Toward the SI System Based on Fundamental Constants: Weighing the Electron

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***Abstract*—A modified International System of Units (SI) based on simply specifying exact numeric values of seven physical constants is described. This "set of constants" approach fixes the scale for all measurements, and the result is that both base units and derived units have equal footing. The seven quantities are the Cs transition frequency, the speed of light, the spectral luminous efficacy, the electron charge, the Avogadro constant, the Planck constant, and the Boltzmann constant. The first three quantities ensure that the second, the meter, and the candela are, in practice, the same as in the present SI. However, this approach requires that the definition of the ampere, mole, kilogram, and kelvin be changed to provide consistency with these constants. A major challenge in ensuring acceptable continuity is in advancing the measure of the kilogram and kelvin in terms of fundamental constants of nature.**

***Index Terms*—Avogadro constant, elementary charge, fundamental constants, International System of Units (SI), kilogram, Planck constant, watt balance.**

I. FOREWORD

THE IDEA of changing the way in which the International System of Units (SI) is defined by simply defining a set of seven constants is appealing to some and likely controversial to others. It is not the intent of this paper to create controversy, but rather to show how this idea is embedded in all of the approaches where constants are used to replace artifacts or other specific natural features that presently define the base units. The metrology and scientific communities may find it preferable to preserve the descriptive structure of the present SI (which defines seven base units), but in this paper, it is shown that this "set of constants" (SC) approach, when combined with our knowledge of physics, can lead to implicit and explicit definitions for the base units. The reverse is also true; the set of definitions for seven base units both defines and, in effect, creates this set of constants, and the distinction between base units and derived units is only semantics. An important point to be made is that this SC approach has clear advantages over a system that defines the kilogram by making the atomic mass unit have an exact mass.

II. BACKGROUND

The SI continues to evolve to meet or anticipate new demands as science and engineering advance. The International Committee for Weights and Measures (CIPM) adopted Recommendation 1 (CI-2005), calling for preparative steps toward new definitions of the kilogram, the ampere, the kelvin, and the mole in terms of fundamental constants, for possible adoption by the 24th CGPM in 2011. The General Conference on Weights and Measures (CGPM) is the highest authority with regard to matters of the International Treaty of the Meter.

In 2004, the CIPM asked the Consultative Committee on Units, through its president, Ian Mills, to investigate and report on the status of redefining the kilogram, presently defined by an artifact, in terms of a fundamental constant of nature. Ian Mills started discussions with Terry Quinn, and at about the same time, Peter Mohr, Barry Taylor, and I started to write a paper showing the very clear advantages to the fundamental constants, should either the Avogadro constant N_A or the Planck constant h be used to define the unit of mass. After much discussion, the five of us decided to publish a joint paper titled "Redefinition for the kilogram: A decision whose time has come" [1], which concluded that the advantages to science through an improved set of fundamental constants outweighed the problems that might possibly occur. The date we suggested for implementing the new definitions was 2007. The aforementioned paper caused a good deal of controversy within our metrological community, and a few papers were written which included comments on the ideas presented there [2], [3].

The problem stems from the difficulty in correctly evaluating the unit of mass in terms of physical constants, which is linked to a very significant discrepancy between the present measurements of h. A number of National Measurement Institutes and all relevant consultative committees to the CIPM concluded that 2007 is too soon to make these changes and that two things must happen before changes to the mass definition can proceed. First, no significant differences should exist in the data that measure the pertinent fundamental constants; and second, at least two independent laboratories need to obtain uncertainties in the range of 20 parts in 10^9 in measurements of a kilogram in terms of those fundamental constants. Whereas there is no guarantee, these data are expected by 2011, which is the next time the CGPM meets. A second paper [4] by the same authors summarizes the present status of this redefinition process. In this paper, there is one alternative for defining the new SI that is the favorite of all five authors. That is, to define the SI without distinguishing between base units and derived units, simply by defining seven quantities of nature that fix the scale for all the

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units in the SI. Even if the CGPM does not choose the SC approach, it is useful to understand it, because this approach is, in fact, the system created by any of the sets of definitions outlined in our paper [4].

