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A measurement of propagation delay*

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Abstract
A measurement method and associated uncertainty analysis have been developed for the measurement of propagation or group delay in electrical transmission lines and optical fibres. The measurement method and uncertainty analysis were applied to measurements of optical fibre delay lines that introduce tens of microseconds of delay (1 km, 2 km and 4 km fibre lengths) and trombone lines that introduce sub-nanosecond delays. For the measurement method described here, uncertainties in the measured delay are as low as 90 fs (95% confidence interval) for a propagation delay of 660 ps and less than 200 ps (95% confidence interval) for a propagation delay of 20 μs.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Propagation delay arises because of the finite group velocity of signals in transmission media. For example, a pulse signal exiting a transmission line is delayed relative to the instant that it was launched by an amount equal to the physical length of the line divided by the group velocity of the propagating pulse. As data and telecommunications links and integrated circuit clock speed increase, knowing the delay of one signal with respect to another becomes increasingly important for proper operation and error-free communications.

There have been many methods demonstrated for determining the propagation delay of electrical and optical transmission lines [1–3]; each has its strengths and weaknesses. Many of these methods may be divided into two groups: pulse methods and phase shift methods [1]. We have developed a measurement method for the accurate measurement of pulse propagation delay in optical and electrical delay lines. The associated uncertainty analysis may be used to identify and reduce factors that contribute to measurement uncertainty. The method presented here is a pulse method that does not rely on a time interval counter or frequency counter to determine the propagation delay but instead relies on measuring a phase shift of an acquired sinewave to determine the propagation delay. As with all pulse methods, the delay determined is valid only for pulses similar to those used in the delay measurement because of frequency dependent propagation effects. This is particularly important to note for optical fibre delay lines. Due to chromatic dispersion, the delay will change with the centre wavelength of the optical pulse. An application of the reported measurement method is determining the chromatic dispersion of an optical fibre by using this method to accurately measure the change in delay as a function of the centre wavelength of the optical pulse propagated.

Propagation delay, as used here, is the difference in the occurrence instants of a reference level on the measured waveform with and without the delay line or transmission media.

2. Measurement method

A pulse generator, either optical or electrical, that is synchronized to a microwave synthesizer is used to launch a pulse down an optical or electrical transmission line (the delay line), respectively. The experimental arrangement is relatively simple and is depicted for the optical fibre delay line in figure 1. The sinewave signals from the microwave synthesizer are acquired by the oscilloscope. It is the phase of the acquired sinewaves that will be used to determine the delay (see section 3). The oscilloscope is triggered by the pulse that has propagated through the delay line. The effects of the transmission media on the properties of the trigger pulse must be considered (section 2.2).

The lowest uncertainties in the measurement of the delay of an electrical transmission line were obtained using the arrangement depicted in figure 2. The electrical pulses were generated by driving a comb generator with the microwave synthesizer. The output of the comb generator was split;

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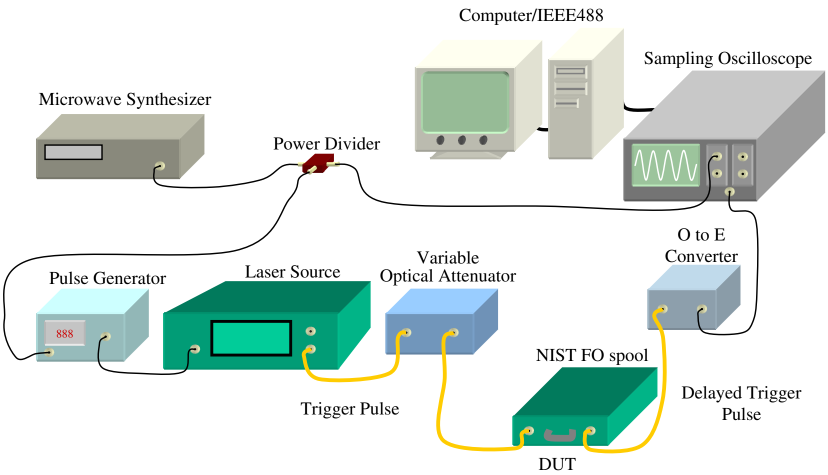

Figure 1. Optical fibre delay line measurement system.

