On-Wafer Measurement of Transistor Noise Parameters at NIST

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***Abstract*—The National Institute of Standards and Technology has developed the capability to measure noise parameters on a wafer in the 1–12.4-GHz range. We describe the measurement method and the uncertainty analysis and present results of measurements on a highly reflective transistor. Typical standard uncertainties are within the range of 20–25 K in $T_{\min}$, which is the minimum transistor noise temperature, and about 0.03 in the magnitude of Γ_{opt}, which is the reflection coefficient for which $T_{\min}$ occurs.**

***Index Terms*—Noise measurement, on-wafer measurement, transistor noise parameters, uncertainty analysis.**

I. INTRODUCTION

MEASUREMENT of the noise parameters of a transistor on a wafer is both challenging and important. Factors that make it challenging arise due to the on-wafer environment and the properties of the transistor itself. Measurements at an on-wafer reference plane require on-wafer vector network analyzer (VNA) calibrations, which are less accurate than typical coaxial VNA calibrations. Similarly, the noise temperature of an input nonambient noise source must be measured or corrected to an on-wafer reference plane. Losses in the probes can restrict the range of reflection coefficients available for input states. The device itself may have a very low noise figure, significantly lower than that of a packaged amplifier, which has added noise from its embedding circuitry. Often the device is highly reflective; the scattering parameters S_{11} and S_{22}, as well as Γ_{opt}, can have magnitudes in excess of 0.5 and often above 0.9. There are also practical difficulties due to possible fragility of devices, difficulty achieving and maintaining repeatable contact between probe and contact pads, wear of the contact pads, and possible exposure to outside microwave radiation.

At the same time, knowledge of a bare transistor's noise parameters, either from measurements or from the predictions of a model (based on measurement results), is of great practical importance, due to the ubiquitous use of transistors in modern electronics and their ever smaller size and lower noise. It is necessary to know a transistor's noise parameters in order to design a circuit or system containing the transistor, both to predict and to optimize the circuit's noise performance.

Commercial systems for the measurements have been in widespread use for some time, but the connection to fundamental standards has been rather tenuous. In principle, the measurements can be traceable to national standards through the calibration of the diode noise source typically used and through the VNA calibration. However, the intervening steps between fundamental standards and the noise parameters that are measured are sufficiently numerous and complicated, and replete with opportunities for error, that there is a real need for a means of verifying on-wafer noise-parameter measurements, as would be provided by comparison to measurements at a National Measurement Institute (NMI). At least two NMIs can perform noise-parameter measurements on packaged amplifiers [1]–[3], but there is no corresponding capability for transistors on a wafer.

The U.S. National Institute of Standards and Technology (NIST) has now developed the ability to measure the noise parameters of a transistor on a wafer and has demonstrated that ability in measurements on an n-channel metal–oxide–semiconductor (NMOS) device with 0.12-μm gate length [4]. This paper presents our measurement method and uncertainty analysis, which were not given in [4]. The next section presents the theoretical framework and uncertainty analysis. Section III describes the measurements and shows results, and Section IV contains conclusions. A shorter version of this paper can be found in [5].

II. THEORETICAL FRAMEWORK AND UNCERTAINTIES

A. Theory

The formalism is essentially the same as that used for amplifier noise-parameter measurements [6]. It is based on the wave representation of the noise matrix [7]. A linear two-port device, such as a transistor or amplifier, can be represented by

$$\boldsymbol{b} = \boldsymbol{S}\boldsymbol{a} + \boldsymbol{c} \tag{1}$$

where $\boldsymbol{S}$ is the usual scattering matrix, $\boldsymbol{a}$ and $\boldsymbol{b}$ are 2-D vectors whose components are the amplitudes of the incident $(\boldsymbol{a})$ and emerging $(\boldsymbol{b})$ waves at the input (the gate, plane 1) and output (the drain, plane 2) of the device, and $\boldsymbol{c}$ is the 2-D vector whose components are the wave amplitudes due to the intrinsic noise of the two-port device. Our normalization is such that the magnitude of the wave amplitude squared gives the spectral power density.

