An Empirical Model for the Warm-Up Drift of a Commercial Harmonic Phase Standard

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***Abstract*—We develop an empirical model for the warm-up drift in a harmonic phase standard used to calibrate the phase distortion of a nonlinear vector network analyzer. The model enables us to estimate the time at which the standard reaches stability.**

***Index Terms*—Drift, empirical, harmonic, model, phase, standard, warm-up.**

I. Introduction

Nonlinear vector network analyzers (NVNAs) are capable of characterizing nonlinear devices under realistic large-signal operating conditions [1], [2]. To do this, complex traveling waves are measured at the ports of a device both at the stimulus frequency (or frequencies) and at other frequencies that are part of the large-signal response. These include harmonics and intermodulation products created by the nonlinearity of the device, in conjunction with impedance mismatches between the system and the device. The calibration of a commercial NVNA consists of three steps: a relative calibration identical to that used in a linear vector network analyzer, an amplitude calibration that makes use of a power meter, and a phase distortion calibration that makes use of a harmonic phase standard (HPS). All are performed on a frequency grid related to the source tones and the anticipated nonlinear response of the device.

A commercial HPS is driven at a fundamental frequency and produces a harmonic-series output signal. The HPS, which is used as a transfer standard, is characterized by a sampling oscilloscope, which in turn is characterized by a nose-to-nose calibration [3]. In this way, we transfer the phase-dispersion calibration of an oscilloscope to "knowing" the phase relationship of each harmonic of the HPS.

In a previous paper [4], we presented a repeatability study of two commercial HPSs measured by an NVNA. By performing multiple calibrations and measurements, we determined the repeatability bounds for the phases and magnitudes of each harmonic component by utilizing the propagation-of-errors (POEs) method to compute expanded uncertainties. We also studied the possibility of warm-up drift in the two devices and discovered considerable drift as a function of time, with an estimated $1/e$ time constant of around 1000 s, which is much longer than the warm-up time of 120 s set by the manufacturer's control software.

In this paper, we expand upon our work done in [5] and develop an empirical model for the warm-up drift of an HPS device, which enables us to estimate its phase-angle response stability time point.

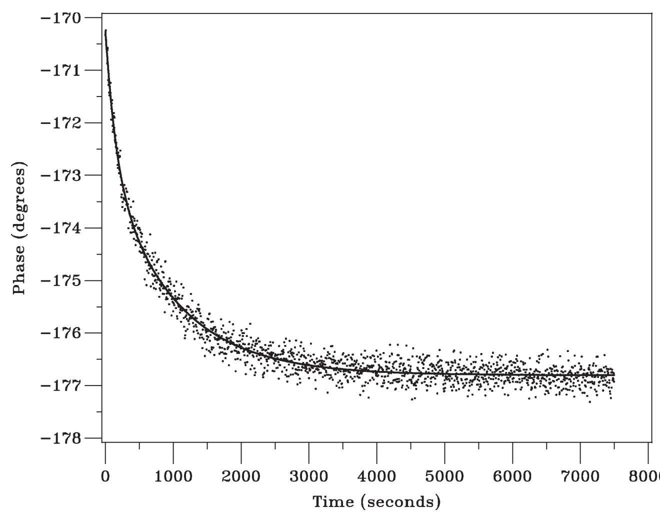

Fig. 1. Phase angles of the fifth harmonic of the HPS along with the estimated curve using an exponential decay model for a typical run.

II. Drift Model of the Phase Angle Response

Fig. 1 shows a typical set of measured phase angles of the fifth harmonic, along with an estimated curve for the true phase angle, which is based on an empirical drift model discussed in the following. In this experiment, we obtained 1500 repeated measurements of the 20-GHz HPS using a fundamental frequency of 600 MHz, with a 5-s pause between each measurement. The deviations or residuals of the measured values from the fitted curve are shown in Fig. 2. The apparent randomness and homogeneity of variance of the residuals are consistent with the assumptions that we make about random noise in the drift model.