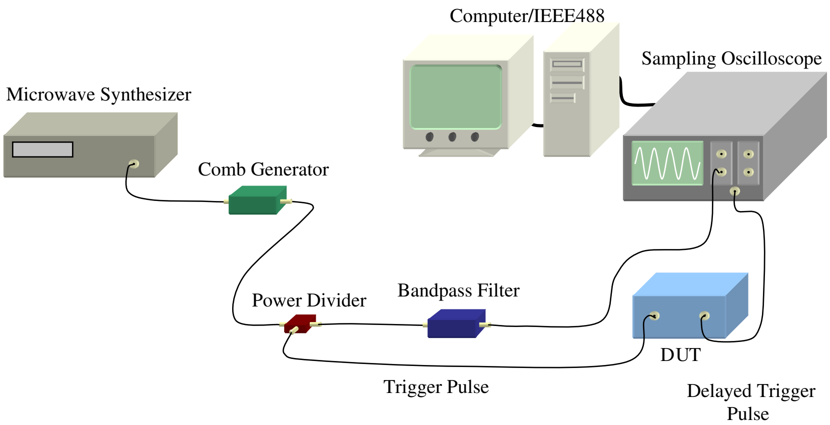

Figure 2. Electrical delay line measurement system.

one pulse was sent to the delay line and the other was bandpass-filtered to yield a spectrally pure sinewave. This sinewave was acquired by the oscilloscope and its phase determined. This arrangement also resulted in the lowest jitter, approximately 0.8 ps rms, that we have observed between pulses and sinewaves. This small amount of jitter is thought to be the limit of the trigger circuitry of the oscilloscope.

The uncertainty in these measurements was determined using the uncertainty analysis described in section 3. It will be shown that the frequency of the sinewaves employed will significantly impact the uncertainty with higher frequencies producing lower uncertainties. Of course, as the frequency is increased, the number of 2π phase shifts must also be accounted for. A lower frequency sinewave with a delay-induced phase shift less than 2π is used first to determine the delay with a coarse resolution. This measurement is followed by another measurement using a higher frequency sinewave, which may have multiple 2π phase shifts, to obtain higher resolution and lower uncertainty in the value of the delay.

2.1. Measurement procedure

The measurement procedure involves five steps. First, the amplitude of the trigger pulse is measured after propagating in the delay line under test (DUT). With the delay DUT inserted between the output of the pulse generator and the input channel (for example, CH1) of the oscilloscope, a set of trigger pulse waveforms is acquired and stored. The average amplitude of these trigger pulses is then determined and recorded. This value will be used for subsequent adjustment of the amplitude of the trigger pulse when the DUT is not inserted between the output of the pulse generator and CH1. Second, the output of the DUT is attached to the trigger input of the oscilloscope and the delayed pulses from the pulse generator are used to trigger the oscilloscope. The synthesizer is connected to CH2 of the oscilloscope, and sinewaves from the synthesizer are acquired and stored. Third, the delay line is disconnected from the oscilloscope trigger input and the pulse generator output connected directly to CH1. Trigger pulse waveforms are acquired and compared with the previously acquired trigger pulses. An adjustment to the trigger pulse amplitude is made as necessary so that the trigger pulse amplitude as observed on CH1 is the same as that when the delay line was inserted between CH1 and the pulse generator. Once the trigger pulse amplitude has been properly adjusted, a set of trigger pulse waveforms is acquired and stored. Fourth, the trigger pulse generator is connected to the oscilloscope trigger input. The sinewaves at CH2 are then acquired and stored. The last step is the data analysis. The phases of the acquired sinewaves are calculated using a three-parameter sinefit routine and the average phase change introduced by the delay line is calculated. The average delay is calculated from the average phase change.

To determine the propagation delay of a 4 km length of optical fibre (delay of approximately 20 µs), a coarse measurement of delay was obtained using a 20 kHz sinewave. This frequency was selected because the delay line would introduce a phase shift less than one cycle. The measurement was then refined by measuring the phase shift of a 10 MHz sinewave. The use of a higher frequency sinewave significantly lowered the uncertainty in the measurement results. When determining the propagation delay of a coaxial trombone line (delay range: 0 ps to 660 ps), the delay was first determined using a 1 GHz sinewave and then the estimate of the delay was refined using a 20 GHz sinewave. The range of frequencies that could be used is only limited by the bandwidth of the oscilloscope used to acquire them. The curve fitting routine used is independent of frequency.