III. INTRODUCTION

Most of the electrical metrology community already are familiar with the following basic concept. Our electrical units are defined through mass, length, time, and the base unit ampere defined as follows: "the ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed one meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length." This definition, along with the physical law that describes the force between two such wires, uniquely defines the magnetic permeability of free space to be $\mu_0 = 4\pi \times 10^{-7}$N/A^2. When defining the ampere in 1960, the authors of the definition understood that choosing this constant fixes the scale for all electrical quantities, even though it takes two experiments to realize the electrical units completely. At that time, they used an absolute ampere experiment and an absolute ohm experiment. In the 1970s, the X-ray crystal density (XRCD) measurement and the calculable capacitor experiment were the best methods. Today, a measure of the fine-structure-constant combined with the quantum Hall effect (QHE) best defines the SI value for resistance, whereas the watt balance best defines the SI electrical watt, and these provide a measure of the SI volt and current as well. This same philosophy, defining constants and using the best physics to realize all units, is what I am describing in the SC approach.

IV. IMPROVING THE SI

The definition for the SC approach is taken directly from [4], except that I have changed the order of the list of quantities. The definition would read as follows:

The International System of Units, the SI, is the system of units scaled so that the

1) ground state hyperfine splitting transition frequency of the cesium 133 atom $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$ is 9 192 631 770 Hz;
2) speed of light in vacuum c_0 is 299 792 458 m/s;
3) spectral luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz, $K(\lambda_{540})$, is 683 lm/W;
4) elementary charge e is $1.60217653 \times 10^{19}$ C;
5) Planck constant h is $6.6260693 \times 10^{-34}$ J · s;
6) Boltzmann constant k is 1.3806505×10^{23} J/K;
7) Avogadro constant N_A is 6.0221415×10^{23} per mol.

(Note: the numeric values used in items (4), (5), (6) and (7) are the 2002 Committee on Data for Science and Technology recommended values. The values that will be chosen to base the new SI would likely differ slightly from these values.)

The seven base units form a time-tested measurement system. In changing the SI, it is essential that we maintain the key property of the SI that all units of measure are uniquely defined. We can show that the aforementioned seven quantities have a one-to-one correspondence with the base units and, thus, will have the same time-tested properties. The first three quantities previously defined have the same effect as the present definitions for the base units of the second, the meter, and the candela. However, for the four constants e, h, k, and N_A, for which an exact value is given in the previous definition, there is a corresponding definition of a quantity or base unit that must be changed. If the electron charge e is to have an exact value, then the magnetic permeability μ_0 will have an uncertainty $\mu_0 = 4\pi \times 10^{-7}(1 \pm 0.7 \times 10^{-9})$N/A^2; similarly, if the Planck constant h is to have an exact value, then the mass of the artifact kilogram that resides at the BIPM $m(\mathrm{K})$ will now have an uncertainty $m(\mathrm{K}) = (1 \pm 2 \times 10^{-8})$ kg; if the Boltzmann constant k is to have an exact value, then the triple point of water is no longer exactly 273.16 K, but has an uncertainty of a few mK; and if the Avogadro constant N_A is to have an exact value, then the mass of one mole of carbon 12 will have an uncertainty $m(\text{mole of } ^{12}\mathrm{C}) = 12(1 \pm 1.4 \times 10^{-9})$ g. Of course, at the time of redefinition, the value of the constants will be chosen so that the units remain unchanged, but the SI uncertainty is transferred to the old quantity. For example, as previously shown, the kilogram artifact $m(\mathrm{K})$ would have a 20 parts in 10^9 relative standard uncertainty when h becomes exact. In this scenario, base units and derived units would be on an equal footing. In fact, all units are derived from the "set of seven constants" and the known physics, just as derived units are in today's SI. If the conditions suggested by the CIPM are satisfied, that is, if the data are consistent and accurate, then the major advantages of having a measurement system based on an invariant set of constants of nature greatly outweigh the inconvenience of the aforementioned loss of exactness to the base units, as now defined.

V. ALL THE UNITS IN THE SI

Given the seven constants previously defined, our knowledge of physics can be used to derive any unit in the SI, including the seven "base units" in the old system. The definitions for the second, the meter, and the candela would be defined the same way as they are in the present SI, although other ways of stating the equivalent definitions would be equally valid. For example, we could say the meter is defined such that the speed of light has the value 299 792 458 m/s.

