# Error and Transient Analysis of Stepwise-Approximated Sinewaves Generated by Programmable Josephson Voltage Standards

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### Abstract

We are developing a quantum-based 60 Hz power standard that exploits the precision sinusoidal reference voltages synthesized by a programmable Josephson voltage standard (PJVS). PJVS technology was chosen for this application because we need to generate voltages up to  $V_{rms} = 1.2$  V. PJVS systems use many series arrays of Josephson junctions operating as a multibit digital-to-analog converter to produce accurate quantum-based dc voltages. Using stepwise approximation synthesis, the system can also generate arbitrary ac waveforms (i.e., an ACPJVS) and in this application produces sinewaves with calculable rms voltage and spectral content. The primary drawback to this ACPJVS synthesis technique is the uncertainty that results from switching between the discrete voltages due to finite rise times and transient signals. In this paper, we present measurements and simulations that elucidate some of the error sources that are intrinsic to the ACPJVS when it is used for rms measurements. In particular, we consider sinewaves synthesized at frequencies up to the audio range, where the effect of these errors is more easily measured because the fixed transition time becomes a greater fraction of the time in each quantized voltage state. Our goal for the power standard is to reduce all error sources and uncertainty in an ac power standard will be reduced to a few parts in 10<sup>6</sup>.

### 1. Introduction

Following the development of series arrays of intrinsically shunted Josephson junctions in the mid 1990's [1-3], there has been considerable work demonstrating their use as Programmable Josephson Voltage Standards. In addition to applications requiring accurate and stable dc voltages, PJVS systems have also been applied to ac metrology by generating waveforms approximated by a staircase series of constant voltages of equal duration, or samples. PJVS systems have been used to produce a variety of arbitrary waveforms; first, for fast-reversed dc comparisons between Josephson sources and thermal voltage converters [4, 5], and later, as sinewaves for high accuracy ac-dc difference measurements at frequencies up to 1 kHz [6, 7]. These PJVS systems are quite different from the ac Josephson voltage standard (ACJVS) based on high-speed pulse-driven arrays, which are presently limited to frequencies above 1 kHz [8, 9]. Although the high accuracy of the pulse-driven ACJVS would be desirable, it is unsuitable for this application because it can presently produce only  $V_{rms} = 0.2$  V maximum voltage (and the existing pulse generator imposes a minimum sinewave frequency of 1.2 kHz).

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By using an ACPJVS we hope to improve one of the dominant sources of uncertainty in 60 Hz power calibration, namely the accuracy of the voltage and current sources. This work is one component of NIST's goal to implement quantum-based standards throughout electrical metrology and for fundamental measurements. This work also complements a EUROMET project with the similar goal of a quantum-based power standard [10, 11]. In a previous paper we demonstrated accurate waveform synthesis at 60 Hz, showed important contributions of digitization harmonics to the total rms voltage, and raised concerns regarding uncertainties due to transients [12]. In this paper, we focus on errors caused by the finite transitions between quantized voltages when stepwise approximated sinewaves are generated. We demonstrate that the overall magnitude of the errors introduced by these transitions is sufficiently small (at 60 Hz) to reach our desired uncertainties of a few parts in  $10^7$ . However, we also show that it is essential to precisely model the behavior of every transition to achieve these uncertainties. Now that we have a more complete understanding of the nature of the transitions, we plan to simplify the NIST implementation of a 60 Hz power standard by use of sampling techniques instead of rms-based comparisons. This allows us to discard high-uncertainty measurement data at the transitions and to directly compare the quantized voltages of the PJVS waveform with a stabilized sinewave from a semiconductor reference source. Modern sampling voltmeters are able to achieve the required accuracy for such measurements, and there have already been demonstrations at other labs of successful application of these techniques with Josephson devices [13].

## 2. PJVS Quantum Accuracy

PJVS systems at NIST are based on multiple series arrays of SNS (superconductor normal-metal superconductor) Josephson junctions, and have been continually refined for more than a decade. Details of the Josephson circuits and systems have been described elsewhere [14, 15, 16]. The PJVS output voltage at any given time is precisely  $V = Mf/K_{J-90}$ , where *M* is the digitally programmed step number of all combined series arrays, *f* is the applied microwave frequency, and  $K_{J-90}$  is the Josephson constant (483 597.9 GHz/V). As an intrinsic quantum standard, the accurate programmable output voltage depends only on *M* and *f*. For each series-connected sub-array (or "cell"), all bias parameters such as dc bias current, microwave power level, and chip temperature must produce no measurable change in the output voltage as each parameter is varied over a finite range. We refer to these bias ranges as operating margins, and to measurements that verify this required behavior as "flat-spot" demonstrations [14].

