An Assessment on the Accuracy of Time-Domain Reflectometry for Measuring the Characteristic Impedance of Transmission Lines

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Abstract—**An assessment of time-domain reflectometry (TDR) for measuring the characteristic impedance Z_{TL} of transmission lines is performed. The assessment includes the accuracy of measuring Z_{TL} as a function of TDR electrical impedance Z_0, the ability to measure impedance perturbations as a function of Z_{TL} and Z_0, and the ability to differentiate between transmission lines of similar Z_{TL}. The information presented herein will be especially useful for those using 50 Ω TDR systems to characterize transmission lines having characteristic impedances less than about 30 Ω.**

Index Terms—**Characteristic impedance, high speed/high frequency, impedance discontinuity, time-domain reflectometry, transmission line.**

I. Introduction

TIME-DOMAIN reflectometry (TDR) is a measurement tool that is used for a variety of applications in electrical characterization of electronic and electrical circuits, such as determining the location of opens and shorts in circuits, determining the characteristic impedance of transmission lines, performing impedance profiles of populated and unpopulated circuits, measuring the electrical impedance of circuit elements, measuring pulse propagation delay times, and pulse propagation velocities, etc. The application considered here is the determination of the characteristic impedance of transmission lines. Describing a set of paired conducting lines as transmission lines is applicable in high-frequency and high-speed circuits where the shortest wavelength of the signal propagating in the conducting lines is approximately equal to or smaller than the length of the conductors.

High-speed electrical circuits typically are designed to operate in an electrical impedance environment of 50 Ω, and the electrical impedance of TDR systems has been designed to match those circuits. Consequently, most, if not all, commercially-available TDR systems have an electrical impedance of 50 Ω. Recently, however, high-speed and high-performance circuits are being designed for specific applications that do not have a 50 Ω environment. These circuits must still be evaluated using a TDR system, and there is concern regarding the ability of present 50 Ω TDR systems to perform adequately. Furthermore, bus bars used for high-power switching circuits may contain noise components in the tens of megahertz range [1]–[3] and radiation from impedance discontinuities is a concern. The purpose of this paper is to determine the accuracy limitations of 50 Ω TDR systems for characterizing non-50 Ω electrical systems. A brief description of TDR pertinent for this paper is provided in Section II. In Section III, TDR systems are assessed for accurate characterization of non-50 Ω transmission lines.

Manuscript received September 24, 1999; revised May 25, 2001. This work was supported by the Advanced Technology Program of the National Institute of Standards and Technology.

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Publisher Item Identifier S 0018-9456(01)08734-4.

II. Background

A TDR system resembles a sampling oscilloscope except that the TDR head contains both a sampling device (the sampler) and a pulse generator. A TDR setup for measuring the characteristic impedance of a transmission line is shown in Fig. 1. The generated pulse or incident pulse is a rectangular pulse with a fast transition between its low-voltage state or baseline (nominally 0 V) and its high-voltage state or topline (nominally 0.25 V). The transition duration (rise time) is typically less than 30 ps. The pulse duration, on the other hand, is very long, typically much longer than the period over which the TDR waveform is observed. Because the pulse duration is much longer than the waveform epoch, the TDR pulse effectively appears like a step pulse (topline continues forever). Reflections are caused as the propagating pulse encounters impedance discontinuities along the transmission line (TL). Consider the simple TDR waveform shown in Fig. 2. This TDR waveform corresponds to a continuous uniform lossless transmission line (of length L and characteristic impedance Z_{TL}) that is connected to the TDR head at one end and unterminated (open circuit) at the other end. This TDR waveform is the result of the reflections occurring at the TDR/TL interface and at the TL/open-circuit interface; the reflected pulses add to the incident pulse. The levels labeled L_0 and L_1 in Fig. 2 are the baseline and topline values of the incident pulse. The amplitude of the pulse that is reflected from the impedance discontinuity is dependent on the impedances on either side of the discontinuity. A good way of envisioning how and which pulses will add to create the TDR waveform is through the reflection-transmission (RT) diagram shown in Fig. 3. The level L_2 in Fig. 2 is the result of the addition of the incident pulse and the first reflected pulse, as indicated in the figure and diagrammatically by the top two leftward-directed arrows in the RT diagram. The level L_3 is the result of the addition of the incident pulse, the first reflected pulse, and the second reflected pulse. The value of L_2 will be greater than L_1 if Z_{TL} is greater than the electrical impedance Z_0,of the TDR head and

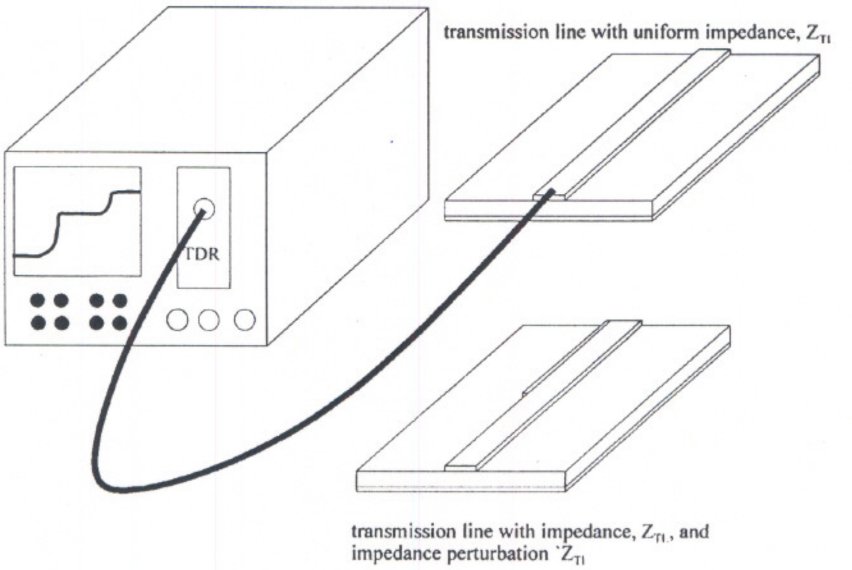

Fig. 1. TDR setup showing two different transmission line structures for testing. One line is uniform and continuous and is described with one Z_{TL} value. The other line contains a perturbation and must be described by two Z_{TL} values.

