Dissipation Factors of Fused-Silica Capacitors in the Audio Frequency Range

Yicheng Wang, *Senior Member, IEEE*, Andrew D. Koffman, *Member, IEEE*, and Gerald J. FitzPatrick, *Member, IEEE*

***Abstract*—We describe dissipation factor measurements of 10-pF fused-silica capacitance standards from 50 Hz to 20 kHz using a toroidal cross capacitor and a 10-pF nitrogen-filled capacitor as the references. The relative combined standard uncertainties are 0.56×10^{-6}, 0.16×10^{-6}, and 0.26×10^{-6} at 100 Hz, 1 kHz, and 10 kHz, respectively.**

***Index Terms*—Capacitance, cross capacitor, dielectric loss, dissipation factor, Farad, frequency dependence, fused-silica capacitor.**

I. Introduction

WE HAVE recently reported progress on determining the frequency dependence of capacitance standards in the audio frequency range [1]. This effort is in part a response to industrial needs. Recently, ultraprecision multifrequency (from 50 Hz to 20 kHz) capacitance bridges have become commercially available, and secondary calibration laboratories have started using this new type of bridge for their impedance calibrations. Another closely related calibration need is the determination of the dissipation factors of capacitance standards, which are required not only for calibrations of capacitance bridges but also for inductance–capacitance–resistance meters and network analyzers. The new measurement capabilities of dissipation factors will also enable improved traceability of energy and power measurements and better characterization of dielectric materials.

The dissipation factors of capacitors have previously been studied at the National Institute of Standards and Technology (NIST) for various applications. Astin [2] studied the loss mechanisms of air capacitors at 60, 200, and 1000 Hz, achieving uncertainties as low as 0.5×10^{-6}. Shields [3] established a dissipation factor standard using a 0.5-pF toroidal cross capacitor $C_{0.5}$ with an estimated uncertainty of 0.02×10^{-6} at 1592 Hz. So and Shields [4] used a variable parallel-plate guard-ring capacitor as the reference for dissipation factor measurements, achieving uncertainties as low as 0.01×10^{-6} at 1592 Hz. However, these previous studies have yet to be extended to other frequencies in the audio frequency range. Absolute determinations of the dissipation factor of capacitors have also been carried out in other National Metrology Institutes. Inglis [5] performed a thorough study of electrode surface film effects on the frequency dependence and dissipation factor of parallel-plate capacitors in the frequency range from 11 Hz to 52 kHz. He found that the dissipation factor of a well-cleaned parallel-plate capacitor with a 1-mm vacuum gap is less than 1.5×10^{-7} over the frequency range. His work was motivated by the need of a new electrostatic ac–dc transfer standard, and unfortunately, his dissipation factor measurements have apparently not been transferred to other capacitance standards. With a variable parallel-plate guard-ring capacitor as the reference, Eklund [6] has determined the dissipation factors of 100-pF capacitors at 1, 4, and 10 kHz. Using a combination of equivalent circuit modeling for capacitor and ac resistor, a programmable two-channel ac voltage source, and a sampling voltmeter, Ramm and Moser [7] have developed a multifrequency method for simultaneously determining the dissipation factor of capacitors and the time constant of resistors below a few kilohertz with uncertainties of 6×10^{-7}.

II. Kronig–Kramers Relations

For a simple parallel-plate capacitor, the dominant loss mechanisms include the dielectric loss between the electrodes and the resistance of the electrodes and their leads. The conductivity of a dielectric can be written as the sum of two components: $\sigma_o + \omega\varepsilon''(\omega)$, where the first term results from the dc conductivity, and the second term is due to dielectric relaxation with ε'' being the imaginary part of the dielectric constant. When the dc conductivity is negligible, as is the case for fused-silica capacitors, the dissipation factor is

$$\tan\delta = \frac{G}{\omega C} = \frac{\varepsilon''}{\varepsilon'} \tag{1}$$

where C is the capacitance, G is the conductance of the dielectric, and ε' is the real part of the dielectric constant. ε'' and ε' are related via the Kronig–Kramers relations as

$$\varepsilon'(\omega) - \varepsilon_\infty = \frac{2}{\pi}\int_0^\infty \frac{\varepsilon''(u)u^2}{\omega^2 - u^2} d\ln u. \tag{2}$$

