Nanosecond delay with subpicosecond uncertainty

Donald R. Larson and Nicholas G. Paulter, Jr.
National Institute of Standards and Technology, 100 Bureau Drive, Gaithersburg, Maryland 20899-8102

(Received 8 May 2007; accepted 27 June 2007; published online 2 August 2007)

We have combined a commercially available, variable-length coaxial delay line (trombone line) with a high-resolution linear translation system. The result is better resolution and lower uncertainty in the achievable delays than previously available. The range of delay is 0 ps to approximately 1250 ps, the bidirectional resolution is 2.0 ps, the unidirectional resolution is 0.2 ps, and the uncertainty (95% confidence interval) in the measured delay is ±0.09 ps. Drift, temperature dependence, repeatability, linearity, and hysteresis were also examined. [DOI: 10.1063/1.2760982]

INTRODUCTION

The variable delay lines that are used to control the delay of a pulsed signal are critical to many communications systems. A variable delay line may also be used to control the phase of a microwave or rf signal. These applications require high-resolution delay control and low delay uncertainty. The variable-length, coaxial delay lines, also known as trombone lines, provide a convenient method of introducing a variable delay with minimal impact on the pulse parameters[1] of the signals delayed. The pulse parameters examined were amplitude and transition duration. Being a mechanical system, the range of delays achievable is limited but this can be somewhat overcome by using more delay lines in series. However, this increases the impact on the pulse parameters due to frequency dependent propagation and insertion losses, which ultimately limit the number of delay lines that can be serially added. The utility of a variable delay line instrument for producing programmable delays is dependent on a number of parameters such as drift, temperature dependence, repeatability, linearity, and hysteresis.

DELAY SYSTEM

The delay system consists of four variable-length coaxial delay lines mounted on a high-resolution linear translation stage. According to the manufacturer, each variable-length coaxial delay line had a delay range of 312.5 ps, a nominal impedance of 50 Ω, and a 3 dB attenuation bandwidth of 18 GHz. The linear translation stage, obtained from another manufacturer, included a glass scale position sensor with a resolution of 50 nm, a 100 mm travel, and a minimum incremental motion of 50 nm. Figure 1 depicts the delay system including custom aluminum mounting plates for the linear translation stage and trombone lines. The position of the translation stage and thus the amount of delay are controlled using a personal computer (PC) and a servomotor drive. The range of delay achievable using a series connection of four trombone lines is 0 ps to approximately 1250 ps. For positive and negative changes in delay (bidirectional motion of the linear translation stage), the resolution is 2.0 ps. For either positive or negative changes in delay (unidirectional motion of the linear translation stage), the resolution is approximately 0.2 ps. The uncertainty (95% confidence interval) in the measured delay is ±0.09 ps. The maximum delay could be halved or quartered with a doubling or quadrupling of the resolution, respectively.

MEASUREMENT METHOD

There have been many methods demonstrated for determining the propagation delay of electrical and optical transmission lines.[2–4] Many of these methods may be divided into two groups: pulse methods and phase shift methods.[2] We have previously described the measurement method used to determine the propagation delay results reported here.[1] Briefly, a microwave synthesizer is used to drive a comb generator, producing an electrical pulse. The electrical pulse is split into two pulse signals. One of these two pulse signals is launched into the input connector of the first trombone line of the delay system; this pulse is displayed in Fig. 2. The other pulse signal is filtered to produce a spectrally pure (all harmonics are less than −100 dB) microwave sinewave signal at a multiple of the repetition rate of the comb generator. This sinewave is acquired using a 20 GHz bandwidth, sampling oscilloscope triggered by the electrical pulse that is output from the last trombone line of the delay system. The phase of the acquired sinewave is determined using a three parameter sinefit routine.[5,6] The delay is then calculated from the phase of the fitted sinewave.[1] The experimental arrangement is relatively simple and is depicted in Fig. 3. Even though a sinewave is acquired and its phase is used to determine the delay, this is a pulse method. It is a pulse that propagates in the delay line and triggers the sampling oscilloscope. The phase of the acquired sinewave is established at the instant the oscilloscope is triggered.

This method of generating a pulse signal synchronized with a sinewave signal resulted in the smallest amount of jitter, approximately 0.8 ps rms, we measured between pulses and sinewaves.[1] This small amount of jitter is near the limit of the trigger circuitry of the sampling oscilloscope.