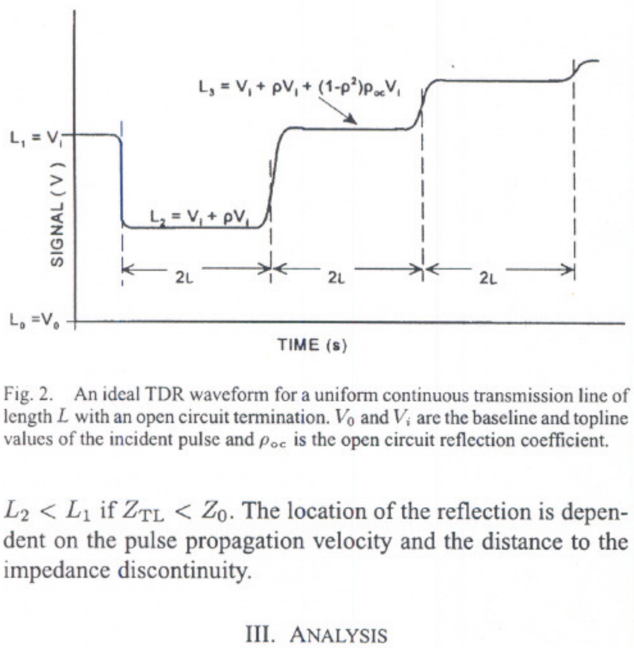

Fig. 2. An ideal TDR waveform for a uniform continuous transmission line of length L with an open circuit termination. V_0 and V_i are the baseline and topline values of the incident pulse and ρ_{oc} is the open circuit reflection coefficient.

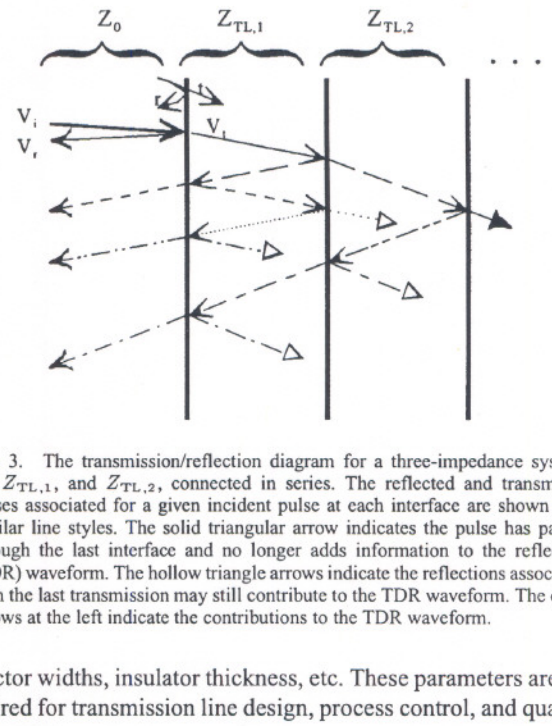

Fig. 3. The transmission/reflection diagram for a three-impedance system, Z_0, $Z_{\mathrm{TL},1}$, and $Z_{\mathrm{TL},2}$, connected in series. The reflected and transmitted pulses associated for a given incident pulse at each interface are shown with similar line styles. The solid triangular arrow indicates the pulse has passed through the last interface and no longer adds information to the reflection (TDR) waveform. The hollow triangle arrows indicate the reflections associated with the last transmission may still contribute to the TDR waveform. The curly arrows at the left indicate the contributions to the TDR waveform.

$L_2 < L_1$ if $Z_{\mathrm{TL}} < Z_0$. The location of the reflection is dependent on the pulse propagation velocity and the distance to the impedance discontinuity.

III. Analysis

There are basically two problems to consider for assessing the utility of an unmatched-impedance TDR system for measuring the Z_{TL} of a transmission line. The first problem is the accuracy with which the characteristic impedance of a uniform continuous transmission line can be determined when using a TDR system where Z_0 is much different than Z_{TL}. The second problem is the ability to measure perturbations in Z_{TL} when using a TDR system with $Z_0 \neq Z_{\mathrm{TL}}$. The first problem is very important because TDR is frequently used to determine the characteristic impedance of uniform continuous transmission lines. Furthermore, the characteristic impedance thus determined is used to verify or extract certain transmission line parameters, such as the permittivity of the insulating layer, conductor widths, insulator thickness, etc. These parameters are required for transmission line design, process control, and quality assurance.

In Section 3.1, the uncertainty in the measurement of Z_{TL} is used to assess TDR for measuring Z_{TL}; in Section 3.2, the uncertainty in measuring changes in Z_{TL} is used to assess TDR for performing impedance profiles of the transmission line. In Section 3.3 and 3.4, the signal-to-noise ratio (SNR) and amplitude discretization are used to assess the ability of TDR to differentiate between transmission lines of approximately the same Z_{TL} and to measure perturbations in Z_{TL}. The percent impedance error shown in the Figs. 4 through 8 represents one standard deviation.