Approximating the integral factor $u^2/(\omega^2 - u^2)$ by the unit-step function [8], we obtain

$$\varepsilon'(\omega) - \varepsilon_\infty = \frac{2}{\pi}\int_\omega^\infty \varepsilon''(u) d\ln u. \tag{3}$$

Manuscript received July 11, 2006; revised November 1, 2006.
The authors are with the Quantum Electrical Metrology Division, National Institute of Standards and Technology, Gaithersburg, MD 20899 USA.
Digital Object Identifier 10.1109/TIM.2007.891144

This derivation effectively assumes that the distribution of the dielectric relaxation times is very broad, leading to relatively flat curves of $\varepsilon''(\omega)$ and $\varepsilon'(\omega)$. Differentiating (3), we have

$$\tan\delta = -\frac{\pi}{2}\frac{d\varepsilon'(\omega)}{\varepsilon' d\ln\omega}. \tag{4}$$

Equation (4) is useful for estimating the dissipation factor of a capacitance standard from its frequency dependence of capacitance. For example, the relative capacitance change due to electrode surface films of a parallel-plate capacitor tends to follow a $\ln(\omega)$ law over the audio frequency range [5]. Using this frequency dependence for $\varepsilon'(\omega)$ in (4), we conclude that the dissipation factor $\tan\delta$ of the capacitor is a constant in the frequency range, in agreement with the experimental observations.

III. Experiment

The ultimate reference for dissipation factor measurements at NIST is the toroidal cross capacitor $C_{0.5}$, which is made of stainless steel and sealed in a vacuum housing [3]. It has been shown that the net contributions of thin dielectric films on the electrodes of such a cross capacitor to the dissipation factors of the two cross capacitances are negligible to first order. The toroidal arrangement also contains another cylindrical 10-pF capacitance C_{10} between two of the four active electrodes. The electrode separation of C_{10} is about 3 mm. Since all electrodes were made from the same material and were finished and cleaned in the same manner, Shields was able to determine that C_{10} has a dissipation factor of less than 0.02×10^{-6} at 1592 Hz with comparable uncertainty. The dissipation factor due to dielectric films is inversely proportional to the electrode separation and is less than 0.15×10^{-6} in the audio frequency range when the electrodes are well cleaned and their separation is 1 mm or more [3], [5]. Comparing C_{10} with an identically made capacitor shows that the difference of their dissipation factors is within the detection limit in the audio frequency range. Comparing with another 10-pF nitrogen-filled cylindrical capacitor whose frequency dependence of capacitance had been determined earlier with respect to a 1-pF cross capacitor shows that the frequency dependence of C_{10} is no more than 0.2×10^{-6} per decade change in frequency. Using this estimate of frequency dependence in (4), we conclude that the dissipation factor of C_{10} is less than 0.14×10^{-6} in the audio frequency range. Simple substitution techniques are employed to measure dissipation factors of 10-pF fused-silica capacitors with respect to C_{10} using ac bridges that have been described previously [1].

In principle, the calculable capacitor could also serve as the reference for dissipation factor measurements at various frequencies. However, this approach is very tedious to cover the entire audio frequency range. The NIST calculable capacitor involves difference measurements between 0.2 and 0.7 pF; calibration of the bridge transformer used for comparison with other standards involves multiple steps at each frequency. The ac bridge system for the NIST calculable capacitor has been calibrated only for in-phase capacitance measurements at 1000 and 1592 Hz. Nevertheless, the bridge transformer errors at 1592 Hz appear stable within a few parts in 10^9 over a period of more than 20 years, and the twice annual calculable capacitor measurements at NIST have recorded relative variations of dissipation factors of several fused-silica capacitors. Typical results will also be discussed in the next section.