While searching for a suitable empirical drift model, we found that two first-order decay terms with an intercept produced excellent fits to all drift data collected to date in repeated calibration runs on the HPS device. The nonlinear decay model for drifting phase measurements of a given harmonic is

$$E(p|t) = \phi + \alpha_1 e^{-\beta_1 t} + \alpha_2 e^{-\beta_2 t} \quad (1)$$

Manuscript received October 15, 2005; revised October 31, 2006.

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Digital Object Identifier 10.1109/TIM.2007.894886

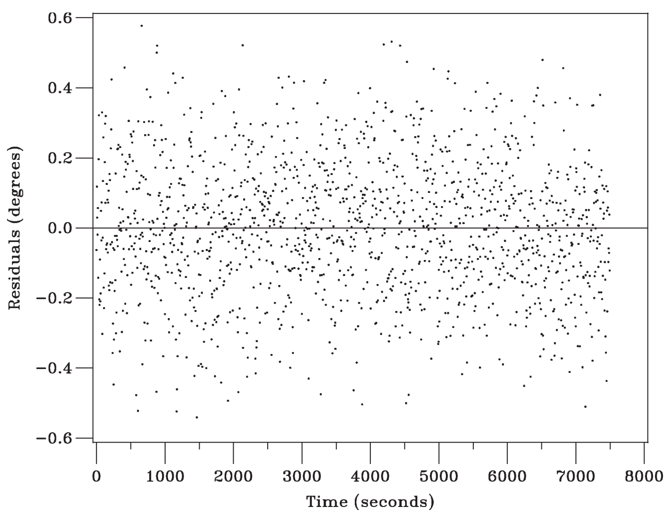

Fig. 2. Residuals from the fitted curve in Fig. 1.

where $E(p|t)$ denotes the expected phase angle at time $t > 0$, ϕ is the true stable phase value after warm up, $\alpha_1 > 0$ and $\beta_1 > 0$ are the unknown parameters of the first decay term, and $\alpha_2 > 0$ and $\beta_2 > 0$ are the unknown parameters of the second decay term. It is not too surprising that this particular empirical model for the expected phase works well, considering that each HPS contains two nonlinear components, namely, an amplifier and a step-recovery diode.

Given a set of repeated measurements (m_1, p_1), $(m_2, p_2), \ldots, (m_n, p_n)$ of magnitude and phase, taken at time points $t_1, t_2, \ldots, t_n$, our statistical model for phase measurements is

$$p_i = \phi + \alpha_1 e^{-\beta_1 t_i} + \alpha_2 e^{-\beta_2 t_i} + w_i^{1/2}\varepsilon_i, \qquad i = 1, \ldots, n \tag{2}$$

where the $\varepsilon_i (i = 1, \ldots, n)$ are random errors with mean zero and unknown standard deviation σ, and the $w_i (i = 1, \ldots, n)$ are weights that may be specified to account for possibly different error variances of drifting phase measurements.

In weighted least squares, weights should be proportional to the inverse of the variance of the measured values. For our phase measurements, a reasonable choice of the weights is based on a general POE formula presented in [4] for the approximate variance of a phase measurement derived from a complex-valued measurement $z = x + jy$. Our experience with several data sets like the one shown in Fig. 1 suggests that it is reasonable to assume that the random errors in the real and imaginary parts of z are uncorrelated and have equal variances, in which case, the POE variance of a measured phase angle p is inversely proportional to the true squared magnitude of the measurement [4]. Since we do not know the true magnitudes, we substitute the estimated weights

$$\hat{w}_i = m_i^2, \qquad i = 1, \ldots, n \tag{3}$$

using the measured magnitudes. Estimates of the unknown parameters $(\phi, \alpha_1, \beta_1, \alpha_2, \beta_2)$ in (2) can be obtained from the usual weighted nonlinear least square solution that minimizes the error sum of squares

$$S(\hat{\phi}, \hat{\alpha}_1, \hat{\beta}_1, \hat{\alpha}_2, \hat{\beta}_2) = \sum_{i=1}^{n} \hat{w}_i (p_i - \hat{p}_i)^2 \tag{4}$$

where

$$\hat{p}_i = \hat{\phi} + \hat{\alpha}_1 e^{-\hat{\beta}_1 t_i} + \hat{\alpha}_2 e^{-\hat{\beta}_2 t_i} \tag{5}$$

is the predicted value of the ith measured phase angle. Here, we have used the convention of denoting the least square estimates of the parameters by placing a caret over the respective symbols for the unknown parameters.