The intrinsic noise correlation matrix is defined by

$$\boldsymbol{N}_{ij} \equiv \left\langle c_i c_j^* \right\rangle \tag{2}$$

where the brackets indicate an average over time or ensemble (assumed to be equivalent). For convenience, we define and

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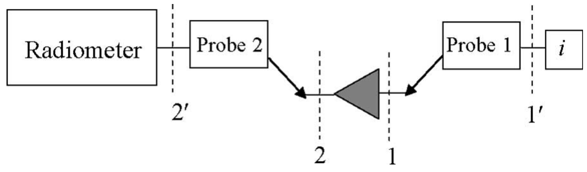

Fig. 1. Configuration for forward measurements.

work with X parameters, which are elements of $\boldsymbol{N}$ scaled, so that they refer to the input

$$k_B X_1 \equiv \left\langle |c_1|^2 \right\rangle \quad k_B X_2 \equiv \left\langle |c_2/S_{21}|^2 \right\rangle$$
$$k_B X_{12} \equiv \left\langle c_1 \left(c_2/S_{21}\right)^* \right\rangle \tag{3}$$

where k_B is Boltzmann's constant. The relationship of the Xs to the usual IEEE parameters [8] can be easily inferred from the relationship of the elements of the noise correlation matrix to the IEEE parameters, as given in [7], and we do not reproduce it here. For reference here, we note that the form of the IEEE parameters that we use is

$$T_e = T_{\min} + t \frac{|\Gamma_{\mathrm{opt}} - \Gamma_G|^2}{|1 + \Gamma_{\mathrm{opt}}|^2 \left(1 - |\Gamma_G|^2\right)} \tag{4}$$

where T_e is the effective input noise temperature, Γ_G is the reflection coefficient of the input termination (the "generator"), and the noise parameters are $T_{e,\min}$, t, and the complex reflection coefficient Γ_{opt}. It is common to write t in terms of the noise resistance R_n, i.e., $t = 4R_n T_0/Z_0$, where $T_0 = 290$ K and Z_0 is the reference impedance, which is taken to be 50 Ω.

Fig. 1 indicates the location of relevant reference planes. In terms of the Xs, the noise temperature at the device output (drain), i.e., the on-wafer reference plane 2, for an input termination i is given by

$$T_{2,i} = \frac{|S_{21}|^2}{\left(1 - |\Gamma_{2,i}|^2\right)} \left\{ \frac{\left(1 - |\Gamma_{1,i}|^2\right)}{|1 - \Gamma_{1,i} S_{11}|^2} T_{i,1} \right.$$
$$\left. + \left| \frac{\Gamma_{1,i}}{1 - \Gamma_{1,i} S_{11}} \right|^2 X_1 + X_2 + 2\mathrm{Re}\left[\frac{\Gamma_{1,i} X_{12}}{1 - \Gamma_{1,i} S_{11}} \right] \right\} \tag{5}$$

where $\Gamma_{1,i}$ and $\Gamma_{2,i}$ are the reflection coefficients looking to the right at planes 1 and 2, with termination i connected at plane $1'$. In addition to the forward configuration shown in Fig. 1, we also measure the reverse configuration, in which planes 1 and 2 are interchanged, so that we measure the noise emerging from the input (gate), with the output terminated in Γ_2, which is typically but not necessarily a matched load. For the reverse configuration, the noise temperature at the measurement plane (plane 1, on the wafer) is given by

$$T_{1,i} = \frac{1}{\left(1 - |\Gamma_{1,i}|^2\right)} \left\{ \frac{|S_{12}|^2 \left(1 - |\Gamma_{2,i}|^2\right)}{|1 - \Gamma_{2,i} S_{22}|^2} T_{2,i} + \left| \frac{S_{12} S_{21} \Gamma_{2,i}}{1 - \Gamma_{2,i} S_{22}} \right|^2 X_2 \right.$$
$$\left. + X_1 + 2\mathrm{Re}\left[\frac{S_{12} S_{21} \Gamma_{2,i} X_{12}^*}{1 - \Gamma_{2,i} S_{22}} \right] \right\}. \tag{6}$$

