Complex Permittivity Measurements at Millimeter Wave Frequencies

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***Abstract*—Contemporary circuit designers require large bandwidths from microsystems to accomodate the ever-increasing demand for denser content. The electrical performance of these microsystems is intrinsically related to the electromagnetic properties of the materials. Methods using the Fabry-Perot resonator (FPR) have found widespread use in systems developed for measuring material properties at millimeter wave frequencies. This paper will provide a brief overview of the theory and methodology of Fabry-Perot resonators and present measurements of selected materials made with the FPR system currently in use at the National Institute of Standards and Technology (NIST).**

I. Introduction

Designing circuits at microwave and millimeter wave frequencies requires a knowledge of the complex permittivity of the dielectric substrates used in the fabrication of those circuits. At microwave frequencies, the most accurate methods for measuring complex permittivity utilize resonators constructed with closed cavities. However, for operating frequencies greater that 40 GHz, this type of resonator becomes impractical due primarily to the conductor losses in the cavity walls, and the difficulties posed by the cavity's small size.

Early research on Gaussian beam theory [1] provided a theoretical foundation for the development of open Fabry-Perot resonators with spherical mirrors to be used in the measurement of complex permittivity at millimeter wave frequencies. This effort was initiated by Cullen [2]–[4] and further developed by researchers at the National Physical Laboratory (NPL) [5]–[8], and has led to the basic theory currently in use worldwide. A good general review article covering Fabry-Perot resonators is given in [9].

Throughout this paper, several basic assumptions will be made about the nature of the electromagnetic fields discussed. All field excitations are considered to be sinusoidal, and expressions of the field components are phasor quantities. As usual, the complex time dependence $e^{\jmath\omega t}$ will be suppressed. The complex permittivity is defined as

$$\epsilon = \epsilon_0 \left(\epsilon_r{}' - \jmath \epsilon_r{}'' \right), \tag{1}$$

where ϵ_0 is the permittivity of free space and ϵ_r' and ϵ_r'' are the real and imaginary parts of the relative permittivity. The loss tangent is given by

$$\tan\delta = \frac{\epsilon_r{}''}{\epsilon_r{}'}. \tag{2}$$

This paper will briefly summarize the salient points of measuring ϵ_r' and $tan\delta$ using a Fabry-Perot resonator with the goal of providing a basic understanding of the theory and measurement technique involved.

II. A Brief Overview of Gaussian Beam Theory

Although Fabry-Perot resonators can be constructed with plane reflectors, spherical mirrors have been found to greatly reduce losses due to diffraction [10]. However, the fields of the standing waves in the volume between the spherical reflectors are no longer planar, but instead possess a spatial distribution that is approximately Gaussian. The mathematical description of these fields can be obtained through the theory of Gaussian beams. The evolution of Gaussian beam theory begins with the Helmholtz wave equation

$$\nabla^2 u + k^2 u = 0, \tag{3}$$

where u is any scalar component of the field, $k = 2\pi/\lambda$ is the wave number and λ is the wavelength. In Cartesian coordinates, a quasi-plane wave solution is assumed in the form

$$u(x, y, z) = \psi(x, y, z) e^{-\jmath k z}, \tag{4}$$

which is substituted into equation (3) to arrive at the reduced wave equation

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} - \jmath 2k \frac{\partial \psi}{\partial z} = 0. \tag{5}$$

At this point, the so-called paraxial approximations are applied to equation (5) that result in the elimination of the $\partial^2\psi/\partial z^2$ term, reducing equation (5) to the paraxial wave equation

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} - \jmath 2k \frac{\partial \psi}{\partial z} = 0. \tag{6}$$

Solutions to this equation then provide the results used to predict the resonant mode frequencies and their Q factors [1], [2].

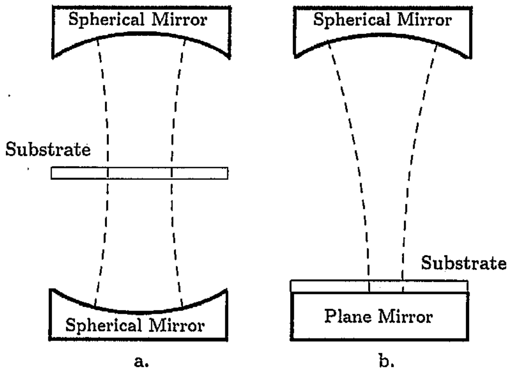

Fig. 1. a. Confocal Configuration b. Hemispherical Configuration

Although the results of Gaussian beam theory are based on an approximate scalar theory as outlined above, Cullen *et al.* [2], [3] showed that for permittivity measurements of moderately lossy materials (*i.e.* $\tan\delta \approx 10^{-4}$ to 10^{-3}), the theory works remarkably well.

III. Measurement System Design

Since the Fabry-Perot resonator is an open cavity, the spatial distribution of the electromagnetic fields between the end mirrors depends entirely on the design and construction of those mirrors because there are no metallic side walls to guide the fields. There are two basic designs currently in use, and each have advantages dependent on the type of material under investigation.

The confocal design (Fig. 1a) employs two spherical mirrors placed, at most, at their radius of curvature from the opposing mirror. One of the major challenges of this type of construction is the parallel alignment of both mirrors and the sample under test in order to minimize any problems due to the effects of diffraction or scattering. Excitation and sampling of the fields are achieved in each mirror through a single aperture placed in the center of the mirror [2].