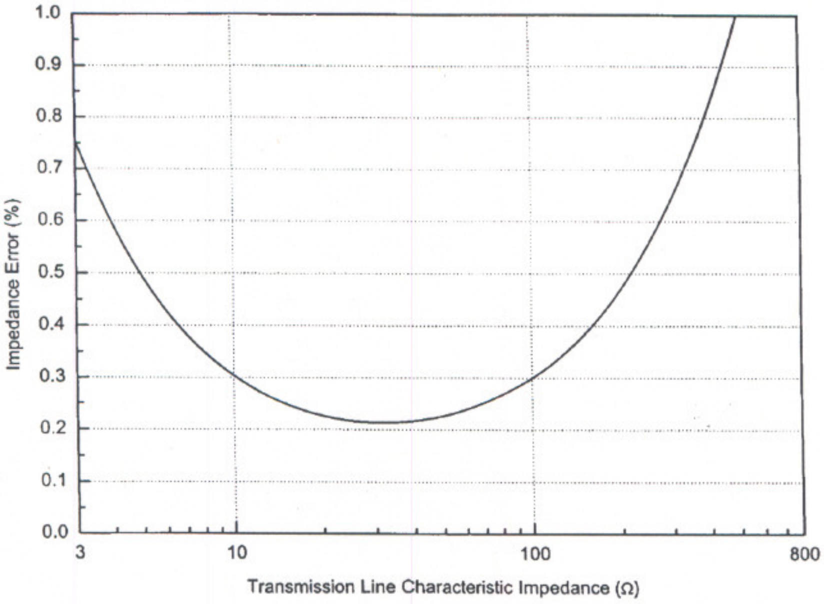

Fig. 4. The percent error in $Z_{\rm TL}$ referenced to a 50 Ω system versus $Z_{\rm TL}$. The impedance errors are obtained from the square root of the sum of the squares of (10) and (11) for $V_i = 0.25$ V and $u_v = 2 \times 10^{-4}$ V and multiplied by 100.

A. Measuring $Z_{\rm TL}$ Uncertainty Limitations

The first problem is addressed by calculating the uncertainty in the characteristic impedance $Z_{\rm TL}$ of the transmission line (TL). The $Z_{\rm TL}$ can be written in terms of the voltage reflection coefficient, ρ [4]

$$Z_{\rm TL} = \frac{1+\rho}{1-\rho} Z_0 \tag{1}$$

where Z_0 is the electrical impedance of the TDR head and is typically about 50 Ω. The reflection coefficient ρ can be written in terms of the voltage incident on the TDR/TL interface V_i and the voltage reflected from that interface, V_r [4]

$$\rho = \frac{V_r}{V_i}. \tag{2}$$

The measured reflected voltage V_2 is a sum of V_r and V_i, so that $V_r = V_2 - V_i$ Using this for V_r in (2) and then (2) in (1) gives

$$Z_{\rm TL} = \frac{V_2}{2V_i - V_2} Z_0. \tag{3}$$

The V_i, V_2, and Z_0 are the measurable quantities and ρ is derived from V_i and V_2. The uncertainty in $Z_{\rm TL}$ due to uncertainties in the measurement of V_i, V_2, and Z_0 can be calculated using a standard propagation of uncertainties method [5], and these uncertainties then used to determine the measurement limitations for estimating $Z_{\rm TL}$ when Z_0 differs significantly from $Z_{\rm TL}$. The uncertainties in $Z_{\rm TL}$, $u_{Z_{\rm TL}}$, due to uncertainties in the measurement of V_i, V_2, and Z_0 are given by the following equations

$$u_{Z_{\rm TL},V_i} = \left|\frac{\partial Z_{\rm TL}}{\partial V_i}\right| u_{V_i} = \frac{2|V_2|Z_0}{(2V_i - V_2)^2} u_{V_i}, \tag{4}$$

$$u_{Z_{\rm TL},V_2} = \left|\frac{\partial Z_{\rm TL}}{\partial V_2}\right| u_{V_2} = \frac{2|V_i|Z_0}{(2V_i - V_2)^2} u_{V_2} \tag{5}$$

and

$$u_{Z_{\rm TL},Z_0} = \left|\frac{\partial Z_{\rm TL}}{\partial Z_0}\right| u_{Z_0} = \left|\frac{V_2}{2V_i - V_2}\right| u_{Z_0}. \tag{6}$$

To simplify (4) and (5), we can set $u_{V_i} = u_{V_2} = u_V$, where u_V is the uncertainty in measuring the incident or reflected voltages and should be the same for both. The absolute uncertainty in the characteristic impedance of a transmission line is not as critical for circuit design as is the relative or percent uncertainty. For example, an absolute uncertainty of ±10 Ω in a 300 Ω system corresponds to a relative uncertainty of ±3%, whereas the same absolute uncertainty in a 20 Ω system corresponds to a relative uncertainty of ±50%. The relative uncertainties are obtained by dividing (4), (5) and (6) by (3) to get

$$\frac{u_{Z_{\rm TL},V_i}}{Z_{\rm TL}} = \left|\frac{2}{(2V_i - V_2)}\right| u_V \tag{7}$$

$$\frac{u_{Z_{\rm TL},V_r}}{Z_{\rm TL}} = \left|\frac{2V_i}{V_2(2V_i - V_2)}\right| u_V \tag{8}$$

and

$$\frac{u_{Z_{\rm TL},Z_0}}{Z_{\rm TL}} = \frac{1}{Z_0} u_{Z_0}. \tag{9}$$

First consider (9), which describes the uncertainty in $Z_{\rm TL}$ caused by the uncertainty in Z_0. If u_{Z_0} is a fixed fraction of Z_0, that is $u_{Z_0} = aZ_0$, where $0 \le a \le 1$, then the relative uncertainty in $Z_{\rm TL}$ according to (9) is dependent on a and not on Z_0. This assumption is reasonable for most transmission line structures because geometries can be scaled both up and down to achieve an impedance change. For most TDR systems, $u_{Z_0} \approx 0.5$ Ω, or $a \approx 0.01$.