A set of input terminations i is connected in succession at plane $1'$. For each termination, the reflection coefficients $\Gamma_{1,i}$ and $\Gamma_{2,i}$ are measured, as is the noise temperature at plane 1 for any nonambient input termination. The output noise temperature $T_{2,i}$ (or $T_{1,i}$ for the reverse configuration) is measured for each termination i, and a weighted least squares fit is performed to (5) and (6), with the fitting parameters taken to be the four X parameters $(X_1, X_2, \mathrm{Re}X_{12},$ and $\mathrm{Im}X_{12})$ and $G_0 \equiv |S_{21}|^2$. S-parameters other than $|S_{21}|^2$ are taken from VNA measurements.

B. Uncertainty Analysis

Type-A uncertainties in the X parameters and G_0 are given by the square roots of the diagonal elements of the covariance matrix determined in the fitting procedure

$$u_A(X_i) = \sqrt{V_{ii}(X)} \tag{7}$$

where X_i represents any of the five fitting parameters (Xs and G_0), and $V_{ij}(X)$ is the covariance matrix of the X parameters. A similar expression holds for the IEEE parameters, but the fit is done in terms of the X parameters, and therefore it does not determine the covariance matrix for the IEEE parameters. The covariance matrix for the IEEE parameters, which we denote as $V_{ij}(\mathrm{IEEE})$, can be computed from $V_{ij}(X)$ by means of the Jacobian matrix of the transformation between the two sets of parameters. If we use I_i to represent one of the five IEEE parameters (including G_0), then the type-A uncertainties in the IEEE parameters are given by

$$u_A(I_I) = \sqrt{V_{ii}(\mathrm{IEEE})}$$
$$V_{ij}(\mathrm{IEEE}) = \sum_{i',j'=1}^{5} \frac{\partial I_i}{\partial X_{i'}} \frac{\partial I_j}{\partial X_{j'}} V_{i'j'}(X). \tag{8}$$

Calculation of the elements of the Jacobian matrix $(\partial I_i / \partial X_{i'})$ is straightforward but tedious, and the results are lengthy and unenlightening. We do not reproduce them here.

Type-B uncertainties are estimated with an expanded and improved version of the Monte Carlo program developed for amplifier noise-parameter measurement uncertainties [6]. Because the program calculates both the X parameters and the IEEE parameters for each simulated measurement set, it directly computes the type-B uncertainties for both parameter sets, and we do not have to resort to the Jacobian contortions necessary for the type-A uncertainties.

An extension of the Monte Carlo program (beyond what was used for amplifiers) was necessary in order to deal with unphysical results, which can arise in the simulated measurements—or in actual measurements for that matter. Because the true values of some of the device properties may be very near the physical limit ($T_{\min}$ near zero, $|\Gamma_{\mathrm{opt}}|$ near one, etc.), small measurement errors can lead to unphysical values of the fitted parameters. This problem did not arise in the amplifier case because the amplifier properties were not close to physical limits. The way that we deal with this problem is to test the noise parameters from each simulated set of measurements to see whether they satisfy certain physical bounds. If any of the physical bounds is violated, that simulated measurement set is discarded and not used in the uncertainty computation. We also test the goodness of fit for each simulated measurement set, requiring that the χ^2 per degree of freedom be less than 1.5. If it is greater than 1.5, that simulated measurement set is discarded. These new features also allow us to estimate the likelihood of "bad" measurement results for a given set of underlying uncertainties, which is an issue that we intend to investigate further.

We have also improved the weighted fits in the analysis of the simulated data sets. In the amplifier work, we assigned a fractional uncertainty to the measurements of the output temperatures based only on whether or not the output temperature was near ambient; one value was used for temperatures near ambient, and a different value was used for all others. We now use a continuous variation with output noise temperature, which more accurately reflects the uncertainties in our actual measurements. We also account for the increase in the uncertainty in measuring $T_{2,i}$ for cases in which $|\Gamma_{2,i}|$ is near one, which did not occur in the amplifier work.