Given the least square solution, the variance of the random error ε in (2) is estimated in the usual way from the residual sum of squares as

$$\hat{\sigma}^2 = \sum_{i=1}^{n} \hat{w}_i (p_i - \hat{p}_i)^2 / (n - 5) \tag{6}$$

where the degrees of freedom of the estimated noise variance is $n - 5$, after adjusting for the five parameters estimated in (4). The least square analysis also provides estimated variances and covariances of the decay parameter estimates, which may be used to approximate the uncertainties of relevant functions of the parameter estimates. In the next section, we use this approach to estimate an approximate standard deviation of the least square estimate of the actual phase-angle error at any time during the warm-up period.

The weighted nonlinear least square estimates of the model parameters are listed in Table I for each of seven repeated runs at harmonics 5, 10, ..., 30. Estimates of each of the four decay parameters versus harmonic number are displayed graphically in Figs. 3–6, respectively. The plots of $\hat{\alpha}_1$ and $\hat{\alpha}_2$ suggest that the parameters increase almost linearly with harmonic number. That the variance of estimates of these two parameters increases with harmonic number is not surprising since the magnitudes of the harmonics are decreasing. The estimates of $\hat{\beta}_1$ and $\hat{\beta}_2$, which are shown in Figs. 4 and 6, are fairly stable across harmonic numbers.

III. Estimation of Stable Phase-Angle Time Point

When a phase measurement is consistent with the decay model of (2), the measured phase differs from the stable value ϕ by a systematic offset that depends on time, plus a random error. The actual offset of a measurement at time t, which is denoted by $\delta(t)$, is

$$\delta(t) = E(p|t) - \phi = \alpha_1 e^{-\beta_1 t} + \alpha_2 e^{-\beta_2 t} \tag{7}$$

where $E(p|t)$ is the expected value of a phase angle measurement at time t. Given the estimates of the decay parameters from a warm-up experiment, an appropriate estimate of $\delta(t)$ is

$$\hat{\delta}(t) = \hat{\alpha}_1 e^{-\hat{\beta}_1 t} + \hat{\alpha}_2 e^{-\hat{\beta}_2 t}. \tag{8}$$

TABLE I
WEIGHTED NONLINEAR LEAST SQUARE ESTIMATES OF DECAY-MODEL PARAMETERS FROM SEVEN WARM-UP EXPERIMENTS FOR SELECTED HARMONICS

