

# Current Ripple Effect on $n$ -Value

L. F. Goodrich and J. D. Splett

**Abstract**—We systematically studied the effect of current ripple on the determination of  $n$ -value, which is the index of the shape of the electric field-current ( $E-I$ ) curve. Commercial Nb<sub>3</sub>Sn wires were measured with controlled amounts of ac ripple. Substitution-box, superconductor simulator circuits were also measured. A battery-powered current supply was used to provide the dc with ripple currents. The ripple frequencies were 60, 120, and 360 Hz, to represent common electrical power harmonics in high-current power supplies. A previous study focused on the effect of ripple on the determination of dc critical current ( $I_c$ ); the current study focuses on how ripple changes the  $n$ -value and shows that ripple has a larger effect on  $n$ -value than on  $I_c$ . We examined models and measurements on simulators to reproduce and explain the effects observed in measurements on superconductors. Current ripple and spikes may be sources of differences in  $n$ -values measured at different laboratories.

**Index Terms**—Critical current measurement, current ripple, niobium compound, superconducting wires.

## I. INTRODUCTION

ALL high-current power supplies have some amount of current ripple and spikes. High-current power supplies with the lowest level of current ripple and spikes are often more than a factor of ten times more expensive than conventional supplies. In addition, current ripple and spikes are more of a problem for short-sample critical current testing than for magnet operation because of the difference in load inductance. Recent high-performance Nb<sub>3</sub>Sn wires have currents as high as 750 A and  $n$ -values as high as 80 at 12 T [1], which are difficult to measure. Therefore, we need to understand the effects of ripple and spikes on the measured critical current ( $I_c$ ) and  $n$ -value, the index of the shape of the electric field-current ( $E-I$ ) curve [2].

This study focuses on how ripple changes the  $n$ -value and shows that ripple has a larger effect on  $n$ -value than on  $I_c$ . Interlaboratory comparisons often show that the percentage variation in  $n$ -value is much larger than the percentage variation in  $I_c$ . We think that current ripple and spikes are sources of differences in  $n$ -values measured at different laboratories.

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The authors are with the National Institute of Standards and Technology (NIST), Boulder, CO 80305 USA (e-mail: goodrich@boulder.nist.gov).

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## II. THEORETICAL MODEL

We modeled the electric field versus current curve according to

$$E = E_0(I/I_0)^{n_0} \quad (1)$$

where  $E_0$  is an electric field constant,  $I_0$  is a current constant, and  $n_0$  is the  $n$ -value without ripple. We examined two models. The current ripple with *proportional* ripple amplitude is modeled as

$$I = i + i \cdot a \cdot \sin(\omega t) \quad (2)$$

where  $i$  is the dc current variable,  $a$  is the proportionality fraction of the ripple,  $\omega$  is the angular frequency ( $2\pi f$ , where  $f$  is the frequency), and  $t$  is the time. The current ripple with a *fixed* amplitude of ripple is modeled as

$$I = i + a \cdot \sin(\omega t) \quad (3)$$

where  $a$  is the *fixed* amplitude of the ripple. The instantaneous  $E$  will then be approximated by

$$E = [E_0/(I_0)^{n_0}] \cdot [i + a \cdot \sin(\omega t)]^{n_0} \quad (4)$$

for the *fixed* amplitude ripple case. The time-averaged (dc)  $E$  will be the integral of the instantaneous  $E$  over one cycle. Using these equations, we mathematically determined  $E-I$  curves for various ripple amplitudes and  $n_0$  values. From the calculated  $E-I$  curves, we determined  $I_c$  and effective  $n$ -value for various ripple amplitudes. The results show that the change in  $I_c$  with ripple amplitude is almost the same for the two cases of *proportional* amplitude ripple (2) and *fixed* amplitude ripple (3). The results for *fixed* amplitude ripple are shown in Fig. 1 for various values of  $n_0$ .  $I_c$  with zero ripple is defined as  $I_{c0}$ , and the ripple amplitude is shown as a percentage of  $I_{c0}$ . For an  $n_0$  of 80 and *fixed* amplitude ripple of 3% of  $I_{c0}$ , the change in  $I_c$  is  $-1.38\%$ . In the case of *proportional* amplitude ripple, the calculated change in  $I_c$  is  $-1.35\%$ . There is no effect of ripple frequency in this model. The changes in  $I_c$  for this model are very similar to those of a previous study [3] that focused on the change in the measured  $I_c$  as a function of ripple. However, the previous study did not investigate the change in  $n$ -value.