After propagation through the four trombone lines, the changes in the transition duration and pulse amplitude were lower than the uncertainty in our measurements.

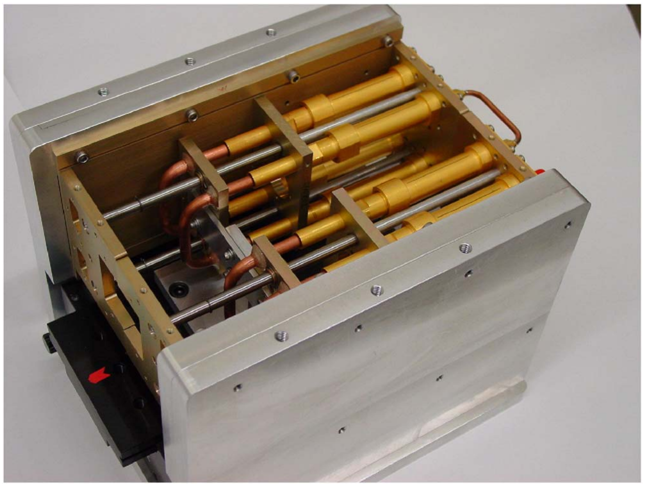

FIG. 1. (Color online) Delay lines and linear translation stage.

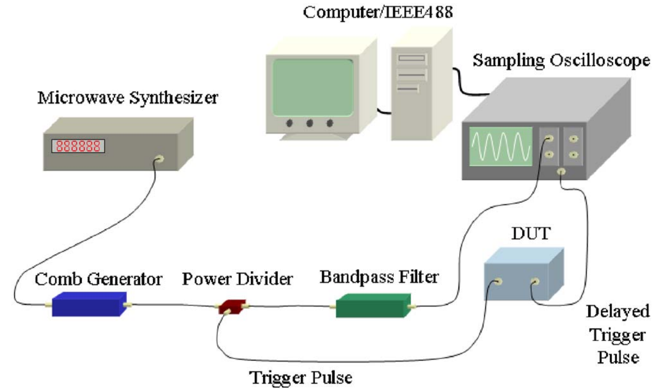

FIG. 3. (Color online) Electrical delay line measurement system.

The uncertainty in these measurements was determined using the uncertainty analysis described in Ref. 1. The uncertainty analysis established that the combined uncertainty is inversely proportional to the frequency of the sinewaves used. However, as the frequency is increased, the delay in the trigger signal path may introduce phase changes that are greater than 2π radians, making it necessary to unambiguously determine the number of 2π phase shifts. To do this, we have used two different frequency sinewaves when measuring long delays.[1] The lower frequency sinewave is selected to introduce a phase shift that is less than π radians. Measurements made with a sinewave of this frequency provide a coarse delay value. We then choose a higher frequency sinewave to obtain measurements with the desired resolution and with lower delay uncertainty. Using this procedure, the number of 2π phase shifts does not need to be tracked for the higher frequency sinewave since the delay determined by the higher frequency is used as a refinement to the coarse delay measurement.

MEASUREMENT RESULTS

The utility of the delay system for producing programmable delays is dependent on a number of parameters such as drift, temperature dependence of delay, hysteresis, repeatability, and linearity. Any backlash present in the linear translation stage, coupled with static friction in the trombone line, would appear as drift or hysteresis and result in poor repeatability of programmed delays.

We examined the performance of the measurement system by first making a set of 15 measurements without the trombone line in the pulse signal path. The total time for acquisition of the 15 measurements was 10 min. The standard deviation of the measurements was 54 fs (see Fig. 4.) There was no apparent drift in the delay. These measurements were repeated with the trombone line inserted in the signal path. Again, there is no indication of drift, but the standard deviation of these measurement results was 100 fs, which is nearly twice the variation of the reference measurement.