To examine the uncertainties in $Z_{\rm TL}$ caused by V_i and V_2, Eqs. (7) and (8) are first simplified using (2) with the substitution $V_r = V_2 - V_i$

$$\frac{u_{Z_{\mathrm{TL}},V_2}}{Z_{\mathrm{TL}}} = \frac{2}{V_i(1-\rho^2)} u_V \tag{10}$$

and

$$\frac{u_{Z_{\mathrm{TL}},V_i}}{Z_{\mathrm{TL}}} = \frac{2}{V_i(1-\rho)} u_V. \tag{11}$$

To examine the adequacy of commerically available TDR systems for the measurement of Z_{TL}, the square root of the sum of the squares of (10) and (11) is used. Commercially-available TDR systems typically have $Z_0 = 50\ \Omega$, V_i ranging between 0.2 V to 0.25 V, and $u_V \geq 2 \times 10^{-4}$ V. The u_V corresponds to about 14 effective bits of resolution for a fixed-gain amplitude input with $a \pm 2$ V range, or 12 effective bits for a variable-gain amplitude input with the gain set to observe the entire amplitude range of the TDR waveform. Substituting these values for V_i and u_V in (10) and (11), and plotting the percentage error in Z_{TL} as a function of Z_{TL} give the results shown in Fig. 4. From Fig. 4, it can be seen that the characteristic impedance of a uniform continuous transmission line has to be significantly different from 50 Ω to cause u_V to contribute appreciably to $u_{Z_{\mathrm{TL}}}$. For example, if the TDR measurement uncertainty is required to be less than 0.5% (excluding that caused by uncertainties in Z_0), then the TDR electrical impedance must be in the range between 10 Ω and 500 Ω, that is, $10\ \Omega \leq Z_0 \leq 500\ \Omega$. This impedance range is not at all restrictive except for the bus bars used in power transmission and delivery. The characteristic impedance of these power transmission lines may be 2 Ω or less.

B. Measuring Perturbations in Z_{TL} Uncertainty Limitations

This assessment is based on the uncertainties in the characteristic impedance of the transmission line due to uncertainties in the measured parameters and, in addition, in Z_{TL}. The measured parameters are Z_0, V_i, V_2, and V_3, where V_2 and V_3 are the signal amplitudes before and after the transmission line discontinuity. To simplify the discussion, V_Δ, where $V_\Delta = V_3 - V_2$, will be used. V_Δ is the amplitude of the reflected pulse caused by the impedance discontinuity in the transmission line. This examination will be simplified by assuming that the perturbation in Z_{TL} continues uniformly after the discontinuity. Although this assumption may not always be true, this assessment is still valid as long as the duration of V_3 is long enough to be measured. That is, the perturbation must be long enough in extent so that it exists for a period exceeding the temporal resolution of the TDR and is represented by a sufficient number of data. The sufficiency requirement is dependent on data analyzes which are not within the scope of this paper.

To start the assessment, let V_Δ be the incremental voltage change caused by the change in transmission line characteristic impedance from Z_{TL} to ${}^{\backprime}Z_{\mathrm{TL}}$

$$V_\Delta = V_3 - V_2 = \frac{{}^{\backprime}Z_{\mathrm{TL}} - Z_{\mathrm{TL}}}{{}^{\backprime}Z_{\mathrm{TL}} + Z_{\mathrm{TL}}} V_t \tag{12}$$

where V_t is the amplitude value of the pulse transmitted through the TDR/TL interface and is given by

$$V_t = (1+\rho)V_i. \tag{13}$$

The pulse transmitted through the TDR/TL interface becomes the incident voltage for the impedance discontinuity in the transmission line. Solving (12) for ${}^{\backprime}Z_{\mathrm{TL}}$ and substituting (13) for V_t gives

$${}^{\backprime}Z_{\mathrm{TL}} = \frac{(1+\rho)V_i + V_\Delta}{(1+\rho)V_i - V_\Delta} Z_{\mathrm{TL}} \tag{14}$$

and using (1) in (14) yields

$${}^{\backprime}Z_{\mathrm{TL}} = \frac{2Z_{\mathrm{TL}}V_i + Z_{\mathrm{TL}}V_\Delta + Z_0V_\Delta}{2Z_{\mathrm{TL}}V_i - Z_{\mathrm{TL}}V_\Delta - Z_0V_\Delta} Z_{\mathrm{TL}}. \tag{15}$$