A typical result from such a measurement, which is shown in Fig. 1, demonstrates a "flat" dc voltage step from a PJVS generating 1.115 066 9 V with almost 2 mA of current margin (i.e. the current range over which no change in output voltage can be detected at the noise floor of the digital voltmeter, typically about 0.1  $\mu$ V in this measurement). We define the dither current as an offset current from the chosen step centers that is simultaneously applied to all cells. For dc metrology, such flatness and operating range measurements for every cell are generally performed weekly (if not daily) to confirm that the dc bias current "set points" are centered within the bias current range. This procedure ensures that the PJVS produces accurate quantized voltages every time it is programmed into a new "state". We refer to the PJVS "state" as the combination of the 13 cell polarities that produces a particular total output voltage. The polarity of each cell *i* is determined by its step number  $n_i$  and can be positive (P,  $n_i=+1$ ), zero (0,  $n_i=0$ ), or negative (N,  $n_i=-1$ ). The precise voltage of each cell,  $V_i = n_i N_i f/K_{J-90}$ , is determined by its number of junctions ( $N_i$ ), the applied frequency, and its step number which in turn is determined by the bias current setpoint. Typical values are shown in Table 1. For example, the state "000PPPNPNPPPN" in Fig. 1 has the cells 1, 2, 3 on the zero step, cells 4, 5, 6, 8, 10, 11, 12 on the

positive step, and the remaining cells on the negative step. The step number M = 29958 for this state is the sum of  $n_i N_i$  over all cells.



Fig. 1 (a) Measured PJVS output voltage deviation from the expected quantized voltage vs. dither current for a dc output of 1.115 066 9 V and 18.0 GHz. The dither current flows simultaneously through all cells. (b) High resolution plot of the same measured data in (a), showing that the truly "flat" region is from -0.8 mA to +1.0 mA. The scatter in the data relative to the PJVS output level is a few parts in  $10^7$ , and the uncertainty bars shown are  $1\sigma$ .

	NI 1 C			D	D	D	0.11
Cell	Number of	Positive Set	Negative Set	Range	Range	Range	Critical
Number	Josephson	$P_{oint}(\mathbf{mA})$	$P_{oint}(\mathbf{mA})$	+1 State	0 State	-1 State	Current
Number	Junctions			(mA)	(mA)	(mA)	(mA)
1	13200	n/a	n/a	n/a	n/a	n/a	n/a
2	13200	n/a	n/a	n/a	n/a	n/a	n/a
3	13197	n/a	n/a	n/a	n/a	n/a	n/a
4	13200	11.79	-11.76	2.38	7.00	2.36	11.01
5	13194	11.67	-11.64	2.28	5.90	2.30	10.67
6	13200	11.37	-11.36	2.38	5.18	2.32	10.82
7	13188	11.16	-11.21	2.28	3.72	2.22	10.45
8	5832	11.02	-11.08	3.32	5.70	3.24	10.64
9	1944	10.87	-10.92	3.52	6.36	3.46	10.53
10	24	10.38	-10.54	4.60	8.54	4.68	10.48
11	72	10.93	-11.04	4.58	10.08	4.60	10.82
12	216	10.97	-11.12	4.58	9.04	4.60	10.88
13	648	11.06	-11.20	4.52	9.86	4.48	10.84

 TABLE 1

 Chip Operating Parameters

Measured bias parameters for a PJVS chip operating at 18.3 GHz. This particular chip is a triple-stacked ternary 3.9 V circuit. Our present current bias generator is limited to a 3 V maximum voltage; thus we cannot use three of the largest cells (1, 2, 3). An important detail to observe is the matching between the measured bias current values for positive and negative voltage states. This is a rough indication of how precisely we might expect the applied biases to agree with the target setpoint biases in an ac waveform. With many cells biased simultaneously, we estimate that the applied set points would lie within a  $\pm 0.2$  mA range.