Although this method uses the phase change of acquired sinewaves to determine the delay, the trigger pulse is the delayed signal. Therefore, this method does determine the group delay of the trigger pulse as opposed to the phase delay of the sinusoidal signal.

2.2. Consideration for propagation losses and distortion

When used to acquire the sinewaves, the oscilloscope is triggered by the pulse that is launched in the delay line. The delay line may be somewhat lossy or attenuating and the pulse amplitude may experience some reduction that is dependent on the length of the delay line being calibrated. When the delay line is removed from the trigger pulse propagation path, the pulse amplitude must be reduced in order to maintain a constant amplitude trigger signal for the oscilloscope. This was accomplished for the optical fibre delay lines tested here by increasing the selected attenuation in the variable optical attenuator and monitoring the pulse amplitude using the oscilloscope. The electrical delay lines examined here did not introduce significant attenuation of the pulse amplitude (within the uncertainty of the pulse amplitude measurement, ±1.5 mV).

In addition to loss, the pulse may become distorted as a result of propagating down the delay line. For optical fibre delay lines, modal, chromatic and polarization-mode dispersions are dependent on the spectral width of the launched pulses and careful control of the pulse temporal and spectral properties is needed to minimize this distortion. The full duration at half maximum (FDHM) of the optical pulses propagated in the fibre tested here was approximately 1.2 ns. This duration, with its attendant narrow spectral width, caused no observable distortion in the acquired waveforms. If shorter duration optical pulses were used, for example 50 ps, which are typically used in high-speed communications, the amount of pulse distortion that would have been observed would have caused additional uncertainty in our delay measurements. For electrical transmission lines, pulse distortion was also minimized by using pulses of appropriate durations.

3. Uncertainty in propagation delay

The delay, D, between two pulses is given by

$$D = t_{2,\mathrm{R}} - t_{1,\mathrm{R}} + \Delta D_T, \tag{1}$$

where $t_{1,\mathrm{R}}$ and $t_{2,\mathrm{R}}$ are the reference level instants for the first or reference pulse (P1) and the second or delayed pulse (P2) and ΔD_T is any temperature-induced change in delay. The reference levels are user defined, typically as a fraction (or percentage) of the pulse amplitude. A measurement instrument (oscilloscope, waveform recorder, etc) is used to acquire the pulse waveforms, P1 and P2, and the reference levels and reference level instants are determined from these acquired pulses using the algorithms defined in the IEEE Standard on Transitions, Pulses, and Related Waveforms (IEEE Std 181-2003) [4].

Another method of determining D uses the measurement instrument to acquire two spectrally pure sets of sinewaves. One set is acquired with the measurement instrument triggered by the undelayed or reference pulse, P1, and another set is acquired with the delayed pulse, P2, providing the trigger. The trigger level for the measurement instrument is set to a fixed level (for example, 125 mV for a pulse amplitude of 250 mV). If the amplitude of P1 is equal to P2, then the fixed level is equivalent to a common reference level (for example, 50% of pulse amplitude). The trigger instant, t_{tr}, is the instant when the trigger pulse attains the trigger threshold (reference level). When these conditions are satisfied, the reference level instants are equivalent to the trigger instant plus a constant, t_0, and may be described by

$$t_{i,\mathrm{R}} = t_{i,\mathrm{tr}} + t_0, \tag{2}$$

where the i subscript may be either 1 or 2, for P1 or P2. t_0 is either zero or a fixed measurement-instrument-dependent delay after the trigger instant. Since t_0 is fixed, it may be taken as zero.