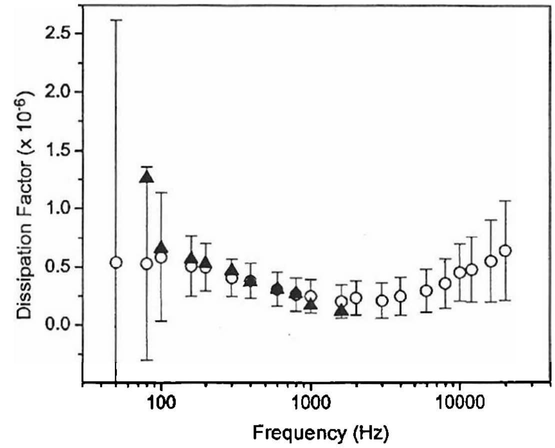


Fig. 1. Measured dissipation factor of C_{112} as a function of frequency (open circles), with 1σ uncertainty bars, and calculated dissipation factor (solid triangles).

TABLE I
CONTRIBUTION OF COMPONENT UNCERTAINTIES TO THE TOTAL UNCERTAINTY AT FOUR REPRESENTATIVE FREQUENCIES FOR C_{112}

Source of uncertainty	Relative standard uncertainty ($\times 10^{-6}$)			
	100 Hz	400 Hz	1 kHz	10 kHz
Type A	0.53	0.05	0.03	0.03
Reference capacitor C_{10}	0.14	0.14	0.14	0.14
Contact resistance of C_{112}	0.01	0.01	0.02	0.2
Bridge linearity errors	0.05	0.05	0.05	0.05
standard uncertainty	0.56	0.16	0.16	0.26

IV. Results and Uncertainty Analysis

Shown in Fig. 1 is the measured dissipation factor of a 10-pF fused-silica transfer standard C_{112} as a function of frequency from 50 Hz to 20 kHz. The main sources of uncertainties for the measurements are listed in Table I for four representative

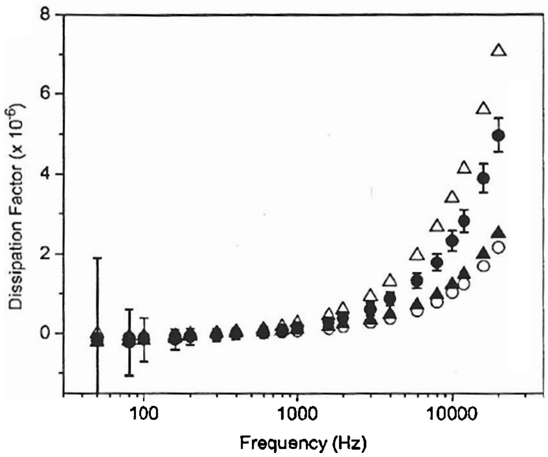


Fig. 2. Measured dissipation factors of four commercial 10-pF capacitance standards (s/n 1421: Open triangles, s/n 1422: Open circles, s/n 1423: Solid triangles, s/n 1424: Solid circles) as a function of frequency, with 1σ uncertainty bars.

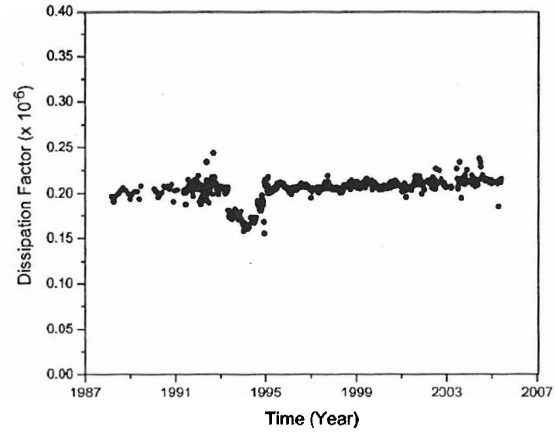


Fig. 3. Dissipation factor of C_{112} at 1592 Hz as a function of time from 1988 to present.