Harmonic	Run	ϕ (deg.)	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_2$	$\hat{\beta}_2$	RMSE
5	1	-176.803	2.4924	0.00636	4.0530	0.00102	0.00409
	2	-177.675	2.2178	0.00604	4.3258	0.00108	0.00399
	3	-177.583	2.3707	0.00628	4.2789	0.00102	0.00398
	4	-177.800	2.0052	0.00686	4.4128	0.00117	0.00412
	5	-177.562	2.2004	0.00689	4.4551	0.00109	0.00391
	6	-176.633	2.8028	0.00516	3.6735	0.00098	0.00403
	7	-177.428	2.4148	0.00568	3.9706	0.00105	0.00387
10	1	-164.732	5.2254	0.00719	9.2687	0.00103	0.00764
	2	-166.620	4.5374	0.00622	9.7733	0.00109	0.00713
	3	-166.506	5.2385	0.00589	9.3249	0.00099	0.00729
	4	-166.349	4.5796	0.00674	9.6508	0.00113	0.00749
	5	-166.432	4.8242	0.00665	9.7503	0.00107	0.00726
	6	-164.412	6.1653	0.00486	8.0099	0.00095	0.00740
	7	-166.079	5.4684	0.00517	8.5053	0.00102	0.00729
15	1	-164.992	8.6648	0.00659	14.4151	0.00101	0.00820
	2	-168.004	7.3066	0.00591	15.5031	0.00108	0.00783
	3	-167.838	8.1338	0.00619	15.1775	0.00101	0.00811
	4	-168.156	7.0437	0.00672	15.4835	0.00116	0.00814
	5	-167.792	7.5703	0.00692	15.7844	0.00108	0.00801
	6	-164.479	9.6527	0.00553	13.4238	0.00099	0.00801
	7	-167.116	8.9229	0.00508	13.4105	0.00100	0.00796
20	1	-163.619	11.0100	0.00651	19.0894	0.00102	0.00830
	2	-167.864	9.2425	0.00645	20.7266	0.00109	0.00807
	3	-167.500	11.0225	0.00670	20.1165	0.00101	0.00793
	4	-167.657	7.9412	0.00797	21.5783	0.00123	0.00823
	5	-167.856	10.4897	0.00666	20.2175	0.00107	0.00812
	6	-163.185	13.4246	0.00496	16.7396	0.00095	0.00806
	7	-166.520	11.8092	0.00482	17.2738	0.00101	0.00804
25	1	-153.922	14.7653	0.00734	24.9713	0.00102	0.00778
	2	-159.114	13.1057	0.00512	24.6900	0.00104	0.00745
	3	-159.263	13.5675	0.00687	26.0361	0.00102	0.00769
	4	-159.769	14.7815	0.00461	22.8576	0.00101	0.00768
	5	-158.309	11.9196	0.00694	26.9199	0.00108	0.00739
	6	-153.238	17.1131	0.00462	20.9699	0.00094	0.00759
	7	-157.713	14.3197	0.00532	23.3012	0.00103	0.00747
30	1	-157.714	16.0941	0.00878	32.1068	0.00107	0.00898
	2	-163.643	15.6276	0.00621	32.0725	0.00108	0.00878
	3	-163.939	13.7922	0.00717	33.3107	0.00106	0.00879
	4	-164.569	13.2042	0.00700	32.3659	0.00117	0.00879
	5	-163.156	15.8687	0.00682	32.2369	0.00107	0.00876
	6	-156.278	22.0002	0.00412	24.1717	0.00090	0.00888
	7	-162.384	20.2411	0.00380	24.1595	0.00091	0.00881

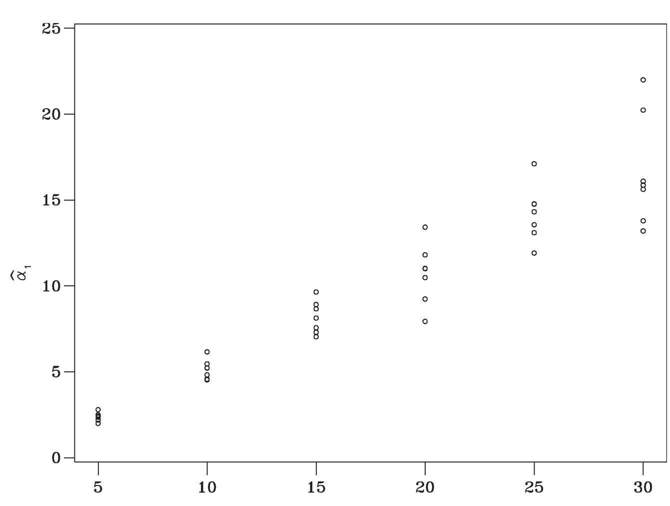

Fig. 3. Parameter estimate $\hat{\alpha}_1$ versus harmonic index for seven repeated runs.

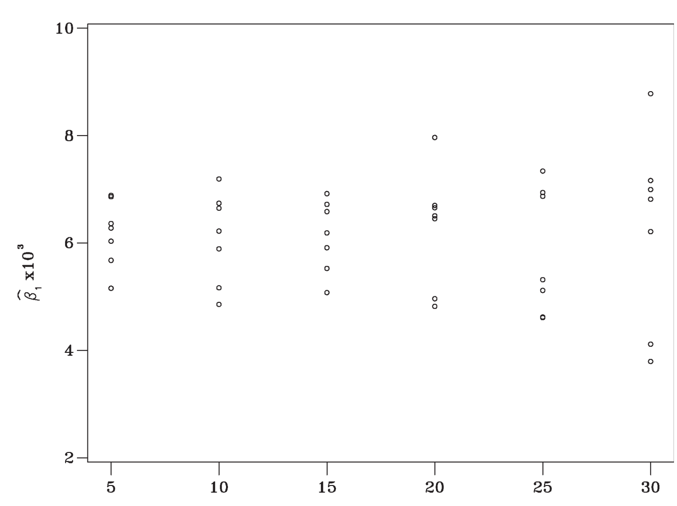

Fig. 4. Parameter estimate $\hat{\beta}_1$ versus harmonic index for seven repeated runs.