DRIFT

The delay system position stability, or drift, was obtained using the following measurement procedure. The delay system was positioned to approximately midrange in its delay. The translation stage remained powered, as was the case for all measurements presented here unless specifically stated otherwise. Five sets of data with five measurements per set were made at approximately 15 min intervals. Each set of five measurements took approximately 3 min to acquire and the total elapsed time for this experiment was ap-

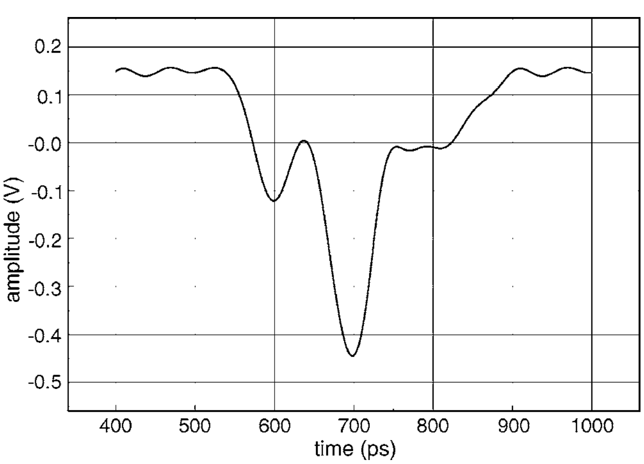

FIG. 2. Signal from comb generator, input to delay line.

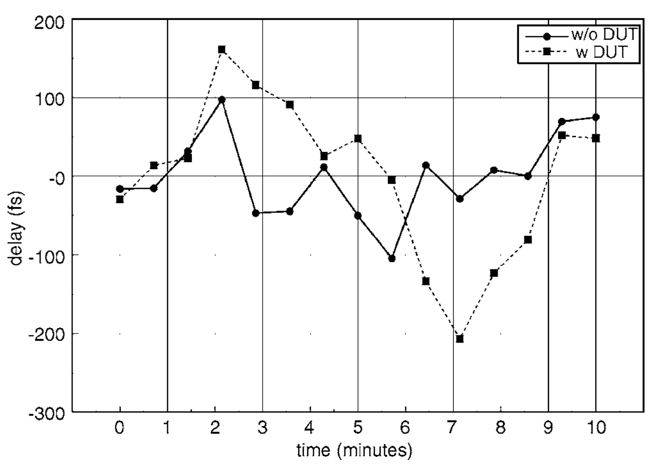

FIG. 4. Reference measurements.

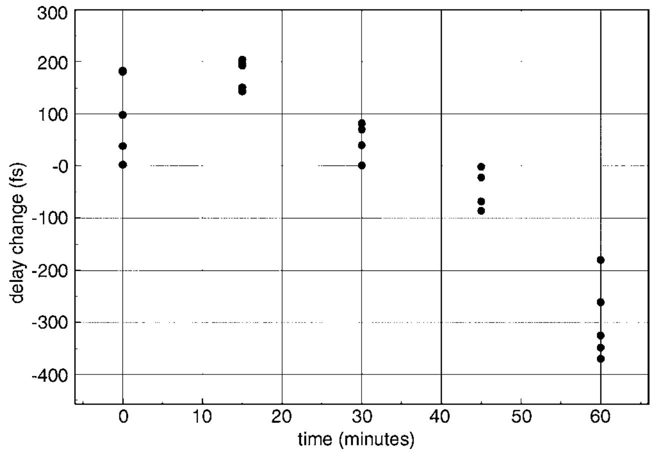

FIG. 5. Change in delay due to drift.

proximately 63 min. The change in delay due to drift is plotted in Fig. 5. The drift is less than ±0.3 ps over the time period observed and the standard deviation of all measurements is 175 fs. The causes of the drift may include thermal expansion, translation stage dither, or translation stage drift. In an attempt to determine the cause of the drift, measurements of delay drift were made with the power to the linear translation stage removed. The delay system was again positioned to approximately midrange in its delay. Power was removed from the translation stage and five delay measurements were obtained at 15 min intervals. The stability of the delay system under this special condition is depicted in Fig. 6. There appeared to be no indication of drift and the standard deviation of all measurement results was 80 fs. The stage was again powered and another five delay measurements were obtained at 15 min intervals. The resulting changes in delay were not plotted due to their similarity with the data in Fig. 5. It appears that the servomotor drive of the linear translation stage contributes to the variability of the measurement results.