The acquisition of the sinewaves uses a sampling process so that the sinewave may be described by

$$s[n\delta t] = A\sin(2\pi f n\delta t + \theta) + V_{\mathrm{offset}}, \tag{3}$$

where A is the amplitude of the sinewave, f is the frequency, n is the sample index, δt is the sampling interval, θ is the phase relative to $n = 0$ and V_{offset} is the voltage offset. The phase, θ, is established by the level of the first sampled element of the sinewave which occurs at $t_{i,\mathrm{R}}$ as defined in equation (2). From examining equation (3), it is apparent that the phase of the acquired sinewave may be described by

$$\theta_i = 2\pi f t_{i,\mathrm{R}}. \tag{4}$$

Solving equation (4) for $t_{i,\mathrm{R}}$ and replacing the 'i' subscript with a '1' to signify the undelayed or reference trigger instant and a '2' to signify the delayed trigger instant yields

$$t_{1,\mathrm{R}} = \frac{\theta_1}{2\pi f} \tag{5}$$

and

$$t_{2,\mathrm{R}} = \frac{\theta_2}{2\pi f}. \tag{6}$$

Substituting equations (5) and (6) in equation (1), the expression for the delay, D, may now be rewritten as

$$D = \frac{\theta_2 - \theta_1}{2\pi f} + \Delta D_T. \qquad (7)$$

The sets of measurements consist of M or N acquired sinewaves, and the delay is found using the difference in the average of the computed phases of the two sets of acquired sinewaves. For each acquired waveform, the signal and trigger connections are broken and remade so the effects of connector repeatability are automatically included in the observed measurement variation.

$$D = \frac{(1/M)\sum_{i=1}^{M}\theta_{2,i} - (1/N)\sum_{i=1}^{N}\theta_{1,i}}{2\pi f} + \Delta D_T = \frac{\bar{\theta}_2 - \bar{\theta}_1}{2\pi f} + \Delta D_T. \qquad (8)$$

The phase, θ_1, is computed using a sinefitting routine applied to the sinewaves acquired using the non-delayed trigger pulse. Similarly, the phase, θ_2, is determined by a sinefitting routine applied to the sinewaves acquired with the pulse delayed by the DUT. The last term, ΔD_T, is the effect of temperature on the delay introduced by the DUT.

From equation (8), the uncertainty in the delay D is given by

$$u_D = \left[\left(\frac{\partial D}{\partial\bar{\theta}_2}\right)^2 u_{\bar{\theta}_2}^2 + \left(\frac{\partial D}{\partial\bar{\theta}_1}\right)^2 u_{\bar{\theta}_1}^2 + \left(\frac{\partial D}{\partial f}\right)^2 u_f^2 + \left(\frac{\partial D}{\partial\Delta D_T}\right)^2 u_{\Delta D_T}^2\right]^{1/2}. \qquad (9)$$

The partial derivatives (sensitivity coefficients) [5] in (9) are

$$\frac{\partial D}{\partial\bar{\theta}_2} = \frac{1}{2\pi f}, \qquad \frac{\partial D}{\partial\bar{\theta}_1} = -\frac{1}{2\pi f}, \qquad \frac{\partial D}{\partial f} = -\frac{(\bar{\theta}_2 - \bar{\theta}_1)}{2\pi f^2}, \qquad \frac{\partial D}{\partial\Delta D_T} = 1. \qquad (10)$$

The various uncertainty components will be considered below.

4. Phase uncertainty

Since separate measurements are used to determine θ_1 and θ_2, each will have a separate uncertainty. However, the measurement method is the same for each and so the uncertainty analysis for one is applicable to both. Equation (4) defines the phase in terms of frequency and the reference level instant. The reference level instant is defined in equation (2). Substituting gives the phase in terms of frequency and the trigger instant, $t_{i,\text{tr}}$, and the fixed, measurement-instrument-dependent delay after the trigger instant, t_0:

$$\theta_1 = 2\pi f(t_{1,\text{tr}} + t_0), \qquad \theta_2 = 2\pi f(t_{2,\text{tr}} + t_0). \qquad (11)$$