By replacing one of the spherical mirrors with a plane mirror, the hemispherical design (Fig. 1b) offers a practical solution to the placement of the sample under test, but requires careful construction and design of the coupling holes since they are usually placed next to each other on the spherical mirror [8], [11]. The main disadvantage with this approach is the increased risk of cross-coupling of electromagnetic energy between the apertures. Because the wavelengths of the fields are relatively small, machining tolerances of all aspects of the mirror and aperature construction must be tight. For the same reason, alignment supports should receive special attention to ensure successful operation of the resonator system. Currently, this is the design in use at NIST.

An important component of the mirror design and construction is the coupling aperture used to excite and sample the fields. The most common approach has been to couple electromagnetic energy through circular waveguide sections that terminate as circular irises on the mirror's surface. Komiyama *et al.* presented data relating the cavity's insertion loss to the length and diameter of the circular waveguide section [12] .

Sample preparation is also important, but because the fields have constant phase fronts with a spatial Gaussian dependence, the optimum shape for the sample is not planar. Researchers have investigated this possible source of error and developed correction terms that may be used for samples with flat, parallel surfaces [2], [6].

IV. Empty Cavity Measurements

The solution of the paraxial equation (6) provides the necessary equations to accurately predict the frequency of the quasi-TEM_{mnq} modes excited between the empty cavity mirrors. In the following discussion, the equations presented will be for a hemispherical design, but similar results are also available for confocal systems [2].

For a hemispherical system, the equation for the frequencies of the resonant modes is given by

$$f_{mnq} = \frac{c}{2D}\left[q+1+\frac{2m+n+1}{\pi}\tan^{-1}\left(\sqrt{\frac{D}{R-D}}\right)\right], \quad (7)$$

where mnq are the mode numbers of the quasi-TEM_{mnq} resonance, c is the speed of light in air, D is the center-to-center distance between the mirrors, and R is the radius of curvature of the spherical mirror [11].

For the fundamental TEM_{00q} modes, the electric field in spherical coordinates is approximately given by

$$E(\rho) = E_0 e^{-\left(\frac{\rho}{w}\right)^2}, \quad (8)$$

where ρ is the radial coordinate and w is the Gaussian beam radius. This result implies that for the TEM_{00q} modes, the fields have a purely Gaussian spatial distribution. The practical implication is that the sample under test does not have to be very large in order to obtain a good measurement [8]. It is important, therefore, to be able to correctly identify all of the frequencies f_{00q} in the mode spectrum needed for a measurement. Using equation (7), it is a straightforward exercise to calculate a mode table for this purpose.

V. Cavity with Sample

When the sample is inserted into the Fabry-Perot resonator, the empty mode frequencies f_{00q} are shifted lower and their Q factors are decreased. These measured changes are then used to calculate the complex permittivity of the sample through the equations derived from the Gaussian beam theory.

At the air-sample boundary, continuity of the field ratios E_x/H_y derived from the beam theory allow for matching conditions that are used to derive expressions for ϵ. The basic equation for ϵ_r' is given by

$$\frac{1}{\sqrt{\epsilon_r'}}\tan\left(kt\sqrt{\epsilon_r'}-\phi_t\right) = -\tan\left(kd-\phi_d\right), \quad (9)$$

with wave number k, sample thickness t, distance $d = D - t$, and phase shifts ϕ_t and ϕ_d. The expression for loss tangent is

$$\tan\delta = \frac{2k\sqrt{\epsilon_r'}(d+t\Delta)}{Q_e\left[2kt\Delta\sqrt{\epsilon_r'}-\Delta\sin(kt\sqrt{\epsilon_r'}-\phi_t)\right]}. \quad (10)$$

Details for the terms Δ, Q_e, ϕ_t, and ϕ_d are given in [11].

VI. Accuracy of FPR Measurements

As the FPR method began to be considered for metrology-level measurements of ϵ, researchers looked for ways to evaluate their results. Through measurements of well-characterized reference materials, confidence was gained in the determination of ϵ_r', but reliable measurements of $\tan\delta$ presented a challenge. Early on, Cullen [3] and Jones [7] addressed the problem of planar sample surfaces and their effect on loss. Cook *et al.* [5] used polytetrafluoroethylene (PTFE) as a reference material for comparison measurements using a mode-filtered cavity and a hemispherical FPR at 35 GHz, with good results. However, other attempts to measure materials with very high or very low loss tangents proved to be problematic. Clarke and Lynch then conducted research on this issue and ended up proposing different FPR configurations for the measurement of $\tan\delta$, depending upon the anticipated loss of the sample under test [13], [14].

VII. Measurement Data with a Hemispherical FPR

As noted previously, NIST currently uses a Fabry-Perot resonator in a hemispherical configuration that operates at approximately 60 GHz [15]. Measurement data for some common materials are shown in the table below. The estimated Type B uncertainties are based on characterizations of the FPR measurement system with standard reference materials and various analysis published previously in many of the references listed in this survey.

Reference Sample Measurements			
Material	f, GHz	ϵ_r'	$tan\delta \times 10^4$
Polystyrene	60	2.54 ± 0.05	4.8 ± 0.5
Quartz	60	4.45 ± 0.05	0.7 ± 0.5
Fused Silica	60	3.82 ± 0.05	5.8 ± 0.6

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