frequencies. The Type A uncertainty, which is directly linked to the signal-to-noise ratio of the ac bridge systems and the stabilities of the standards, dominates at low frequencies. The reference standard C_{10} is a four-terminal-pair capacitor, and its loss due to leads and contacts is negligible in the frequency range. However, C_{112} is a three-terminal capacitor, and its lead resistance is the dominant loss mechanism at high frequencies. In the frequency range from 300 Hz to 6 kHz, the uncertainties of the reference standard dominate. The relative combined standard uncertainties are shown in Fig. 1 together with the dissipation factor data.

Also shown in Fig. 1 is the estimated dissipation factor below 1592 Hz using (4) and the frequency dependence of capacitance of C_{112} measured earlier [1]. The comparison is restricted to the low frequency region where the dominant source of frequency dependence results from dielectric relaxation. The leads effect becomes significant above 1592 Hz, and we have not attempted to separate the contributions from the two sources.

Shown in Fig. 2 are the measured dissipation factors of four commercial 10-pF fused-silica capacitance standards as a function of frequency from 50 Hz to 20 kHz. The dissipation factors of these standards are uniformly within a few parts in 10^7 below 1592 Hz; they increase with frequency to a few parts in 10^6 at 20 kHz with comparable variations from one standard to another. This increase is partly due to the series lead and electrode resistance. However, if we assume the combined series resistance of 0.1 Ω and the combined stray capacitance of 100 pF, the loss is no more than 1×10^{-6} at 16 kHz. The observed increase and variations of dissipation factors could be partly due to the dielectric loss in the fused silica. The dielectric properties of fused silica have been known to be strongly influenced by the residual hydroxyl content. Dielectric loss peaks as large as several parts in 10^4 have been observed in the audio frequency range in fused-silica samples at low temperatures (below 4.2 K) and were attributed to a distribution of dipolar two-level systems of hydroxyl anions having energy splittings comparable to the thermal energies of the experiments [9]. Such dipolar systems may also make observable contributions

TABLE II
DISSIPATION FACTORS OF FOUR NIST 10-pF CAPACITANCE STANDARDS MEASURED IN 1974, 1990, AND 2006

Standard	Dissipation factor ($\times 10^{-6}$)		
	1974	1990	2006
C_{109}	0.74	0.75	0.77
C_{114}	0.33	0.37	0.39
C_{124}	0.35	0.35	0.35
C_{127}	0.78	0.65	0.65

to the dielectric loss of the room-temperature-fused silica in the present case.

Variations of the dissipation factor of C_{112} relative to the calculable capacitor at 1592 Hz are shown in Fig. 3 as a function of time from 1988 to present. As can be seen in the figure, the dissipation factor remains stable within 5×10^{-8} over the period. Comparisons of the presently measured dissipation factors of four 10-pF fused-silica standards with those measured by So [10] more than 30 years ago for these same standards are shown in Table II. The largest change is seen with C_{127}, which is about 1.3×10^{-7}, while the dissipation factor of C_{124} shows no observable change.

V. CONCLUSION

References have been established for measuring the dissipation factor of 10-pF capacitance standards from 50 Hz to 20 kHz. Measurements of both NIST-fabricated fused-silica capacitors and similar commercial standards have shown that the dissipation factors of these standards are typically within a few parts in 10^7 below 1592 Hz; their dissipation factors may increase with frequency to a few parts in 10^6 at 20 kHz due to their lead and electrode resistances and also possibly due to the dielectric loss resulting from the residual hydroxyl content in the bulk fused silica. The dissipation factors of several typical fused-silica capacitors have remained stable within 2×10^{-7} over 30 years.

REFERENCES

[1] Y. Wang, "Frequency dependence of capacitance standards," *Rev. Sci. Instrum.*, vol. 74, no. 9, pp. 4212–4215, 2003.