Equation (15) describes the change in transmission line characteristic impedance as a function of the measurable parameters and Z_{TL}. The effect of each of these four variables on the ability to accurately measure ${}^{\backprime}Z_{\mathrm{TL}}$ will be ascertained using the uncertainties associated with these variables. This process is similar to that done in Section 3.1. The percentage change will also be used here, but this percentage will be with respect to the value of ${}^{\backprime}Z_{\mathrm{TL}}$. The relative uncertainty values are

$$\frac{u_{{}^{\backprime}Z_{\mathrm{TL}},Z_0}}{{}^{\backprime}Z_{\mathrm{TL}}} = 4\frac{|V_\Delta V_i| Z_{\mathrm{TL}}}{|V_\Delta^2(Z_0+Z_{\mathrm{TL}})^2 - 4Z_{\mathrm{TL}}^2V_i^2|} u_{Z_0} \tag{16}$$

$$\frac{u_{{}^{\backprime}Z_{\mathrm{TL}},V_i}}{{}^{\backprime}Z_{\mathrm{TL}}} = 4\frac{|V_\Delta| Z_{\mathrm{TL}}(Z_{\mathrm{TL}}+Z_0)}{|V_\Delta^2(Z_0+Z_{\mathrm{TL}})^2 - 4Z_{\mathrm{TL}}^2V_i^2|} u_V \tag{17}$$

$$\begin{aligned}\frac{u_{{}^{\backprime}Z_{\mathrm{TL}},V_M}}{{}^{\backprime}Z_{\mathrm{TL}}} &= \left|\frac{\partial {}^{\backprime}Z_{\mathrm{TL}}}{\partial \Delta V}\right| \left|\frac{\partial \Delta V}{\partial V_M}\right| u_V \\ &= 4\frac{|V_i| Z_{\mathrm{TL}}(Z_{\mathrm{TL}}+Z_0)}{|V_\Delta^2(Z_0+Z_{\mathrm{TL}})^2 - 4Z_{\mathrm{TL}}^2V_i^2|} u_V\end{aligned} \tag{18}$$

where V_M represents either V_2 or V_3 and

$$\frac{u_{{}^{\backprime}Z_{\mathrm{TL}},Z_{\mathrm{TL}}}}{{}^{\backprime}Z_{\mathrm{TL}}} = \frac{|V_\Delta^2(Z_{\mathrm{TL}}+Z_0)^2 + 4V_iZ_{\mathrm{TL}}(V_\Delta Z_0 - V_iZ_{\mathrm{TL}})|}{Z_{\mathrm{TL}}\,|V_\Delta^2(Z_0+Z_{\mathrm{TL}})^2 - 4Z_{\mathrm{TL}}^2V_i^2|} u_{Z_{\mathrm{TL}}}. \tag{19}$$

Also, to view the results as before, Eqs. (16) through (19) are further simplified by replacing Z_{TL} with (1), and V_Δ by $\rho_\Delta(1+\rho)V_i$, where ρ_Δ is the reflection coefficient for the perturbation in the transmission line and $(1+\rho)$ is the transmission coefficient of the pulse through the TDR/TL interface. Using these substitutions, (16) through (19) become

$$\frac{u_{{}^{\backprime}Z_{\mathrm{TL}},Z_0}}{{}^{\backprime}Z_{\mathrm{TL}}} = \frac{(1-\rho)|\rho_\Delta|}{Z_0(1-\rho_\Delta)^2} u_{Z_0} \tag{20}$$

$$\frac{u_{{}^{\backprime}Z_{\mathrm{TL}},V_i}}{{}^{\backprime}Z_{\mathrm{TL}}} = 2\frac{|\rho_\Delta|}{|V_i|(1-\rho_\Delta)^2} u_V \tag{21}$$

$$\frac{u_{{}^{\backprime}Z_{\mathrm{TL}},V_M}}{{}^{\backprime}Z_{\mathrm{TL}}} = \frac{2}{|V_i|(1+\rho_\Delta)(1+\rho-\rho_\Delta-\rho\rho_\Delta)} u_V \tag{22}$$

and

$$\frac{u_{{}^{\backprime}Z_{\mathrm{TL}},Z_{\mathrm{TL}}}}{{}^{\backprime}Z_{\mathrm{TL}}} = \frac{(1-\rho)\left(1+\rho\rho_\Delta-\rho_\Delta-\rho_\Delta^2\right)}{Z_0(1+\rho_\Delta)(1+\rho-\rho_\Delta-\rho\rho_\Delta)} u_{Z_{\mathrm{TL}}}. \tag{23}$$

Analysis shows that for reasonable cases, namely where ${}^{\backprime}Z_{\mathrm{TL}}$ is not significantly different from Z_{TL}, the effect due to uncertainties in Z_0 [see Eq. (20)] is not significant. Similarly, analysis

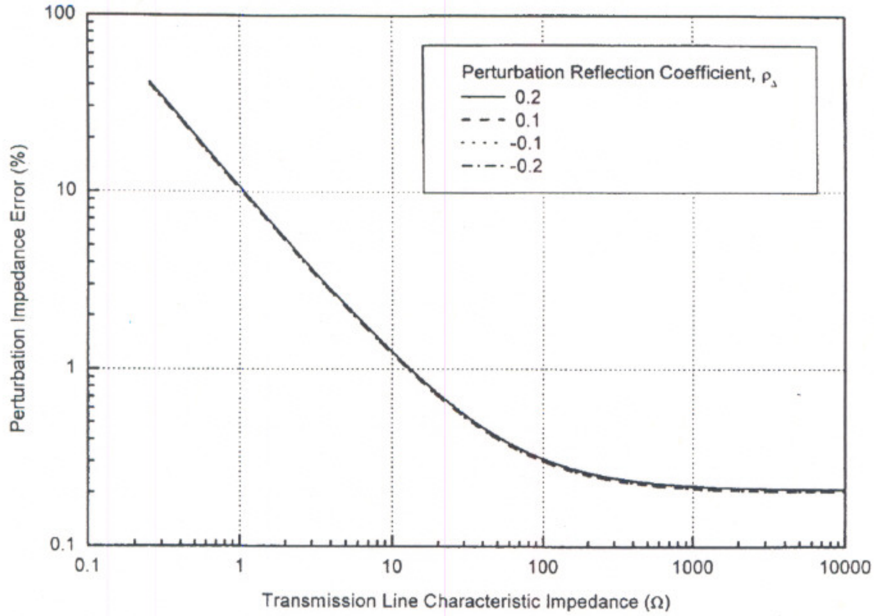

Fig. 5. The percent error in ${}^{\prime}Z_{\rm TL}$ due to uncertainties in V_Δ versus ρ_Δ for various values of ρ and referenced to a 50 Ω TDR system. The errors are obtained using (22) with $V_i = 0.25$ V and $u_v = 5 \times 10^{-4}$ V and multiplied by 100.