TEMPERATURE DEPENDENCE

Changes in the temperature of a coaxial cable may result in changes in delay due to the finite linear coefficient of thermal expansion of copper and brass from which the trombone line was constructed. To determine the temperature de-

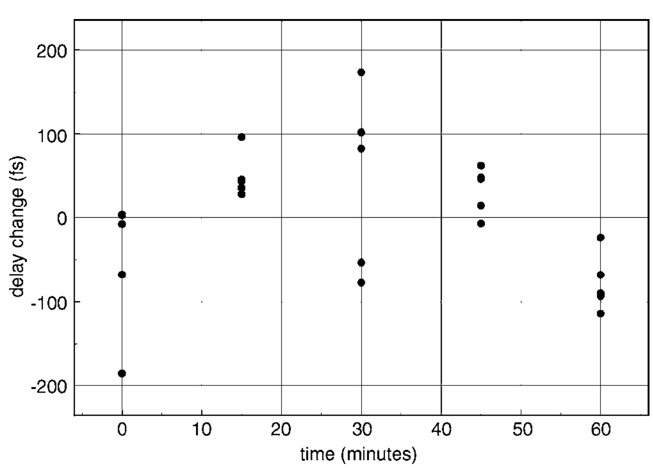

FIG. 6. Change in delay due to drift with translation stage not powered.

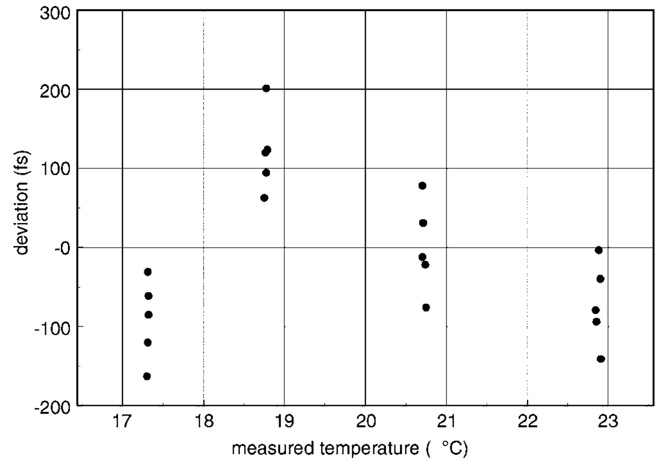

FIG. 7. Temperature effect on delay.

pendence of delay for this trombone line, it was placed in an environmental chamber and the temperature of the trombone line mount was monitored. The temperature ranged from 17.3 to 22.9 °C in approximately 2 °C increments. The temperature of the trombone line was allowed to come to equilibrium by waiting 60 min after each increment of the chamber temperature before acquiring data. The results are depicted in Fig. 7. There is very little correlation between temperature and delay and the results appear very similar to the drift data in Fig. 5. We conclude that the temperature dependence of delay is less than about 0.2 ps, the resolution of this measurement system.

REPEATABILITY

The repeatability of the delay obtained for given stage positions was measured under several conditions. The following procedure was used to check the repeatability for a unidirectional motion. The stage position was set to zero and then changed to 24.8 mm, which resulted in a delay of about 657 ps. While the stage was at this position, five measurements of delay were made. The stage position was again set to zero. This procedure was repeated five times. The maximum standard deviation of the five sets of measurement results was ±0.512 ps.

In another test of repeatability, the stage position was set to midrange and five delay measurements were made. The stage position was then changed by a positive 100 μm (equivalent to a 2.65 ps delay) from midrange and the delay measured five times at this stage position. Next, the stage was returned to midrange and the corresponding delay measured five times. This procedure was repeated five times and the results are depicted in Fig. 8. The measurement results indicate that the delay system provides a very repeatable delay. The total variation is less than ±0.25 ps. However, the actual delay only changed by about 0.5 ps for either positive or negative stage displacement even though the 100 μm displacement should have resulted in a 2.65 ps delay. This is thought to be the result of mounting hardware and trombone line flexure. The linear translation stage uses a position sensor with a position uncertainty of ±0.5 μm per the manufacturer's specifications. The positioning uncertainty would cor-