III. MEASUREMENTS

A. Setup and Procedure

We performed measurements with nine different input terminations, plus one reverse measurement with a matched load connected at plane $1'$. The nine forward states included one hot source (around 1100 K) and eight ambient temperature terminations chosen to produce adequate coverage of the complex plane. We made no formal effort to optimize the set of input states, relying instead on the uncertainty analysis to alert us if the set was deficient.

A two-tier VNA calibration was performed at the on-wafer reference planes 1 and 2, using a multiline TRL [9], [10] calibration set that was fabricated on the wafer. As a byproduct of the two-tier calibration, we obtained the S-parameters of probe 2, which are needed to determine the noise temperature at plane 2 from the measurements at plane $2'$. The calibrated VNA was used to measure the S-parameters of the transistor and the reflection coefficients $\Gamma_{1,i}$ and $\Gamma_{2,i}$ for each of the input terminations i. The noise measurements were performed on the NIST coaxial radiometer [11]. In addition to the output temperature $T_{2,i}$ (or, in the case of the reverse measurement, $T_{1,i}$) for each termination i, we also had to measure the on-wafer noise temperature $T_{1,i}$ for the hot input termination. The radiometer measurements were made at the coaxial plane $2'$, and the noise temperature at the on-wafer plane 2 was obtained by treating probe 2 as an adapter, with its available power ratio $\alpha_{2'2}$ computed from its S-parameters (measured in the on-wafer calibration process) and $\Gamma_{2,i}$. (For simplicity, we suppress the subscript i on $\alpha_{2'2}$.) Knowing $\alpha_{2'2}$, we computed the output noise temperature at plane 2 in terms of $T_{2',i}$ in the usual manner, from $T_{2'} = \alpha_{2'2}T_2 + (1 - \alpha_{2'2})T_a$, where T_a is the temperature of the probe, which is assumed to be ambient.

B. Results

Measurements were made on an NMOS transistor with 0.12-μm gate length, which was designed by RF Micro Devices and fabricated by IBM as part of the Kelvin Project. The device is very reflective, with a value of $|S_{11}|$ above 0.9 at 2 GHz and above 0.5 throughout the measurement range. More details of the device are contained in [4]. Measurements were performed at integer frequencies in the range of 2–12 GHz; however, the measurements at 5 GHz did not admit a good fit, and we discarded them. The results for the X parameters are shown in Fig. 2. All error bars correspond to the standard uncertainty. Since the X parameters all have dimensions of temperature and are all roughly the same order of magnitude, they can be plotted on the same graph. Two features of Fig. 2 warrant comment. The very small uncertainties on X_1 are due to the fact that the reverse measurement is very nearly a direct measurement of X_1; there is very little effect of the fitting process. The other point is that X_2 is the effective input noise temperature for $\Gamma_{1,i} = 0$, which is the matched case. Fig. 3 shows the results for G_0 in decibels. Since G_0 was also measured with the VNA, we plot both the noise and the VNA results. The good agreement between the two independent sets of measurements constitutes a partial check of the results. It is interesting to note that the small disagreement in the G_0 results at 7 GHz coincides with a suspicious low point in the X_2 results.

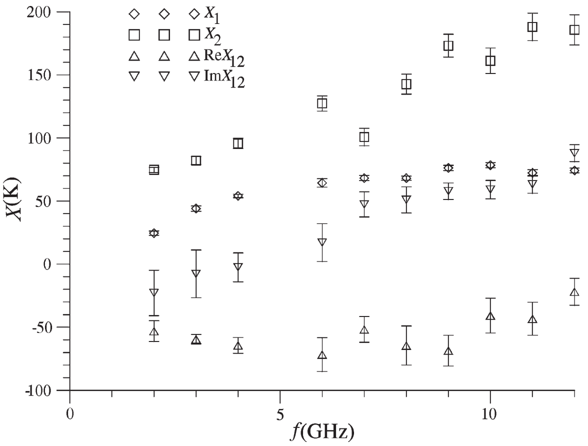

Fig. 2. Measurement results for X parameters.