[2] A. V. Astin, "Nature of energy losses in air capacitors at low frequencies," *J. Res. NBS*, vol. 22, pp. 673–695, Jun. 1939.

[3] J. Q. Shields, "Absolute measurement of loss angle using a toroidal cross capacitor," *IEEE Trans. Instrum. Meas.*, vol. IM-27, no. 4, pp. 464–466, Dec. 1978.

[4] E. So and J. Q. Shields, "Losses in electrode surface films in gas dielectric capacitors," *IEEE Trans. Instrum. Meas.*, vol. IM-28, no. 4, pp. 279–284, Dec. 1979.

[5] B. D. Inglis, "Frequency dependence of electrode surface effects in parallel-plate capacitors," *IEEE Trans. Instrum. Meas.*, vol. IM-24, no. 2, pp. 133–150, Jun. 1975.

[6] G. Eklund, "Frequency dependence of the dissipation factor of capacitors up to 10 kHz," in *Proc. CPEM Conf. Dig.*, 2004, pp. 95–96.

[7] G. Ramm and H. Moser, "New multi-frequency method for the determination of the dissipation factor of capacitors and of the time constant of resistors," *IEEE Trans. Instrum. Meas.*, vol. 54, no. 2, pp. 521–524, Apr. 2005.

[8] C. J. F. Bottcher and P. Bordewijk, *Theory of Electric Polarization*. Amsterdam, The Netherlands: Elsevier, 1978.

[9] G. Frossati, J. L. G. Gilchrist, J. C. Lasjaunias, and W. Meyer, "Spectrum of low-energy dipolar states in hydrated vitreous silica," *J. Phys. C, Solid State Phys.*, vol. 10, no. 18, pp. L515–L519, Sep. 1977.

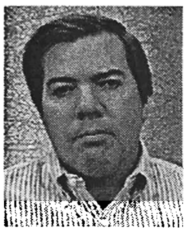
[10] E. So, "Losses in electrode surface films in gas dielectric capacitors," Ph.D. dissertation, George Washington Univ., Washington, DC, 1973.

Yicheng Wang (M'96–SM'03) received the Ph.D. degree in atomic physics from the College of William and Mary, Williamsburg, VA, in 1987.

He was a Post-Doctoral Associate with the College of William and Mary until 1989. From 1990 to 1996, he was a Research Staff with the Notre Dame Radiation Laboratory, U.S. Department of Energy, Notre Dame, IN. In 1996, he joined the National Institute of Standards and Technology, Gaithersburg, MD, where he has been working in the Quantum Electrical Metrology Division and is now the Leader of the Farad and Impedance Metrology Project. His current work focuses on precision ac measurements and impedance standards.

Andrew D. Koffman (S'87–M'88) received the B.S degree from the University of Maryland, Colleg Park, in 1988 and the M.S. degree from Vanderbil University, Nashville, TN, in 1990, both in electrica engineering.

In 1990, he joined the Quantum Electrica Metrology Division (formerly Electricity Division) National Institute of Standards and Technology Gaithersburg, MD. He currently works in the area o developing ac impedance measurement systems an impedance calibration services.



Gerald J. FitzPatrick (M'80–S'85–M'88) was bor in Greenwich, CT, in 1955. He received the B.S degree in physics from Rutgers University, Camden NJ, the M.S. degree in electrical engineering fron the New Jersey Institute of Technology, Newark, an the Ph.D. degree in electrical engineering from th State University of New York at Buffalo in 1988.

From 1979 to 1984, he was with Exxon Researc and Engineering Company, Clinton, NJ, working i research on the dielectric properties of high-voltag insulation. In 1988, he joined the National Institut of Standards and Technology, Gaithersburg, MD, where he has worked in th areas of precision high-voltage ac and impulse measurements. He currentl leads the Applied Electrical Metrology Group, Quantum Electrical Metrolog Division. His other research interests include optical systems for current an voltage measurements, and ac metrology.