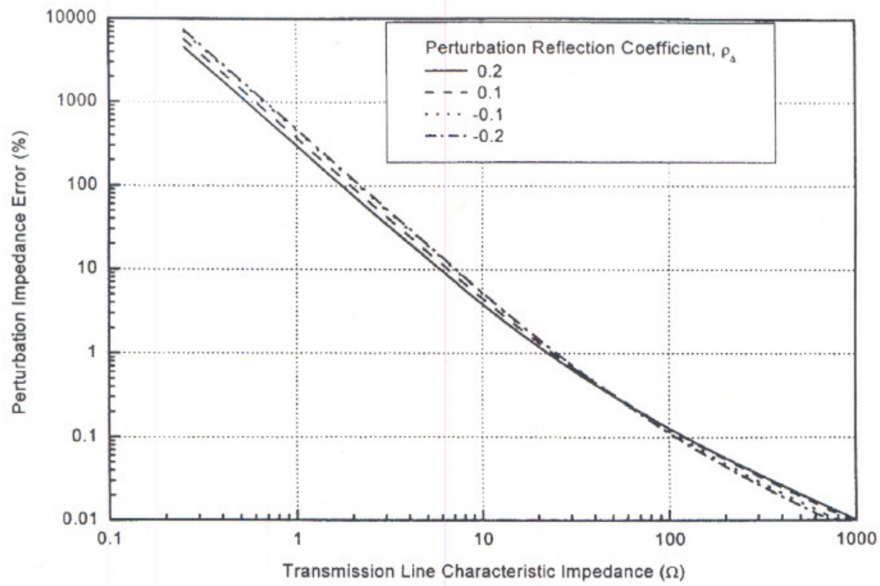

Fig. 6. The percent error in ${}^{\prime}Z_{\rm TL}$ due to uncertainties in $Z_{\rm TL}$ versus ρ_Δ for various values of ρ and referenced to a 50 Ω TDR system. The errors are obtained using (23) with $Z_0 = 50$ Ω and $u_{Z_{\rm TL}} = 0.17$ Ω.

shows that the effect of uncertainties in the measurement of V_i [see Eq. (21)] does not have a significant effect on the errors in the ${}^{\prime}Z_{\rm TL}$ values.

Fig. 5 shows the errors in ${}^{\prime}Z_{\rm TL}$ due to uncertainties in the measurement of V_2 or V_3 as a function of ρ [see Eq. (22)] referenced to a 50 Ω TDR system. These errors are relatively small for $Z_{\rm TL} > 50$ Ω. For example, if the total contribution from uncertainties and errors in ${}^{\prime}Z_{\rm TL}$ from V_Δ must be kept below about 0.5%, then using Fig. 5, the $Z_{\rm TL}$ of the transmission line must be greater than 30 Ω. This is not restrictive for most applications. Moreover, this error affects the ability to accurately measure perturbations in the transmission line, which may not be as critical as measuring the average characteristic impedance of the transmission line.

Fig. 6 shows the errors in ${}^{\prime}Z_{\rm TL}$ caused by uncertainties and errors in $Z_{\rm TL}$ [see Eq. (23)]. The errors in $Z_{\rm TL}$ have the largest contribution to errors in ${}^{\prime}Z_{\rm TL}$ compared to Z_0, V_i, and V_Δ. A three-standard deviation error of under 1% would require that $Z_{\rm TL}$ be greater than about 50 Ω. Consequently, 50 Ω is not the ideal TDR impedance for accurately measuring the characteristic impedance of perturbations in a transmission line with $Z_{\rm TL} < 50$ Ω.

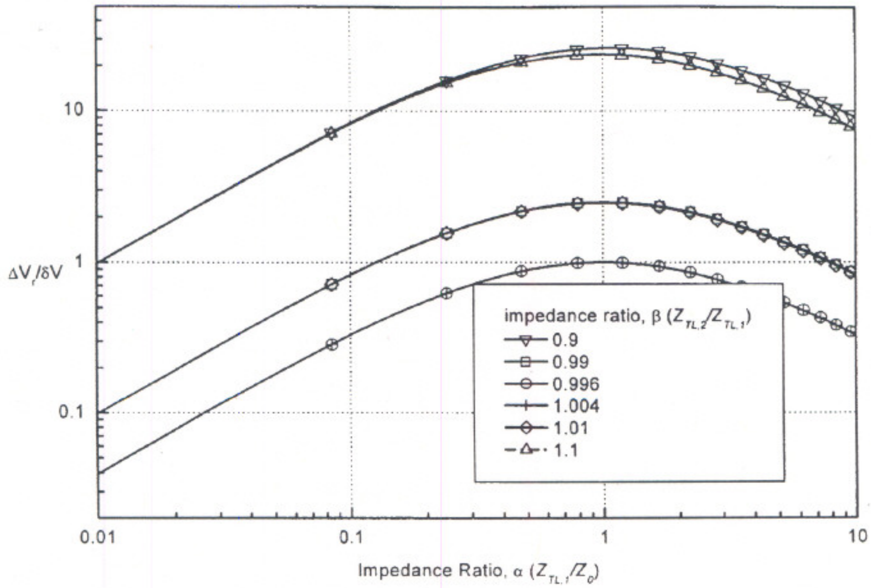

Fig. 7. The $\Delta V_r/\delta V$ ratio versus α for various β [see (26)]. β is the ratio of the characteristic impedance of TL_2 to that of TL_1 and α is the ratio of the characteristic impedance of TL_1 to that of the TDR (Z_0). The input signal is assumed to have a level of 0.25 V and the noise or amplitude discretization, δV, is assumed to have a level of 5×10^{-4} V. The value of ΔV_r is dependent on the input signal level and the magnitude of ρ.