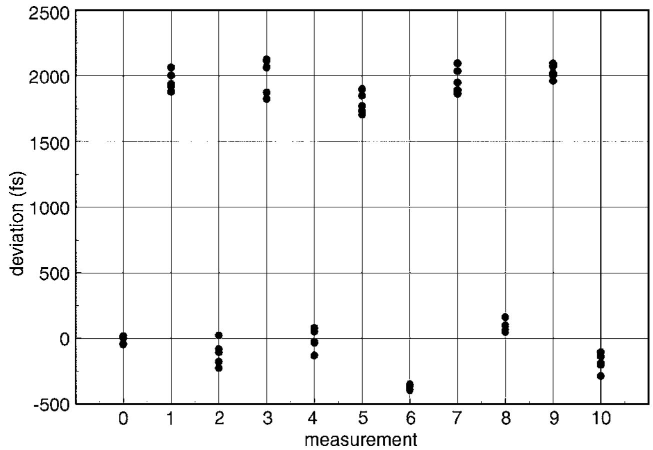

FIG. 8. Repeatability, deviation from commanded delay. The deviation from the reference delay is depicted by measurements 0, 2, 4, 6, 8, and 10. The deviation from a commanded delay of 2.5 ps is depicted by measurements 1, 3, 5, 7, and 9.

respond to an uncertainty in the delay change of approximately ±12.5 fs. This result indicates that the bidirectional resolution is 2.0 ps.

LINEARITY

A measurement of linearity was performed by repeatedly increasing the stage position by 80 μm, which corresponds to a 2.12 ps delay increment. Five successively increasing positions were used. At each of the five position settings, five measurements were made. The mean and standard deviation were calculated for each set of measurements; the maximum standard deviation for any of these five groups was ±55 fs. To determine the linearity of the delay system, a straight line was fit to the data (see Fig. 9). The residuals varied from −156 to 81 fs. Figure 9 displays all the data acquired; phase has been converted to time in picoseconds.

Another check of the linearity was performed using a larger range of motion. The translation stage was used to adjust the trombone line in 936 μm (approximately 24.8 ps delay) increments. The residuals varied from −0.27 to 0.286 ps. Figure 10 is a plot of these data; it is evident from this figure that the translation stage and delay

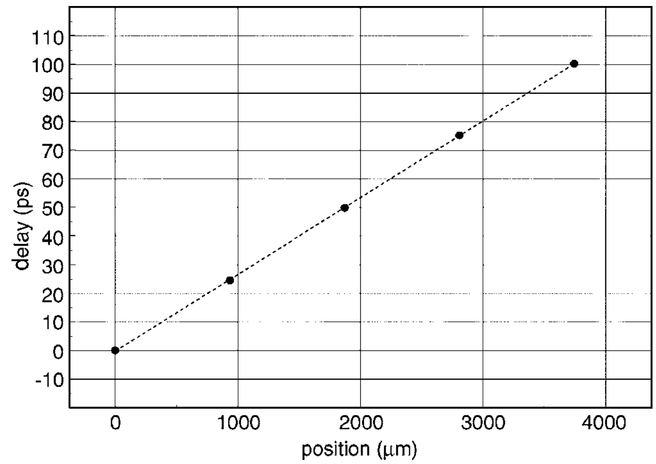

FIG. 10. Linearity demonstrated using unidirectional motion, 936 μm (approximately 25 ps) increments.

are highly linear and the positioning is repeatable. It also appears that any positioning errors are not cumulative.

RESOLUTION

The unidirectional resolution was checked by moving the translation stage in fixed increments. As Fig. 11 illustrates, 8 μm or nominally 0.2 ps increments were clearly resolved. The unidirectional resolution was also checked by moving the translation stage in fixed 4 μm increments corresponding to approximately 100 fs increments. Figure 12 illustrates that this small change was not resolved.

HYSTERESIS

The ability of the variable delay system to return to a reference delay after increasing or decreasing the delay was determined. The delay system was moved to midrange from a more negative delay setting and the delay was measured five times before making any further changes to the delay setting. The delay was then changed by a positive 10.0 mm (approximately 265 ps) from midrange, returned to midrange, and the delay measured five times without further intentional change in delay. The delay was changed to a negative 10.0 mm (approximately −265 ps) from midrange,