To synthesize ac waveforms with the PJVS, we generate a rapid sequence of states that produces a stepwise approximation of the desired signal, as illustrated by the sinewave in Fig. 2. This technique has the potential for high accuracy because each state in the output sequence is a precisely known quantized Josephson voltage (as in Fig. 1). However, the ultimate accuracy depends on the transitions between the quantized voltages where the voltage is inexact. In order to see how well our sampling voltmeter can resolve the ac waveform in Fig. 2, we sampled for several minutes to obtain 400 averages. Then we applied various values of dither current ( $\Delta i$ , an offset current through the entire

array applied to every state in the waveform) in order to verify that the quantum levels from the PJVS are independent of bias current setpoint over some finite range.

To see the effect from this dither current, we closely examined the time-dependent voltage of each state of the stepwise approximated waveform. In Fig. 3 we show the voltage of one of these states as it was sampled over its entire duration and averaged over 400 measurements. The goal of this measurement is to quickly verify that all the bias currents produce the correct voltage in every state. Measurements were made with different dither currents applied over the range  $\pm 0.9$  mA. Over this range of dither currents, the mean voltages at each time agree within the measurement uncertainty. We conclude that the cells must all be on operating margins and that the individual cell bias currents for this state are correct. Similar measurements were made for other states used in the waveform, and the same conclusion was obtained for similar current ranges.

However, there are several features of Fig. 3 that are undesirable. First, the voltage appears to settle after the bias current reaches the set point, and this feature occurs similarly for every transition in the sinewave. Although the first few time samples appear to show a settling transient on this voltage, the mean voltages of all the samples at each time agree within the 1  $\sigma$  uncertainty. Second, the measured voltage disagrees with the calculated quantum voltage. This is probably due to an uncorrected nonlinearity or gain error in the sampling measurement. Nevertheless, such a nonlinearity will significantly increase the uncertainty for rms measurements, and calibration of these nonlinearities is equally important for sampling measurements. Lastly, there is significant scatter in the data that should be optimized in future measurements by experimenting with different sampling techniques and perhaps more modern digital samplers.



Fig. 2. Measured PJVS 60 Hz waveform for a 0.5 V (rms) sinewave with 64 states. The measurement was performed by an integrating sampler using 2000 sample points, repeated 400 times for averaging. Some of the transitions between quantum levels are visible at this scale.



Fig 3. High resolution plot of an individual state in the sampled 60 Hz waveform of Fig. 2. The waveform was measured at several dither currents,  $\Delta i$ , from -0.9 mA to +0.9 mA. We observe that the PJVS quantum level does not change for  $\Delta i$  values in this range (relative to the 1 $\sigma$  error bars). The horizontal line indicates the calculated PJVS output voltage level, which differs from the measured data due to nonlinearity and gain error in this particular digitizer.

#### **3.** Transitions between Quantum Levels

The primary goal for most ac applications of the ACPJVS is to provide a signal with a precisely calculable rms value. Using our knowledge of the quantized voltages and the timing sequence of the

digital synthesis, we model the synthesized voltage waveform in order to compute the total rms value. The transitions between levels affect the accuracy of this model in two ways. First, we use a linear approximation for all transitions between states. In reality, the transitions are certainly not linear and the individual transitions are not identical. Fortunately, we have a new bias current generator [17] with an internal 5 ns rise-time, which greatly increases the time on quantized voltages and decreases the transition-related errors. Secondly, as we will explain in detail below, the timing of the transitions is affected by the precise bias current set point for each cell in each state. Small errors in the setpoints due to output variations from the current drivers shift the transition timing, and the magnitude of this error mechanism is surprisingly large. We discuss both of these sources of transition error, but focus primarily on the latter.

When preparing for precision measurements, we measure and correct the gain and dc-offset on every current driver to ensure that the bias currents accurately reach their target values when different combinations of cell voltages are used for the different output states. After a microwave frequency is selected, our fully automated PJVS system uses a nanovoltmeter to determine the exact center and operating range of each (P, 0, N) voltage step for all 13 cells. A typical bias parameter table from this initialization measurement is shown in Table 1. For dc metrology, we obtain quantized voltages by selecting any current-bias value within the operating current range, as illustrated in Fig. 1. For stepwise approximated ac waveforms, however, the rms voltage is greatly affected by the bias setpoint chosen for every step and by variations in the exact bias current delivered to each cell. The following examples illustrate this point.