In contrast to the change in  $I_c$ , the change in  $n$ -value with current ripple depends on the details of the ripple amplitude. In the case of *proportional* amplitude ripple, the  $n$ -values of the calculated  $E-I$  curves are exactly the same as without ripple. For *fixed* amplitude ripple, the change in  $n$ -value is  $-2.24\%$  for an  $n_0$  of 80 and *fixed* amplitude ripple of 3% of  $I_{c0}$ . This is

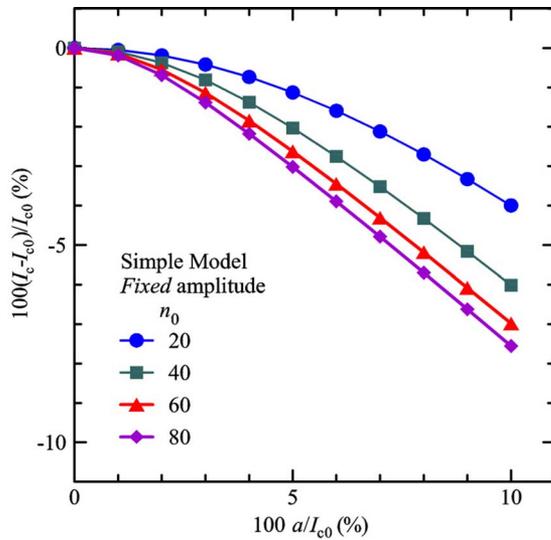


Fig. 1. Simple mathematical model calculations of the percentage change in critical current relative to zero ripple critical current ( $I_{c0}$ ) versus *fixed* amplitude ripple relative to  $I_{c0}$  for various values of  $n_0$ .

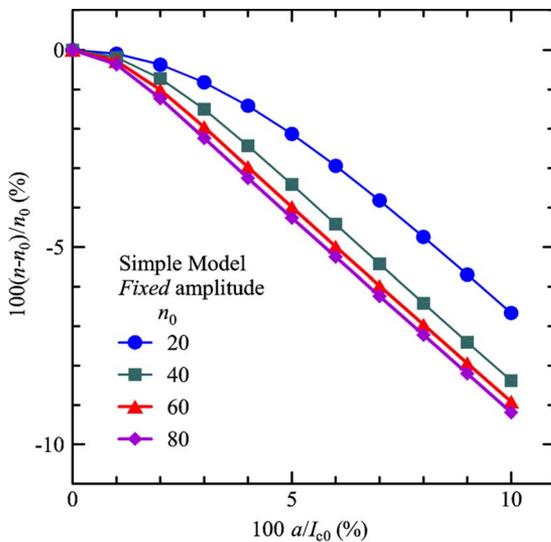


Fig. 2. Simple mathematical model calculations of the percentage change in  $n$ -value relative to  $n_0$  versus *fixed* amplitude ripple relative to zero ripple critical current ( $I_{c0}$ ) for various values of  $n_0$ .

a larger percentage change than the change in  $I_c$  for the same conditions. The changes in  $n$ -value with *fixed* amplitude ripple for various values of  $n_0$  are shown in Fig. 2.

To understand how  $I_c$  and  $n$ -value change with ripple current, one needs to visualize the instantaneous  $E-I$  curve that follows the zero ripple  $E-I$  curve. The time average of  $I$  is the same as the zero ripple case because  $I$  is linear. The time average of  $E$  is higher than the zero ripple case because  $E$  is nonlinear. The resulting measured  $E-I$  curve with ripple is shifted up. For the *proportional* ripple case, the curve shifts and stays parallel to the zero ripple curve; thus, there is a change in  $I_c$  and no change in  $n$ -value. For the *fixed* ripple case, the curve shifts more at the lower currents; thus, the  $n$ -value is lower with ripple.

### III. EXPERIMENTAL MEASUREMENTS

#### A. Measurement System

The current ripple for commercial power supplies is expected to have fairly constant amplitude over a given current range. Thus, the *fixed* amplitude ripple is expected to be the more appropriate model. A battery-powered current supply was used to provide the dc with controlled amounts of ac ripple currents. Our experimental measurements were made with various *fixed* ripple amplitudes and frequencies of 60, 120, and 360 Hz to investigate common electrical power harmonics in high-current power supplies.