From equation (11), the uncertainty associated with each set of phase measurements is given by

$$u_{\bar{\theta}_1} = \left[\left(\frac{\partial\theta_1}{\partial f}\right)^2 u_f^2 + \left(\frac{\partial\theta_1}{\partial t_{1,\text{tr}}}\right)^2 u_{t_\text{tr}}^2 + \left(\frac{\partial\theta_1}{\partial t_0}\right)^2 u_{t_0}^2 + \sigma_{\bar{\theta}_1}^2\right]^{1/2} \qquad (12)$$

and

$$u_{\bar{\theta}_2} = \left[\left(\frac{\partial\theta_2}{\partial f}\right)^2 u_f^2 + \left(\frac{\partial\theta_2}{\partial t_{2,\text{tr}}}\right)^2 u_{t_\text{tr}}^2 + \left(\frac{\partial\theta_2}{\partial t_0}\right)^2 u_{t_0}^2 + \sigma_{\bar{\theta}_2}^2\right]^{1/2}, \qquad (13)$$

where

$$\frac{\partial\theta_1}{\partial f} = 2\pi(t_{1,\text{tr}} + t_0), \qquad \frac{\partial\theta_1}{\partial t_{1,\text{tr}}} = 2\pi f, \qquad \frac{\partial\theta_1}{\partial t_0} = 2\pi f \qquad (14)$$

and

$$\frac{\partial\theta_2}{\partial f} = 2\pi(t_{2,\text{tr}} + t_0), \qquad \frac{\partial\theta_2}{\partial t_{2,\text{tr}}} = 2\pi f, \qquad \frac{\partial\theta_2}{\partial t_0} = 2\pi f. \qquad (15)$$

The uncertainties in θ_2 and θ_1 include the standard deviation, σ_θ, of the measurement of these phases as shown in (12) and (13). Substituting (14) and (15) into equations (12) and (13) yields

$$u_{\bar{\theta}_1} = [(2\pi(t_{1,\text{tr}} + t_0))^2 u_f^2 + (2\pi f)^2 u_{t_{1,\text{tr}}}^2 + (2\pi f)^2 u_{t_0}^2 + \sigma_{\bar{\theta}_1}^2]^{1/2} \qquad (16)$$

and

$$u_{\bar{\theta}_2} = [(2\pi(t_{2,\text{tr}} + t_0))^2 u_f^2 + (2\pi f)^2 u_{t_{2,\text{tr}}}^2 + (2\pi f)^2 u_{t_0}^2 + \sigma_{\bar{\theta}_2}^2]^{1/2}. \qquad (17)$$

It is useful to note that although the trigger process itself is an analogue electronic process and not a sampled process, the sinewave from which the phase is determined is acquired by a sampling process. The uncertainty in the trigger instants would normally be equal to one-half of the sampling instant, but since a fitting routine is used in the sinewave characterization, the uncertainty is arbitrarily assumed to be approximately

$$u_{t_{i,\text{tr}}} = \pm\tfrac{1}{20}\delta t. \qquad (18)$$

Similarly, even though it is assumed that $t_0 = 0$, t_0 has an uncertainty associated with it, the uncertainty in the trigger instant is also assumed to be given by

$$u_{t_0} = \pm\tfrac{1}{20}\delta t. \qquad (19)$$

4.1. Frequency uncertainty

The frequency, f, that is output by the sinewave generator may be offset from the set frequency, f_set, by an amount f_offs. The frequency may also drift with time, Δf_time, and temperature, Δf_T:

$$f = f_\text{set} + f_\text{offs} + \Delta f_\text{time} + \Delta f_T. \qquad (20)$$

The manufacturer of the sinewave generator or synthesizer provides a value for the frequency stability as a function of time and temperature. These parameters may also be determined using proper measurement instruments. The typical uncertainty in the set frequency, u_set, is approximately $10^{-5}f$ and includes any frequency offset. If Δt is the measurement time, σ_T the standard deviation of the temperature during the measurement process, S_time the timebase stability as a function of time, and S_T the timebase stability as a function of temperature then the frequency uncertainty may be written as

$$u_f = [u_{\text{set}}^2 + (S_{\text{time}}\Delta t)^2 + (S_T\sigma_T)^2]^{1/2}. \tag{21}$$

The applied sinewave should be free from harmonics and other distortions, and the measuring instrument should not introduce distortions. However, since the settings of the measuring instrument are the same for both sets of sinewave acquisitions, any harmonics and distortions present will be identical for each set and are assumed to cancel when the difference of the phases is calculated.