The square-wave simulations shown in Fig. 4 illustrate varying transitions for a single cell from the zero state to the +1 state and then returning to the zero state. Our model used 40 ns linear current ramps applied to the simulated voltage-versus-current characteristic shown in Fig. 5 that is typical for SNS junctions. The resulting nonlinear voltage transitions are shown in the PJVS output voltage waveforms of Figs. 4 (a) and 4 (b). The trigger events at 0 ns and 80 ns in the plots indicate the exact start time of the current ramp from one state to the next. Notice that many nanoseconds elapse as the current ramp is first changing before there is any change in the output voltage. As the caption of Fig. 4 describes in detail, increasing the bias current above the centered setpoint for either the zero state (b) or the +1 state (a) shifts the transition edges away from each other in time, thereby increasing the total rms-voltage generated. Conversely, decreasing the bias current from the centered setpoint moves the transitions closer to each other, thus decreasing the rms voltage.

This effect is observed experimentally, as shown in Fig. 6, where we plot the change in rms voltage measured by a Fluke 792A<sup>†</sup> thermal transfer standard for a  $V_{zero-to-peak} = 0.5$  V square wave from an individual cell at various frequencies as a function of dither current from the centered setpoint of the +1 state. These measurements were obtained on the 0.7 V range of the transfer standard, where it utilizes a high-impedance input amplifier that draws no significant load current from the PJVS. The edges of the constant-voltage state are visible at approximately ±1 mA. The striking feature is that rms voltage varies with dither current, and the corresponding slope increases with increasing frequency. The linear slope indicates a correlation between the total rms voltage and the dither current from step center. The simulated results for this example are also shown in Fig. 6 as solid lines which agree well with the measured data. The 0.12  $\mu$ V/mA slope (rms) at 60 Hz for this simple example would translate

<sup>&</sup>lt;sup>†</sup> Commercial instruments are identified in this paper in order to adequately specify the experimental procedure. Such identification does not imply recommendation or endorsement by NIST, nor does it imply that the equipment identified are necessarily the best available for the purpose.

into an uncertainty contribution of  $\pm 2.4$  parts in  $10^8$  for an estimated  $\pm 0.2$  mA setpoint accuracy. The effect scales with frequency such that the uncertainty increases to  $\pm 4$  parts in  $10^7$  at 1 kHz, while at 4 kHz the uncertainty is  $\pm 1.6$  parts in  $10^6$ . Note that these uncertainties are for setpoint variation in a single cell!



Fig. 4. Simulated transitions for an individual cell generating a square wave between the zero state and the +1 state for different current-bias set points. The solid-black points in both plots depict  $V_{out}$  when the cell switches between exactly centered values for the zero state (0 mA), the +1 state (10 mA), and back to the zero state. The open circles in (a) show the effect of increasing the current-bias set point of the +1 state to 11 mA, which results in an increased total rms voltage, since the left and right transition edges move in opposite directions. Similarly, the open diamonds in (b) show an increase in rms voltage when the zero-state bias point is increased 1 mA. The square wave frequency in both examples is 6.25 MHz, which allows us to show both rising and falling edges in the same plot. These same transition-edge shifts occur at low frequencies, as the measurements and simulations in the following section demonstrate.



individual PJVS cell with voltage steps of  $\pm 500 \text{ mV}$  for current-bias set points centered at  $\pm 10 \text{ mA}$ . To model the complete PJVS chip in our simulations, we scaled each cell voltage according to its number of junctions.

Fig. 6. Measured change in total rms voltage of a  $V_{zero-to-peak} = 0.5$  V square wave generated by a single cell as a function of dither current from the center of the +1 step current range. The solid points indicate measured data at various frequencies, and the corresponding lines show the simulated result (as in Fig. 4(a)). The variation in rms voltage with dither current is approximately 2  $\mu$ V/mA (rms) at 1 kHz, but only 0.1  $\mu$ V/mA (rms) at 60 Hz.