The filter frequency of the sample voltmeter was also an experimental parameter that we changed to determine whether this would affect the results. The voltmeter analog filter frequencies investigated were 3.2, 35, and 700 Hz. The lower filter frequencies reduced the variation of the voltage readings, as expected. The  $E-I$  curves were measured in the usual ramp and hold method where the current was ramped and held at a number of current set points. After a settling time of 3 s at each set point, 15 voltage and 15 current readings were taken and averaged. Each reading has a one power-line cycle integration time. We also monitored the instantaneous voltage and current with a digital processing oscilloscope. The instantaneous voltage signal was taken from the digital nanovoltmeter after the analog filter and amplifier, but before the power-line cycle integration. The instantaneous voltage signal showed the smoothing effect of the various analog filter frequencies.

Another experimental parameter was whether the sample voltage signal had the normal inductive signal of the co-wound voltage taps or the effective inductance was reduced with a bucking coil. The sample voltage taps covered a 50 cm length of the superconducting wire. These taps were co-wound with the sample to reduce mutual inductance to about 50 nH. An adjustable bucking coil was added that could remove most of the symmetric inductive signal to a level of about 4 nH. The effect of the bucking coil was evident on the instantaneous voltage signal, but the only observable effect on the digital reading was slightly lower variation.

A conservative estimate of standard uncertainty in critical current measurements due to systematic effects is 2.5%, and that due to random effects is 0.5%. A conservative estimate of standard uncertainty in  $n$ -value determinations due to systematic effects is the larger of 6% or 1.5, and that due to random effects is the larger of 1.5% or 0.5.

#### B. Measurements on $Nb_3Sn$ Samples

Two nominal 0.8 mm diameter  $Nb_3Sn$  wires were measured: a high-performance/marginally stable wire (P) with an  $I_c$  of 750 A at 10  $\mu V/m$  and 12 T, and a stable wire (S) with an  $I_c$  of 150 A at 10  $\mu V/m$  and 12 T. The specimens were mounted on a coil holder made of Ti-6Al-4V (percent by mass) tube with nine turns ( $\approx 90$  cm) between the current contacts. Only some results from the high-performance wire will be shown here.

Fig. 3 shows the effect of current ripple on  $I_c$  of the high-performance wire at 14 T where the  $I_{c0}$  was 506 A at 10  $\mu V/m$ . All of the data shown were taken with a 3.2 Hz analog voltmeter filter. The ripple frequencies were 60, 120, and 360 Hz.

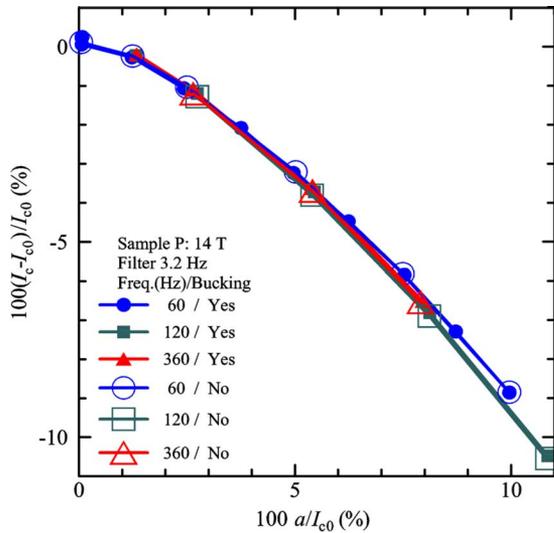


Fig. 3. Measured percentage change in critical current relative to zero ripple critical current ( $I_{c0}$ ) versus constant ripple amplitude relative to zero ripple critical current ( $I_{c0}$ ) for various ripple frequencies and with and without a voltage bucking coil. Sample is the high-performance wire measured at 14 T.