Table I is copied from [4, Table 1], which shows various choices for defining four base units. But instead of giving a choice of one definition for each of the four base units of mass, current, temperature, and the amount of substance, I view these definitions as valid descriptions of how these units are derived from this "set of constants."

There is a simple correspondence between the set of seven constants and the set of seven base units previously defined, and each set can be derived from the other. It follows that we create the same system whichever way we choose to define it formally. It also follows that because the new SI can define the same seven units, it should enjoy the same time-tested success as the old system.

One interesting point that is sometimes not understood is that Single Electron Tunneling (SET) experiments can help verify

TABLE I

THE DEFINITIONS OF THE KILOGRAM, AMPERE, KELVIN, AND MOLE DISCUSSED IN [4] LINK THESE UNITS TO EXACT VALUES OF THE PLANCK CONSTANT h, ELEMENTARY CHARGE e, BOLTZMANN CONSTANT k, AND AVOGADRO CONSTANT N_A, RESPECTIVELY. IN THE "SET OF CONSTANTS" SI, ALL THESE ARE ACCURATE DEFINITIONS. IF INSTEAD THE CGPM CHOOSES TO DEFINE BASE UNITS, THEN ONLY ONE DEFINITION FROM EACH UNIT WILL BE THE DEFINITION FOR THAT BASE UNIT

kilogram	The kilogram is the mass of a body (whose equivalent energy is equal to that of a number of photons whose frequencies sum to exactly) –or– (whose de Broglie-Compton frequency is equal to exactly) $[(299\ 792\ 458)^2/(6.626\ 069\ 3 \times 10^{-34})]$ hertz.	The kilogram, unit of mass, is such that the Planck constant is exactly $6.626\ 069\ 3 \times 10^{-34}$ joule second.
ampere	The ampere is the electric current in the direction of the flow of exactly $1/(1.602\ 156\ 53 \times 10^{-19})$ elementary charges per second.	The ampere, unit of electric current, is such that the elementary charge is exactly $1.602\ 176\ 53 \times 10^{-19}$ coulomb.
kelvin	The kelvin is the change of thermodynamic temperature that results in a change of thermal energy kT by exactly $1.380\ 650\ 5 \times 10^{-23}$ joule.	The kelvin, unit of thermodynamic temperature, is such that the Boltzmann constant is exactly $1.380\ 650\ 5 \times 10^{-23}$ joule per kelvin.
mole	The mole is the amount of substance of a system that contains exactly $6.022\ 141\ 5 \times 10^{23}$ specified elementary entities, which may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.	The mole, unit of amount of substance of a specified elementary entity, which may be an atom, molecule, ion, electron, any other particle, or a specified group of such particles, is such that the Avogadro constant is exactly $6.022\ 141\ 5 \times 10^{23}$ per mole.

assumptions about the Josephson junction and QHE accuracy, but SET is not likely to be the most accurate way to realize the SI unit of current in the near future, except at very low current. We can measure the number of electrons passing in a circuit today [1], assuming the Josephson and Quantum Hall theories are accurate, just by measuring the voltage across a resistor in the 1990 representation. The measured current divided by the charge $e_{90} = 2/(K_{\mathrm{J}-90}R_{\mathrm{K}-90})$ tells the number of elementary charges per second in the circuit. e_{90} is not an exact value in the SI, so today, this is not an SI current, but in the new system, e and h will be fixed, making these measurements SI.

VI. WHY DEFINE h?

The most discussed issue that still remains is the choice between defining the kilogram in terms of the atomic mass unit u or the Planck constant h. There are two non–SI systems of units that would be affected by the choice between h and u. The first system is the 1990 electrical representation of the units of the volt and ohm (SI_{90}). This system is used to maintain all the electrical units, where numeric values are assigned to the Josephson constant, $K_{\mathrm{J}-90}$, and von Klitzing constant, $R_{\mathrm{K}-90}$. The second system is based on the mass of carbon 12, defined to be equal to 12 u. If the CGPM chooses to define e and h, thus $2e/h$ and h/e^2 also have exact values, the SI_{90} type units will neither be needed nor desired, but the unit u would have a relative uncertainty of order 1.4×10^{-9}. If, however, the CGPM decides that u be given an exact value, then it is likely that an electrical representation like the 1990 unit would still be needed, although the uncertainty of that representation in the SI would be of the order 1.4×10^{-9}, much smaller than at present. If the kilogram is defined to make u exact, the atomic mass unit would still be used, because it would still be useful to express atomic mass in units where the atomic weight has a value near the number of nucleons. The effect of the CGPM choosing h over u is that one non–SI system (the 1990 representation) would be eliminated.