The change in voltage ( $\Delta V$ ) versus dither current shown in Fig. 6 are obtained from ac-ac measurements, meaning that each data point in the plot represents the measured difference between  $V_{rms}$  (at each given dither current) and a reference voltage (in this case  $V_{rms}$  at zero dither current, same frequency). The data have been corrected to remove the transfer standard scaling factor and first-order drift, so  $\Delta V$  indicates the actual error (in  $V_{rms}$  in microvolts) introduced in the  $V_{zero-to-peak} = 0.5$  V square wave by the dither current. These measurements could also have been performed by comparing  $V_{rms}$  at each dither current to precise dc levels from the PJVS. This ac-dc difference method has the same ability to determine  $\Delta V$  versus dither current, which was our primary interest here, but it also provides additional information about the "absolute" rms voltages at different frequencies for comparison with each other. Eventually, we will be able to use this "absolute" rms information to calculate formal uncertainties for PJVS ac waveforms generated by stepwise synthesis, but presently our assumed transition model (using the arbitrary choice of centered set point) cannot predict the magnitude of the rms voltage with the necessary precision, as we will discuss later.

### 4. Waveforms with Many Transitions

In the previous example, we analyzed transitions of an individual cell to explain why the rms voltage of a simple square wave depends on the bias current setpoint (which is not the case for dc voltages). Now we consider the effect of transition timing for ac waveforms, such as sinewaves, with many transitions. We used the ACPJVS to generate  $V_{rms} = 0.4$  V sinewaves of various frequencies and numbers of samples, and again used a thermal transfer standard to measure differences. First, we were interested to see how much the rms voltage of the sinewave changed in response to varying the bias setpoint for an individual cell, as would be the situation if a single bias current in Table 1 were shifted. Results of such an experiment are presented in Fig. 7, where we see that for the 60 Hz case in Fig. 7 (a), the error is acceptably small for our application and is hidden within the few hundred nanovolt measurement noise. Fig. 7 (b) shows the dependence at 1 kHz, where we observe that the apparent center position of the smallest slope is a few tenths of a milliamp negative, indicating that for this cell the current bias is slightly above step center. If our bias point on this cell were off by 0.2 mA, the shift in the rms voltage at 1 kHz due to this cell alone would be 0.06  $\mu$ V (rms) which is 1.2 parts in  $10^7$ . When you consider that the other cells have their own current offsets for this waveform, it is clear that exceedingly tight tolerances would be required for both bias currents and rms voltage modeling in order to achieve an uncertainty of a part in  $10^6$  at 1 kHz for a multibit waveform. It is interesting that for such large variations in the number of samples, the slopes for any given sinewave frequency (Figs. 7(a) and (b)) are so similar. We believe that we understand why this occurs, and the example in the following section is a simpler case where the explanation will become clear.



Fig. 7. Measured change in total rms voltage of several  $V_{rms} = 0.4$  V PJVS sinewaves at (a) 60 Hz and (b) 1 kHz as a function of dither current applied to a *single* cell (cell 8). (a) For a 60 Hz waveform the slope is approximately 0.04  $\mu$ V/mA (rms) from -1 mA to +1 mA, while at 1 kHz the slope is approximately 0.28  $\mu$ V/mA (rms) from -0.5 mA to +0.5 mA. At both frequencies, very similar behavior is observed for 64, 256, 512, 1024 samples.

Next, we measured the change in rms voltage of the sinewave in response to simultaneously shifting the bias setpoints to *all* the cells, as would be the case if the microwave power were to drift slightly or if a change in dewar pressure were to shift the chip temperature (thus moving all the values in Table 1 in the same direction). This is a worst-case measurement, because shifting the bias currents in the same direction. The results are presented in Fig. 8 where we see that a "worst case" simultaneous 0.5 mA shift in all cells introduced an error of only 0.1  $\mu$ V rms (2 parts in 10<sup>7</sup>) for a 60 Hz sinewave. This error is acceptably small for our application, and nearly meets our goal of an error no greater than 1 part in 10<sup>7</sup>. However, Fig. 8 (b) shows a much larger dither dependence on rms voltage at 1 kHz, and once again we observe that the apparent center position is not at zero, indicating that for this waveform the current-biases are generally somewhat below step center. In this case if all the bias currents were shifted by the same 0.5 mA, the shift in the rms voltage at 1 kHz would be 1.9  $\mu$ V rms (3.8 parts in 10<sup>6</sup>). At this frequency, in order to place a bound of 1 part in 10<sup>6</sup> on the error due to a systematic shift in the current-biases, we would need to ensure that the operating bias currents are stable and reproducible within ±0.05 mA of the measured initialization values (an extremely challenging task).

As observed in Fig. 7 and previously described, the data from Fig. 8 also confirm that changing the number of samples doesn't affect the slope (the changes in voltage with dither amplitude) for any given sinewave frequency. We believe that this relates to the nature of the timing shifts, illustrated in Fig. 9, where we see that doubling the number of samples doubles the number of time intervals, while the net increase in the  $\int dV dt$  caused by a given shift in the transition timing remains practically unchanged.