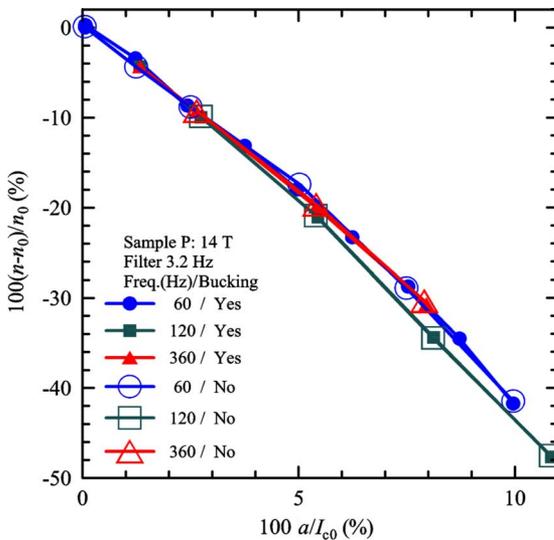


Fig. 4. Measured percentage change in  $n$ -value relative to zero ripple  $n$ -value ( $n_0$ ) versus constant ripple amplitude relative to zero ripple critical current ( $I_{c0}$ ) for various ripple frequencies and with and without a voltage bucking coil. Sample is the high-performance wire measured at 14 T.

The solid symbols indicate that a bucking coil was used on the sample voltage. Fig. 3 shows that same change in  $I_c$  was observed regardless of ripple frequency or whether or not a bucking coil was used. Other results with 35 and 700 Hz analog voltmeter filter settings indicated that the choice of analog filter did not affect the observed change in  $I_c$ . The change in  $I_c$  was about  $-1.5\%$  for a current ripple of 3% of  $I_{c0}$ , which is close to the expected amount based on the model results shown in Fig. 1. The change in  $I_c$  for the superconductor was slightly larger than that of the model at higher ripple values.

Fig. 4 shows the effect of current ripple on the measured  $n$ -value of the high-performance wire at 14 T where the  $n_0$  was 49. All  $n$ -values were determined over the electric field range of

5 to 20  $\mu\text{V}/\text{m}$ . The experimental parameters were the same as in Fig. 3 and again there is no frequency or bucking coil effect. The change in  $n$ -value was about  $-10.6\%$  for a current ripple of 3% of  $I_{c0}$ . This change is about a factor of 4.7 times the expected change based on the model results shown in Fig. 2. Other results with 35 and 700 Hz analog voltmeter filter settings indicated that the choice of analog filter did not affect the observed change in  $n$ -value.

Similar results in the change of  $I_c$  and  $n$ -value occurred at other magnetic fields for this high-performance  $\text{Nb}_3\text{Sn}$  wire and for the more conventional  $\text{Nb}_3\text{Sn}$  wire.

### C. Measurements on a Superconductor Simulator

Additional measurements were made on superconductor simulator circuits [4] using the same system of equipment as was used for the measurement of superconductors. A superconductor simulator is a physical electronic circuit that operates at room temperature, can be substituted for the superconductor, and emulates the extremely nonlinear  $E - I$  characteristic of a superconductor along with its other major electrical properties. We used two types of simulators: passive and active. The passive simulator circuit consists of wire, resistors, and a diode. The circuit basically inverts the typical diode characteristic where the current is zero until a certain voltage is reached to a characteristic where the voltage is zero until a certain current is reached. The total sample current flows through the passive circuit and the diode provides the nonlinear characteristic. An active simulator uses the voltage drop across a sample current shunt resistor as an input, and operational amplifiers with variable gains and set points to provide adjustable  $I_c$  and  $n$ -values. The active circuit also uses a diode for the nonlinear behavior. The response time of both simulators is sufficient for current ripple up to 360 Hz. The mutual inductance of the voltage signal can be adjusted from near zero to a value similar to that of a superconductor sample.

Measurements on passive and active simulators demonstrated that the change in  $I_c$  and  $n$ -value with current ripple agree fairly well with the theoretical model. The best agreement was with the active simulator configured such that the  $n$ -value was fairly constant over a wide range of voltage. For a 10% ripple amplitude, changes in  $I_c$  and  $n$ -value for the active simulator and the model agreed to within 1%. This level of agreement suggests that the measurement system is capable of achieving the theoretical results under equivalent conditions. The same measurement system produces significantly different changes in  $n$ -values for superconductive samples. This indicates that not all of the physical mechanisms have been accounted for by the theoretical model and the simulator. One missing physical mechanism may be the hysteretic loss of the superconducting sample.  $E - I$  characteristics are asymmetric with current ramp direction; a portion of the asymmetry was attributed to hysteretic loss [5], [6]. With an asymmetry in current ramp direction, the ripple current would generate more voltage during the upward portion of the cycle than during the downward portion. This low-level voltage bias is expected to be nonlinear and would provide an additional mechanism to lower the measured  $n$ -value without further reduction in  $I_c$ , except perhaps at the higher ripple values.