Quantum physics is the basis of most modern day metrology. The Planck constant h is important to quantum physics, as c is to relativity and e is to electromagnetic theory. In practical terms, when converting X-ray frequencies into electronvolts, one must add an uncertainty in today's SI. An uncertainty would still be needed if u is exact, but by choosing h no added uncertainty is necessary. In fact, if e, h, k, and N_A are made to have exact values, then the following are also exact: the Faraday constant F, the magnetic flux quantum ϕ_0, the Josephson constant K_{J}, the von Klitzing constant R_{K}, the electronvolt in Joules eV, the molar gas constant R, and the Stefan–Boltzmann constant σ.

It is probably true that a definition of the kilogram based on an exact value for u is easier to explain than one based on an exact value for h; but the metrology community has worked hard to base much of our measurement system on quantum effects, so it seems reasonable to explain the importance of the Planck constant as part of any description of the SI. Choosing u makes sense if you look only at mass, but looking at the entire SI strongly favors choosing h. To explain the "set of constants SI" to a nonscientist, I would describe or define the kilogram as follows:

> The kilogram is the mass of 6.0221415×10^{26} idealized atoms, each of these atoms having a mass such that the Planck constant, the most important constant in quantum mechanics, has the specified value of $6.6260693 \times 10^{-34}$ J · s. Such atoms have a mass very close (within an uncertainty of 1.4 ng/g) to 1/12th the mass of ^{12}C. This means that a mole of ^{12}C weighs $12 \times (1 \pm 1.4 \times 10^{-9})$ g.

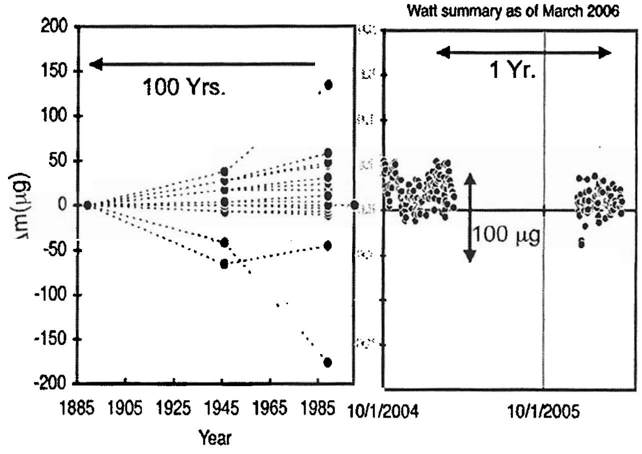

Fig. 1. Left graph plots comparisons of various nations' primary kilogram Pt–Ir standards against the International Prototype Kilogram standard made in 1889, 1946, and 1989. The right graph plots a Pt–Ir mass in vacuum measured using the NIST watt balance apparatus in terms of the Planck constant. The vertical scales are the same in relative magnitude, but the horizontal scales differ by a factor of one hundred.

VII. WEIGHING THE ELECTRON

The most accurate methods with which to measure the Planck constant, the electron mass, the electron charge, and the Avogadro constant are the XRCD method and the watt-balance method. Peter Becker will report the progress and future of the XRCD method, and earlier results have been published [5]. Several papers presented at the 2006 Conference on Precision Electromagnetic Measurements (CPEM06) discussed the prospects for the watt balance. A third method uses the current of an ion beam to count the number of ions deposited on a balance [6]. All three methods measure a macroscopic mass, the kilogram, in terms of quantum effects in nature.