Fig. 8. Measured change in total rms voltage of several  $V_{rms} = 0.4$  V PJVS-synthesized sinewaves as a function of dither current simultaneously applied to *all* cells for (a) 60 Hz and (b) 1 kHz waveforms. At 60 Hz the slope is approximately 0.2  $\mu$ V/mA (rms). Notice that this worst-case slope is about 20 times greater than for the individual cell shown in Fig. 7 (a). At 1 kHz the slope is approximately 3.8  $\mu$ V/mA (rms), which is about 13.5 times greater than for the individual cell variation shown in Fig. 7 (b). At both frequencies, very similar behavior is observed over a wide range of numbers of samples per cycle.



Fig. 9. Example showing how a 1 ns shift in timing of transition edges effects a stepwise approximated sinewave for different numbers of samples. The increased area due to the timing shift is very similar between the first case and the case where the number of samples is doubled.

#### 5. Absolute Accuracy

PJVS systems are intrinsic standards due to their quantized output voltage levels that are determined by the applied microwave frequency and the programmed step number. As we have discussed, when applied as an ACPJVS system, the transitions between the quantized voltages in stepwise approximated ac waveforms cause the absolute accuracy of the rms voltage to depend upon other parameters as well, including the bias current setpoints used for the state each cell, the ability of the bias electronics to accurately produce those setpoints, and also non-timing-related transient features such as overshoot, transmission line reflections, and ringing. In order to correctly calculate the expected rms voltage from the ACPJVS for this synthesis method, these error mechanisms must be carefully modeled (utilizing measured data for *I-V* curve characteristics and settling times) for the entire ACPJVS circuit. A precision measurement procedure must then be used to verify that the calculated voltage is correct. One possible procedure is to perform ac-dc difference measurements with a thermal transfer standard where we program a dc voltage magnitude with precisely the same rms value as the ac waveform. This procedure must be performed continually to ensure (a) that the ACPJVS output waveform falls within the calculated uncertainty, and (b) that the transition models (the parameters of which will change for different microwave frequencies, microwave power levels, and chip designs) are correct at all times. We have just started to implement accurate modeling and precision measurements. Future work on the ACPJVS will be devoted to integrating these procedures into routine operation and improving our understanding of the errors and uncertainties.

## 6. Conclusion

As suggested by the results in this paper, a sampling strategy for the quantum watt system appears highly favorable compared to a rms measurement strategy, especially at higher frequencies. Therefore, we plan to use modern sampling techniques to compare the 60 Hz ACPJVS waveform to an independent reference sinewave of high stability and spectral purity. This will enable us to avoid contributions in the analysis where the ACPJVS is in transition, and directly compare the precisely known quantum voltages with the reference sinewave. However, to more fully understand the limitations of the ACPJVS in generating waveforms by stepwise approximation synthesis, we explored the transitions between quantum voltages both in simulations and experimental measurements. We discovered that for our SNS Josephson arrays, the uncertainty in the total rms voltage due to transitions with timing shifts is significantly larger than was previously appreciated. The primary cause of these timing shifts is that the bias current selected for each PJVS cell affects the overall transition timing and thus the rms voltage, even without considering nonlinear transient behavior. At 60 Hz we could place an upper bound on this error of a few parts in  $10^7$  if we routinely measured the ACPJVS transitions to confirm that they match our rms voltage model. However, because this error mechanism scales with frequency, the constraints become much tighter in the audio range where extremely precise modeling of the output voltage using measured values for I-V curves, bias current setpoints, settling times, etc... will be required to predict the rms voltage to even a few parts in  $10^6$ . Further investigation is necessary to determine how the non-timing-related (transient non-uniformity and ringing) aspects of the transitions affect the rms voltage uncertainty, but we are expecting these contributions to be smaller than the timing shifts. The large output voltages of ACPJVS arrays (potentially up to  $V_{rms} = 10$  V) for ac stepwise approximated waveforms may outweigh the disadvantages in some applications such as 60 Hz power standards. Unfortunately, obtaining low uncertainties for rms voltages at and above audio frequencies will always be a significant challenge. At present, sampling technology appears to be the best measurement technique to circumvent these difficulties.

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