4.2. Uncertainty associated with jitter

Since the acquired signals are sinewaves and signal averaging is used, any jitter present will affect the signal amplitude and not the phase. Therefore, there are no uncertainties associated with jitter. This assumes that the jitter has mean zero.

4.3. Uncertainty due to thermal effects on the delay

The delay will also be a function of the temperature of the delay line. This is due to the temperature dependence of the length and diameter (coefficient of thermal expansion) and of the propagation constant (refractive index or dielectric constant) of the optical fibre or electrical transmission line. If the change in delay as a function of temperature can be determined for a representative delay line, then the following relation can be applied for delay lines of identical construction:

$$\Delta D_T = \Delta D_L \Delta T L. \tag{22}$$

In the above equation, ΔD_L is the change in delay per unit temperature and per unit length, ΔT is the change in temperature and L is the length of the delay line. The uncertainty due to thermal effects is therefore

$$u_{\Delta D_T} = \left[\left(\frac{\partial \Delta D_T}{\partial \Delta D_L}\right)^2 u_{\Delta D_L}^2 + \left(\frac{\partial \Delta D_T}{\partial \Delta T}\right)^2 u_{\Delta T}^2 + \left(\frac{\partial \Delta D_T}{\partial L}\right)^2 u_L^2\right]^{1/2}. \tag{23}$$

For the term ΔD_L what is actually measured is the change in delay, ΔD, as a function of temperature of a delay line of length, L_R. This is described by the equation

$$\Delta D_L = \frac{\Delta D}{L_R}. \tag{24}$$

For the optical fibre delay line used in this work, we have recently reported a value of 33 ps (km °C)$^{-1}$ for ΔD_L [6]. For the coaxial trombone line, a value of 0.1 ps (m °C)$^{-1}$ for ΔD_L was estimated. The uncertainty associated with ΔD_L is

$$u_{\Delta D_L} = \left[\left(\frac{\partial \Delta D_L}{\partial \Delta D}\right)^2 u_{\Delta D}^2 + \left(\frac{\partial \Delta D_L}{\partial L_R}\right)^2 u_L^2\right]^{1/2} = \left[\left(\frac{1}{L_R}\right)^2 u_{\Delta D}^2 + \left(-\frac{\Delta D}{L_R^2}\right)^2 u_{L_R}^2\right]^{1/2}. \tag{25}$$

Substituting (25) into (23) and solving the partial derivatives yields

$$u_{\Delta D_T} = \left[(\Delta T L)^2\left[\left(\frac{1}{L}\right)^2 u_{\Delta D}^2 + \left(-\frac{\Delta D}{L^2}\right)^2 u_L^2\right] + (\Delta D_L L)^2 u_{\Delta T}^2 + (\Delta D_L \Delta T)^2 u_L^2\right]^{1/2}. \tag{26}$$

The combined uncertainty in the delay may now be calculated by substituting the above relations into the original equation (9):

$$u_c = u_D = \left[\left(\frac{1}{2\pi f}\right)^2 u_{\bar{\theta}_2}^2 + \left(-\frac{1}{2\pi f}\right)^2 u_{\bar{\theta}_1}^2 + \left(-\frac{\bar{\theta}_2 - \bar{\theta}_1}{2\pi f^2}\right)^2 u_f^2 + u_{\Delta D_T}^2\right]^{1/2}. \tag{27}$$

4.4. The expanded uncertainty

The expanded uncertainty is

$$U_E = k u_c = t_p(\nu_{\text{eff}}) u_c, \tag{28}$$

where the degrees of freedom, ν_{eff}, are calculated using the Welch–Satterthwaite formula and the coverage factor, k, is determined by setting it equal to the t-distribution for the calculated degrees of freedom to give a 95% confidence interval as recommended in [5].

5. Conclusions

The uncertainty in the measured and processed results is dependent on a wide variety of factors. The most significant component is the variation in the measured phase, which can be reduced by using higher frequencies. The sampling interval of the oscilloscope used to acquire the sinewaves is also a significant contributor. The measurement method examined here resulted in uncertainties (95% confidence interval) of about 90 fs for a delay of 660 ps produced by a coaxial trombone line and less than 200 ps (95% confidence interval) for a delay of 20 µs produced by a 4 km length of single mode optical fibre.

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