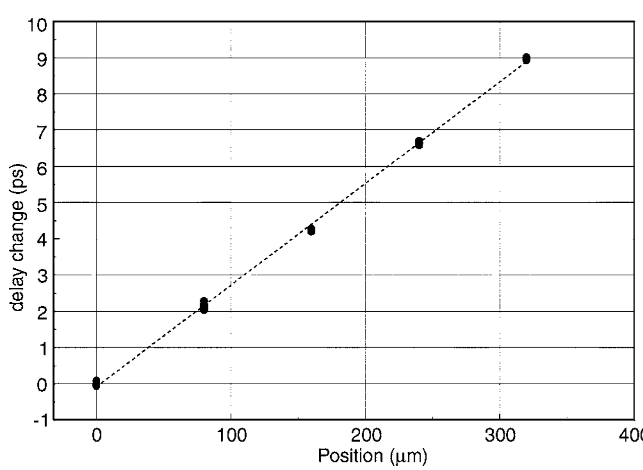

FIG. 9. Linearity demonstrated using a unidirectional motion, 80 μm (nominally 2.12 ps) increments.

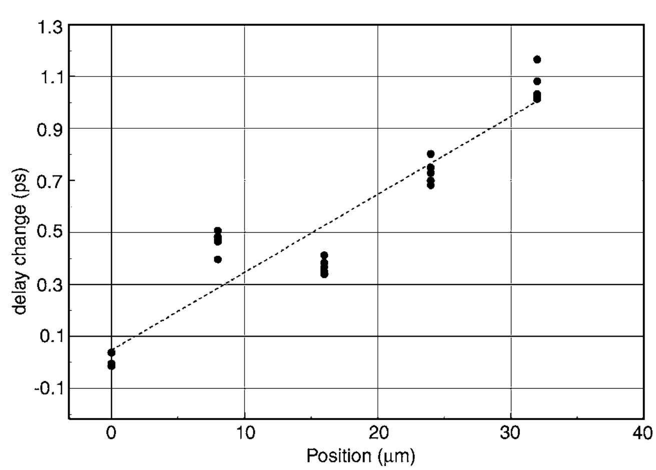

FIG. 11. Check of unidirectional resolution, repeated motion of 8 μm, nominally 0.2 ps increments.

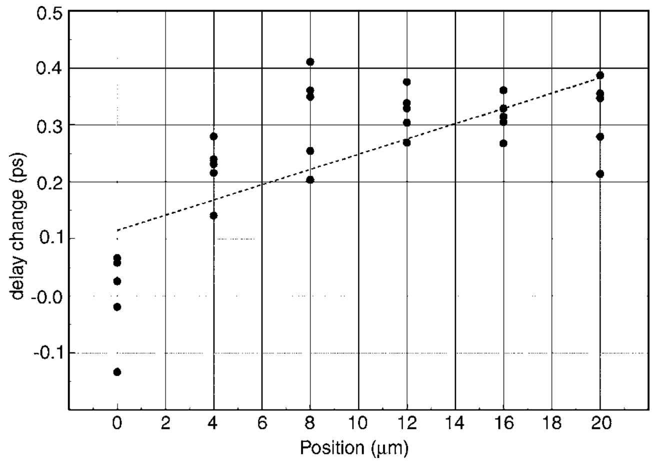

FIG. 12. Check of unidirectional resolution, repeated motion of 4 μm, nominally 100 fs increments.

returned to midrange, and the delay measured five times without further intentional change in delay. This procedure was repeated three times. From these results depicted in Fig. 13, we determined that the delay system exhibits about 3 ps of hysteresis but with a repeatability better than 0.5 ps.

UNCERTAINTY ANALYSIS

The delay D is given by

$$D = \frac{(1/M)\sum_{i=1}^{M}\theta_{2,i} - (1/N)\sum_{i=1}^{N}\theta_{1,i}}{2\pi f} + \Delta D_T = \frac{\bar{\theta}_2 - \bar{\theta}_1}{2\pi f} + \Delta D_T, \tag{1}$$

where θ_i is the measured phase for the reference pulse ($i=1$) and the delayed pulse ($i=2$), f is the frequency of the sinewave derived from the comb generator output, and ΔD_T is the temperature dependent delay. The integers M and N indicate the number of measurements of acquired sinewaves taken with two different stage position settings, one position corresponding to the reference pulse and the other to the delay (or test) pulse. The delay is calculated as the difference between the average of the computed phases of the acquired sinewaves for the reference delay and the average of the computed phases of the acquired sinewaves for the test delay. For each acquired waveform, the signal and trigger connections are broken and remade so the effect of connection repeatability is automatically included in the observed measurement variation.