C. Differentiating Between Two Transmission Lines With Similar Z_{TL}: SNR and Amplitude Discretization Limitations

In this section, the ability to differentiate between two transmission lines, TL_1 and TL_2, one having characteristic impedance $Z_{TL,1}$ and the other $Z_{TL,2}$, is analyzed. The value of level L_2 for a given transmission line is given by

$$V_2 = V_i + \frac{Z_{\mathrm{TL}} - Z_0}{Z_{\mathrm{TL}} + Z_0} V_i. \tag{24}$$

The V_i immediately to the right of the equal sign results from the fact that the TDR measures both the incident and reflected pulses simultaneously. The reflection from both TL_1 and TL_2 can be described by (24). The difference in amplitude between the reflected pulses can be used to determine how well a TDR system can differentiate between the characteristic impedances of two different transmission lines with similar Z_{TL}. This difference is

$$\begin{aligned}\Delta V_r &= V_{2,TL1} - V_{2,TL2} \\ &= \left(V_i + \frac{Z_{TL,1} - Z_0}{Z_{TL,1} + Z_0} V_i\right) - \left(V_i + \frac{Z_{TL,2} - Z_0}{Z_{TL,2} + Z_0} V_i\right) \\ &= 2Z_0 V_i \frac{Z_{TL,2} - Z_{TL,1}}{(Z_{TL,1} + Z_0)(Z_{TL,2} + Z_0)}.\end{aligned} \tag{25}$$

This result can be parameterized by setting $Z_{\mathrm{TL},1} = \alpha Z_0$ and $Z_{\mathrm{TL},2} = \beta Z_{\mathrm{TL},1}$ which gives

$$\Delta V_r = 2V_i \frac{\alpha(\beta - 1)}{(\alpha + 1)(\alpha\beta + 1)}. \tag{26}$$

Let the noise in the measured voltage or the amplitude discretization level be δV. The ability to differentiate between two different transmission lines using a TDR system is dependent on the amplitude difference of the reflected signals from the two transmission lines, given by ΔV_r, and the noise or amplitude discretization error in the measurement process, given by δV. The ratio of these two quantities, $\Delta V_r/\delta V$, can be used to determine this TDR limitation. Fig. 7 shows several plots of $\Delta V_r/\delta V$ versus α for various values of β. The higher the value of $\Delta V_r/\delta V$, the better able the TDR system can resolve two transmission lines with similar characteristic impedances. From Fig. 7, it can be seen that the ideal characteristic impedance for resolving two transmission lines is obtained if $Z_{\mathrm{TL},1} \approx Z_0$ and $Z_{\mathrm{TL},2}$ is significantly different from $Z_{\mathrm{TL},1}$. The latter requirement is not acceptable since the purpose is to differentiate between two very similar Z_{TL} values. For reference, the β values shown in Fig. 7 would correspond to $Z_{\mathrm{TL},2}/Z_{\mathrm{TL},1}$ reflection coefficients of $-0.053, -0.005, -0.002, 0, 0.002, 0.005$, and 0.048. The limit on the ability to resolve and measure the relative difference between two transmission lines is at $\mathrm{SNR} = 1$; below $\mathrm{SNR} = 1$, the two transmission lines cannot be differentiated by TDR.

D. Measuring Perturbations in Z_{TL}: SNR and Amplitude Discretization Limitation

Similar to the analysis in Section 3.3, the expected reflected signal amplitude relative to the signal noise and amplitude discretization can be used to determine whether a TDR system is adequate for measuring perturbations in a transmission line. In this case, however, we will consider the first reflection and the second reflection of a single transmission line. The difference between the amplitudes of the reflection from the transmission line and its perturbation is given by

$$\begin{aligned}\Delta V_{r,2} &= V_{r,\Delta} - V_r \\ &= (V_i + \rho V_i + (1 - \rho^2)\rho_\Delta V_i) - (V_i + \rho V_i) \\ &= V_i(1 - \rho^2)\rho_\Delta \\ &= 2V_i \frac{Z_{\mathrm{TL}} Z_0}{(Z_{\mathrm{TL}} + Z_0)^2} \frac{Z_{\mathrm{TL},\Delta} - Z_{\mathrm{TL}}}{Z_{\mathrm{TL},\Delta} + Z_{\mathrm{TL}}}\end{aligned} \tag{27}$$