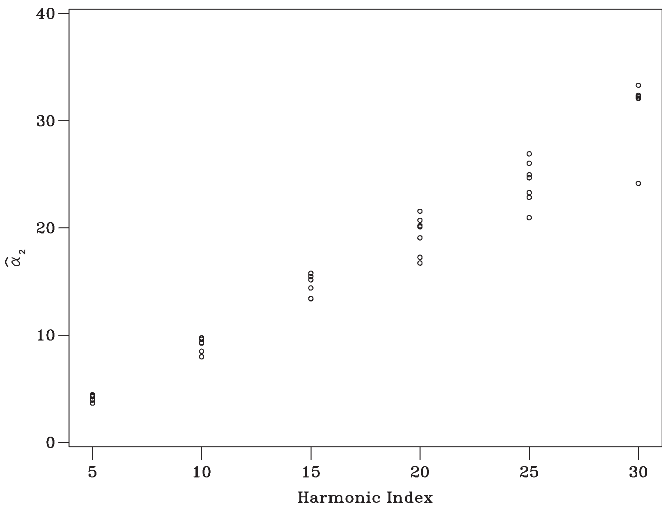

Fig. 5. Parameter estimate $\hat{\alpha}_2$ versus harmonic index for seven repeated runs.

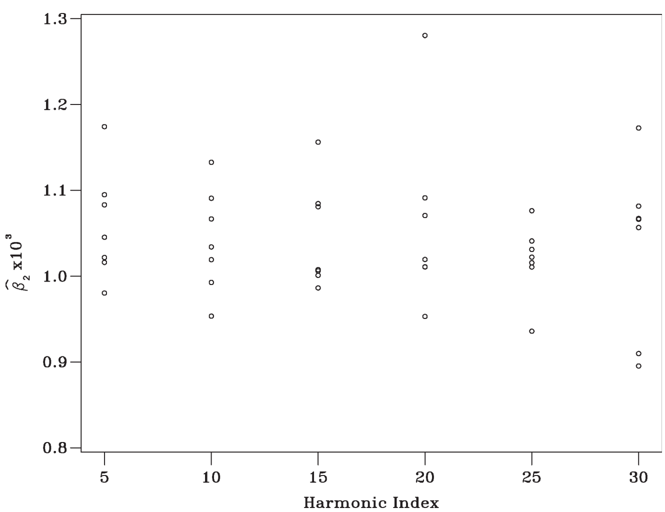

Fig. 6. Parameter estimate $\hat{\beta}_2$ versus harmonic index for seven repeated runs.

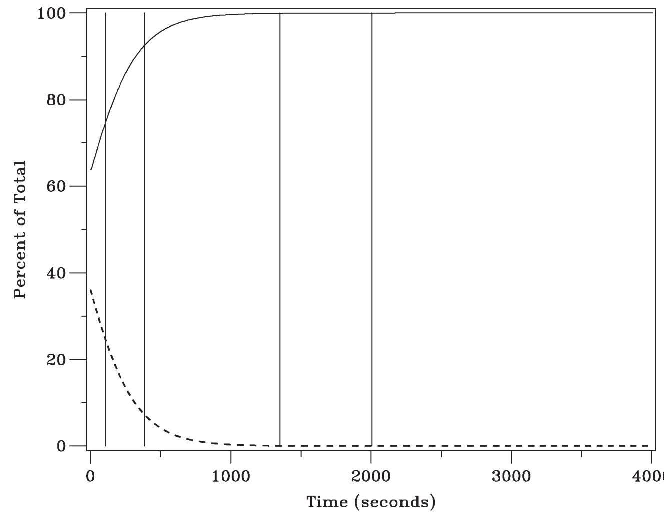

Fig. 7. Percent contribution of the first (dashed curve) and second (solid curve) decay terms to the total error in (8) for the fifth harmonic. Vertical lines indicate $\hat{t}$ for $\Delta = 5, 3, 1, 0.5°$ from left to right.