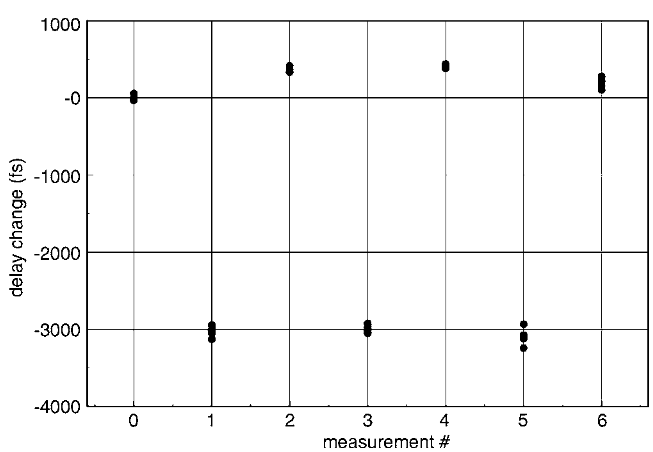

FIG. 13. Hysteresis; measurement numbers 0, 2, 4, and 6 are deviation from reference delay after negative delay change. Measurement numbers 1, 3, and 5 are deviation from reference delay after a positive delay change.

The uncertainty in the delay measurement was analyzed and included contributions from statistical variations of the phase measurements (standard deviation), frequency, jitter, and thermal effects. The total combined uncertainty is given by

$$u_D = \left[\left(\frac{1}{2\pi f}\right)^2 u_{\bar{\theta}_2}^2 + \left(-\frac{1}{2\pi f}\right)^2 u_{\bar{\theta}_1}^2 + \left(-\frac{\bar{\theta}_2 - \bar{\theta}_1}{2\pi f^2}\right)^2 u_f^2 + u_{\Delta D_T}^2\right]^{1/2}, \tag{2}$$

where the u_x is the uncertainty contribution from the reference and delayed phases, the frequency, and the change in temperature.[1] The terms in parentheses are the sensitivity coefficients. The expanded uncertainty, which is reported here, is given by

$$U_E = ku_c = t_p(\nu_{\mathrm{eff}})u_c, \tag{3}$$

where the degrees of freedom ν_{eff} are calculated using the Welch-Satterthwaite formula and the coverage factor k is determined so that a 95% confidence interval is achieved, as recommended in Ref. 7. For a delay of 657 ps, the expanded uncertainty (95% confidence interval) was about ±90 fs.

DISCUSSION

The combination of a variable-length coaxial delay line (trombone line) and a high precision linear translation stage creates a high-resolution delay system with linearity better than 0.286 ps and repeatability better than ±0.512 ps. The unidirectional resolution was determined to be 0.2 ps. The uncertainty in the measured and processed results is dependent on a variety of factors. The most significant component is the statistical variation in measured phases, which can be reduced by using higher frequencies and capturing more cycles. The sampling interval of the oscilloscope used to acquire the sinewaves is also a significant contributor because of the limited number of samples (typically a maximum of 4096) for a given waveform epoch. The uncertainty can be reduced by using shorter epochs with the maximum number of samples, especially at the higher frequencies. The measurement method examined here resulted in uncertainties (95% confidence interval) of about ±90 fs for a delay of 657 ps.

[1] D. R. Larson and N. G. Paulter, Jr., Metrologia **44**, 64 (2007).
[2] W.-C. Liu and M.-H. Lu, Opt. Laser Technol. **36**, 81 (2004).
[3] B. Petrovic, M. Stojcev, and D. Krstic, Meas. Sci. Technol. **6**, 1028 (1995).
[4] P. Zhang, IEEE Trans. Instrum. Meas. **40**, 13 (1991).
[5] IEEE Standard on Transitions, Pulses, and Related Waveforms, IEEE Std 181–2003, New York, NY.
[6] IEEE Standard on Waveform Recorders, IEEE Std-1057-1994, New York, NY.
[7] B. N. Taylor and C. E. Kuyatt, NIST Technical Note 1297, 1994 (NIST, Gaithersburg, MD, September 1994).