The watt balance experiment uses the quantum Hall resistance and the Josephson volt in an experiment that equates electric (quantum) power to mechanical (SI) power. This is accomplished using a balance that compares the force on a coil in a magnetic field to the force of gravity on a kilogram mass. A clever technique suggested by Bryan Kibble [7] allows us to measure the magnetic field and geometrical effects by measuring a voltage in the moving coil and its velocity. These measurements can be used to calculate the Planck constant (see [8, eq. 6]) and many other fundamental constants, including the SI mass of the electron.

There are five active watt balance experiments in progress in the world today. Four are at the national measurement institutes of Great Britain [9], USA [10], Switzerland [11], and France [12], and one is at the International Bureau of Weights and Measures [13]. These five watt balance experiments each involve significant differences in design from the others, and I refer the reader to their respective reports. The most accurate measurement to date was reported by the National Institute of Standards and Technology (NIST), Gaithersburg, MD [8], [10], but others may soon report more accurate results. Results presented here and in the near-future CPEMs will likely determine if the redefinition of the SI will occur in 2011.

Fig. 1 shows two graphs of Pt–Ir artifact kilograms. The left graph shows all three measurements of the International Prototype Kilogram $m(\mathrm{K})$ against sister copies distributed to countries who signed the Treaty of the Meter in 1875. The right graph shows 175 measurements of a similar Pt–Ir kilogram measured in vacuum in terms of the Planck constant using the NIST watt balance. Both have about the same vertical scale, but the horizontal (time) scale is about 100 times expanded in the right graph. This figure says that we can, at present, measure mass standards against constants of nature with similar accuracy to the artifact's stability. It is time to make the change to a kilogram standard based on fundamental constants; being able to calibrate your standard only once every 50 years is not acceptable. It has restricted mass metrology research for years. Once we put this old artifact standard to rest, we will likely see increased activity in high-accuracy measurements, just as we saw increased activity in length measurements when we adopted the constant c as the definition for the meter.

VIII. CONCLUSION

There may not yet be total agreement on how we define the improved SI, but it appears that almost everyone agrees it is good to replace the artifact standards with quantum-based constants as soon as reliable experiments are available. As the CIPM has recommended, the year 2011 is a reasonable target date. I believe that the best way to redefine the SI is to define the constants in a simple statement and include those seven constants previously listed, but whichever way we decide to define the new SI, science and industry will be much better off with a system based on constants of nature. It will take the entire scientific community to communicate these changes, so it is important that we start the process now.

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I. Mills, P. Mohr, T. Quinn, and B. Taylor are major contributors to the SI redefinition discussion in this paper, and similarly, R. Steiner, D. Newell, and R. Liu are major contributors to the watt balance work.

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The highlights of his work are chronologically listed as follows: His thesis result is still among the most accurate tests of Coulomb's law and is pictured in J. D. Jackson's "Classical Electrodynamics" textbook. He worked with Prof. Stuart Crampton at Williams College, Williamstown, MA, on hydrogen maser research and measured the frequency shifts due to spin exchange collisions in hydrogen. He worked with Paul Olsen and William Phillips at the National Institute of Standards and Technology (NIST), Gaithersburg, MD, and accurately measured the dimensions of a precision solenoid to 0.1 ppm and then measured the proton gyromagnetic ratio in water, which led to one of the best values for the fine-structure constant. He worked with Marvin Cage at NIST and was involved in the detection of quantized states in the dissipation of quantum Hall devices carrying high current. He worked with the group in Saclay, France, and was involved in the detection of single-electron oscillations in small islands using an SET transistor as the detector. He worked with Richard Steiner and David Newell and measured the SI Watt using a precision balance and 0.03-ppm measurement of a velocity. (This Watt experiment is expected to help replace the last artifact in the SI system, the Kilogram, with an atomic definition.) He helped start a nanoforce calibration project at NIST. Recently, he has been working to provide a revised SI system with Ian Mills, Peter Mohr, Terry Quinn, and Barry Taylor, and on a new vacuum mass competence project with Zeina Jabbour. His research has involved testing Coulomb's law (setting photon rest mass limit), measuring fundamental constants (e.g., proton gyromagnetic ratio, fine-structure constant, Planck's constant), testing the QHE, and SET experiments. He has published over 50 papers in the field of precision measurements.

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