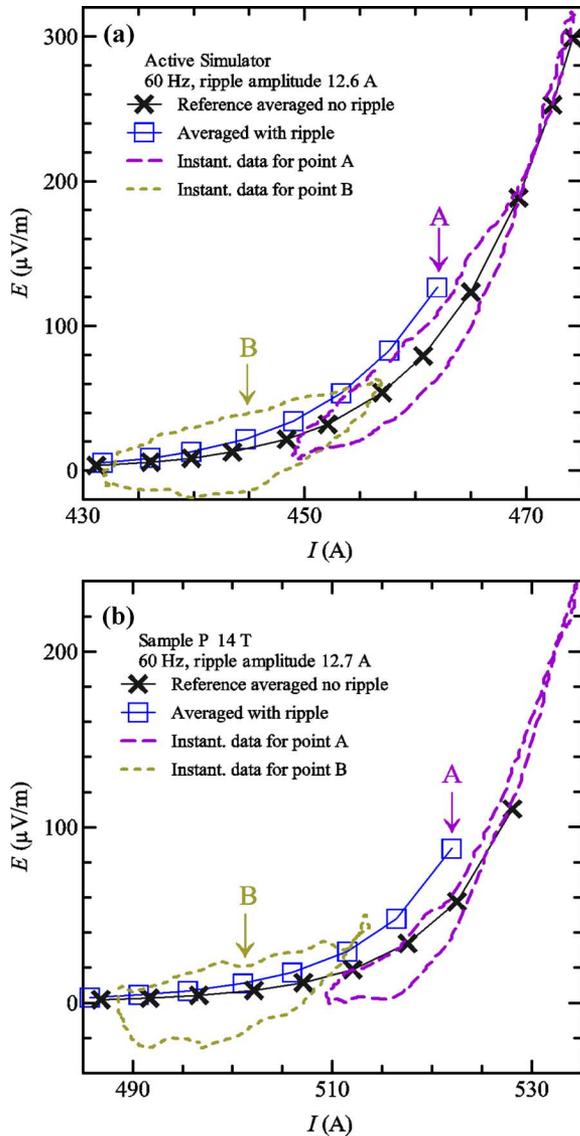


Fig. 5. Comparison of instantaneous and averaged  $E - I$  curves with ripple for (a) simulator measurements and (b) superconductor sample measurements. Reference curves are averaged data without current ripple.

Other evidence of the differences between measurements on the simulator and those on the sample are given in Fig. 5. Time averaged points and instantaneous loops for two points are shown. In Fig. 5(a), both instantaneous loops follow the reference curve without ripple (X), just as the model assumes. The loops open up in the middle where the current's rate of change reaches a maximum and they close near each end where the current's rate of change goes through zero. The inductance of the simulator was adjusted to be about to 5 nH, similar to the 4 nH of the bucked inductance of the sample. In contrast

(Fig. 5(b)), the upper portion of instantaneous loop B for the sample (510 to 513 A) goes above the average curve with ripple ( $\square$ ), even though the inductive voltages are near zero for this portion. The instantaneous loop A of the sample follows the reference curve without ripple, indicating that the effect is not just sample heating and that additional voltage is not observed at the higher currents. The additional voltage for the lower current points will decrease the measured  $n$ -value.

#### IV. CONCLUSION

The changes in  $I_c$  induced by ripple were fairly consistent for the theoretical model, the simulator measurements, and superconductor measurements. The changes in  $n$ -value induced by ripple were similar for the theoretical model and simulator measurements; however, they did not agree with the superconductor measurements. We propose that this difference is due to additional voltage resulting from the hysteretic loss in the superconductor, which could explain the relatively large changes in  $n$ -value with current ripple.

The effect of current ripple changes the measured  $n$ -value of a superconductor by a factor of about 7 times more than it changes  $I_c$ . The changes in  $n$ -value are not a function of ripple frequency, analog filter frequency, or inductive signal. This study indicates that regardless of the mechanism responsible for the lowering of the  $n$ -value of samples, current ripple has a significant effect on the measured  $n$ -value. Current spikes were not studied here; however, they are expected to decrease measured  $n$ -value by much more than they change  $I_c$ . Current ripple and spikes may be factors contributing to differences observed in interlaboratory comparisons of  $I_c$  and  $n$ -value.

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