys. Lett.* B **255** 445–50

[33] Unnikrishnan C S and Gillies G T 2004 The electrical neutrality of atoms and of bulk matter *Metrologia* **41** S125–35

[34] Lafarge P, Pothier H, Williams E R, Esteve D, Urbina C and Devoret M H 1991 Direct observation of macroscopic charge quantization *Z. Phys.* B **85** 327–32

[35] Grabert H, Ingold G-L, Devoret M H, Estève D, Pothier H and Urbina C 1991 Single electron tunnelling rates in multijunction circuits *Z. Phys.* B **84** 153–5

[36] Zimmerman N M, Keller M W and Eichenberger A L 2008 Is the charge of an electron in a metal the same as the charge of a free electron? In preparation

[37] Janssen T J B M and Hartland A 2000 Accuracy of quantized single-electron current in a one-dimensional channel *Physica* B **284–288** 1790–1

[38] Scherer H, Lotkhov S V, Willenberg G-D and Zorin A B 2005 Steps toward a capacitance standard based on single-electron counting at PTB *IEEE Trans. Instrum. Meas.* **54** 666–9

[39] Keller M W, Martinis J M, Zimmerman N M and Steinbach A H 1996 Accuracy of electron counting using a 7-junction electron pump *Appl. Phys. Lett.* **69** 1804–6

[40] Keller M W, Martinis J M and Kautz R L 1998 Rare errors in a well-characterized electron pump: comparison of experiment and theory *Phys. Rev. Lett.* **80** 4530–3

[41] Zimmerman N M, Simonds B J and Wang Y 2006 An upper bound to the frequency dependence of the cryogenic vacuum-gap capacitor *Metrologia* **43** 383–8

[42] Keller M W, Zimmerman N M and Eichenberger A L 2007 Uncertainty budget for the NIST electron counting capacitance standard, ECCS-1 *Metrologia* **44** 505–12

[43] Taylor B N and Witt T J 1989 New international electrical reference standards based on the Josephson and quantum Hall effects *Metrologia* **26** 47–62