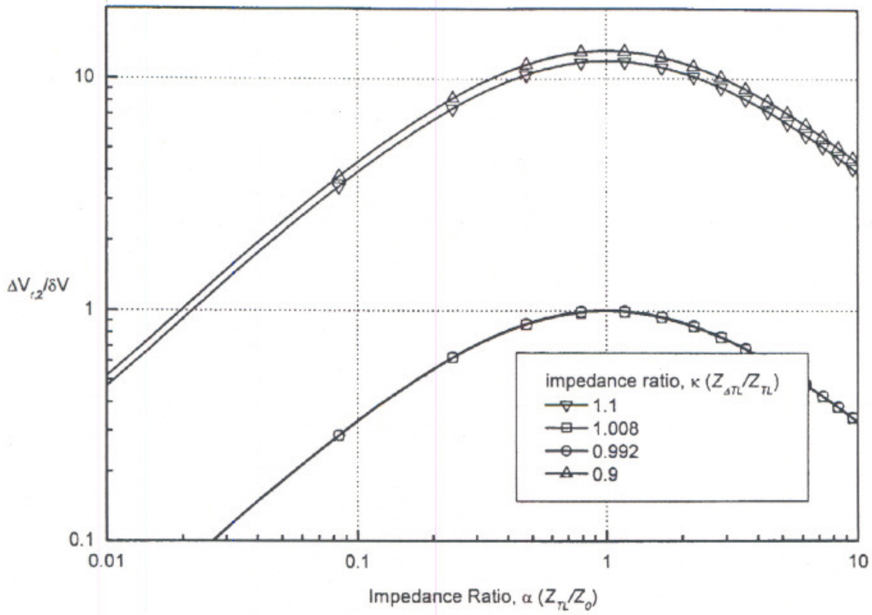

Fig. 8. The $\Delta V_{r,2}/\delta V$ ratio versus α for various κ [see (28)]. κ is the ratio of the characteristic impedance of the transmission line perturbation to that of TL and α is the ratio of the characteristic impedance of TL to that of the TDR (Z_0). The input signal is assumed to have a level of 0.25 V and the noise or amplitude discretization, δV, is assumed to have a level of 5×10^{-4} V. The value of $\Delta V_{r,2}$ is dependent on the input signal level and the magnitudes of ρ and $\Delta\rho$.

where $(1-\rho^2)$ is the product of $(1+\rho)$, the transmission coefficient of the TDR/TL interface for the pulse propagating in the direction of the transmission line, and $(1-\rho)$, the transmission coefficient of the TDR/TL interface for the pulse propagating in the direction of the TDR head. The ρ_Δ is the reflection coefficient from the transmission line impedance discontinuity, and $V_{r,\Delta}$ is the voltage level in the TDR waveform corresponding to the reflection from the discontinuity.

Similar to previous analyzes, substitutions are used to normalize the impedance values to Z_0, namely: $Z_{\rm TL} = \alpha Z_0$ and $Z_{\rm TL,\Delta} = \kappa Z_{\rm TL}$. Equation (27), with these substitutions, becomes

$$\Delta V_{r,2} = 2V_i \frac{\alpha}{(\alpha+1)^2} \frac{\kappa-1}{\kappa+1} \tag{28}$$

and the measurement limitations imposed on the TDR system by either the noise in the measurement process or amplitude discretization can be assessed by the ratio $\Delta V_{r,2}/\delta V$. Fig. 8 shows $\Delta V_{r,2}/\delta V$ versus α for various values of κ. As can be seen from Fig. 8, if the characteristic impedance of the perturbation is less than about 1% of $Z_{\rm TL}$, it is not possible to measure the perturbation. This value of 1% assumes the ideal condition, namely, with $Z_{\rm TL} = Z_0$.

IV. Conclusion

For most applications, a 50 Ω TDR system is more than adequate for measuring the characteristic impedance of a uniform continuous transmission line. The only transmission lines that would not be adequately characterized are the bus lines in power systems. If the purpose is to measure perturbations in the characteristic impedance of the transmission line, then the 50 Ω TDR presents more limitations. These limitations are caused by the noise in the measurement system which puts a lower bound on errors in the measurement of $'Z_{\rm TL}$ and bounds the minimum value of $Z_{\rm TL}$ for which errors on $'Z_{\rm TL}$ are less than 0.5%. For a 50 Ω TDR this lower $Z_{\rm TL}$ bound is about 30 Ω. Total signal noise and amplitude discretization also bound the range of $Z_{\rm TL}$ for which errors in $'Z_{\rm TL}$ are less than 0.5%. However, signal noise and amplitude discretization impose both upper and lower bounds, although the upper bound is high. Signal noise and amplitude discretization limit the ability to measure $'Z_{\rm TL}$ only to those $'Z_{\rm TL}$ that differ from $Z_{\rm TL}$ by more than 1%.

The last assessment performed was the ability to differentiate between two transmission lines of similar $Z_{\rm TL}$. Signal noise and amplitude discretization impose low and high $Z_{\rm TL}$ bounds on the two transmission lines of which a difference in $Z_{\rm TL}$ is desired. The best case is when the $Z_{\rm TL}$ of one transmission line is equal to Z_0 and a difference of about 0.5% between $Z_{\rm TL}$ of the two transmission lines is measurable. In summary, for $Z_{\rm TL} \geq 50$ Ω, a 50 Ω TDR system is sufficient for transmission line characterization. For $Z_{\rm TL} < 50$ Ω, there are two considerations. If the application is impedance profiling and accuracy better than about 1% is required, then $Z_{\rm TL} >\approx 30$ Ω. If the application is to determine the characteristic impedance of a uniform continuous transmission line, then $Z_{\rm TL} >\approx 10$ Ω. Consequently, for most TDR applications for measuring and characterizing transmission lines, a 50 Ω impedance is sufficient. For transmission line bus bars in high-power systems that have a characteristic impedance less than 2 Ω, however, a 50 Ω TDR system is not adequate.

Acknowledgment

The author would like to thank D. R. Larson and A. R. Hefner for technical comments and B. A. Bell for administrative support and editorial suggestions.