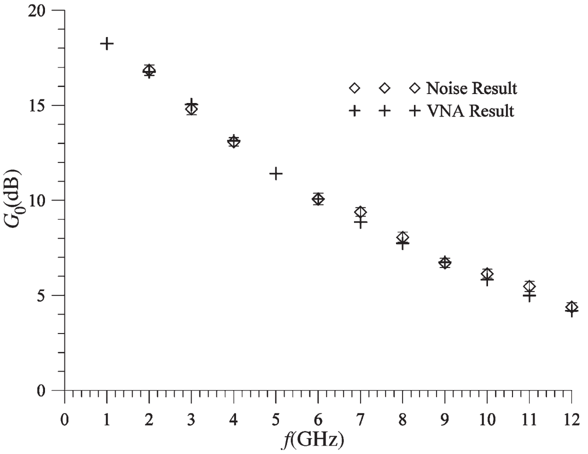

Fig. 3. Measurement results for G_0.

Because the IEEE parameters are more familiar to most readers, we also present the results for $T_{\min}$ and $|\Gamma_{\text{opt}}|$ in Figs. 4 and 5. In the interest of space, we do not show R_n or

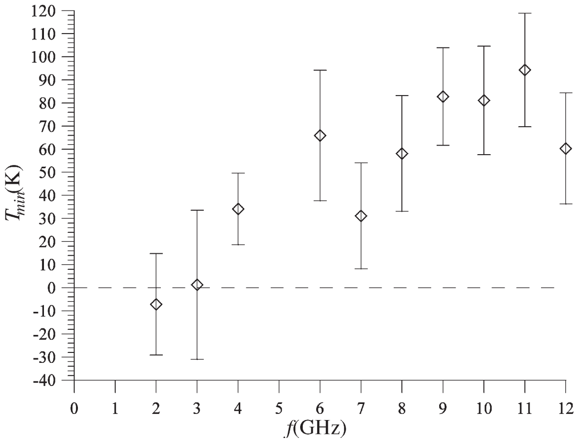

Fig. 4. Measured values of $T_{\min}$.

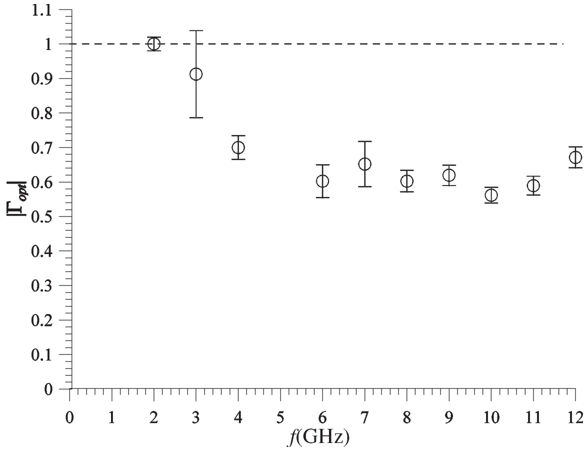

Fig. 5. Measured values for $|\Gamma_{\rm opt}|$.

the phase of $\Gamma_{\rm opt}$. The values of $T_{\min}$ and $|\Gamma_{\rm opt}|$ at the lower frequencies demonstrate the difficulties of the measurements, with $T_{\min}$ approaching zero and $|\Gamma_{\rm opt}|$ very near one. They also demonstrate the need for improved measurements.

IV. CONCLUSION

We have developed the capability to measure the noise parameters of transistors on a wafer and have demonstrated that ability in measurements on highly reflective CMOS devices at frequencies ranging from 2 to 12 GHz. Comparison to our measurements should provide valuable support for measurements performed in industry. The uncertainties in the measurements are sizeable, and, at low frequencies, the uncertainties in $T_{\min}$ and $(1 - |\Gamma_{\rm opt}|)$ are larger than the quantity itself, indicating a need for improved measurements. We are currently pursuing several ideas for such improvements.

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