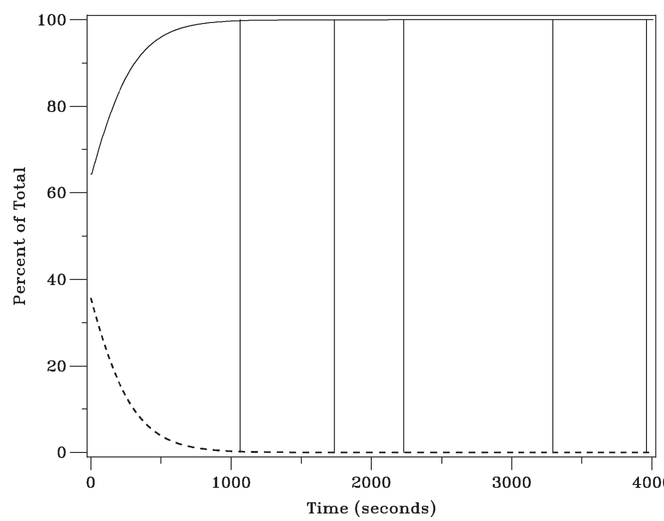

Fig. 8. Percent contribution of the first (dashed curve) and second (solid curve) decay terms to the total error in (8) for the 30th harmonic. Vertical lines indicate $\hat{t}$ for $\Delta = 10, 5, 3, 1, 0.5°$ from left to right.

Since the error $\delta(t)$ is decreasing in time, we define the true stable time point to be the time t when $\delta(t) = \Delta°$, where Δ is a phase-error bound judged to be acceptable for the intended use of a phase-angle measurement. Since $\delta(t)$ depends on unknown parameters, statistical methods are required to estimate the stable time point. An estimate of the minimum time at which phase error is not greater than Δ may be taken to be the solution for $\hat{t}$ in the estimating equation

$$\hat{\alpha}_1 e^{-\hat{\beta}_1 \hat{t}} + \hat{\alpha}_2 e^{-\hat{\beta}_2 \hat{t}} = \Delta. \tag{9}$$

The solution of (9) for the estimated waiting time $\hat{t}$ can be obtained by numerical methods. Monte Carlo methods may be used to estimate a variance of $\hat{t}$ from solutions $\hat{t}_j (j = 1, \ldots, N)$ of (9) calculated from N simulated vectors from a multivariate Gaussian distribution with mean vector ($\hat{\alpha}_1$, $\hat{\beta}_1$, $\hat{\alpha}_2$, $\hat{\beta}_2$) and with variances and covariances for the parameter estimates that are typically produced by nonlinear least square software. The asymptotic or large-sample theory of nonlinear least square estimates suggests that a Gaussian approximation to the distribution of the decay parameters may be acceptable.

An analysis using the decay parameter estimates in Table I shows that the second decay term in (8) rapidly becomes the dominant part of the phase error as the HPS warms up. Figs. 7 and 8 illustrate this point using the estimated error (9) for the fifth and 30th harmonics, respectively. The solid curves in the figures show the increasing percentage $100(\hat{\alpha}_2 e^{-\hat{\beta}_2 t}/\hat{\delta}(t))$ contribution of the second decay term to the total error during warm-up, while the dashed lines show the decreasing contribution of the first decay term. Vertical lines, which are in order, are positioned at the estimated warm-up times for achieving expected error bounds $\Delta = 5, 3, 1, 0.5°$ for the fifth harmonic [Fig. 7], and $\Delta = 10, 5, 3, 1, 0.5°$ for the 30th harmonic [Fig. 8]. For the 30th harmonic, we see that the second decay term accounts for nearly 100% of the error for values of Δ as large as $10°$, but for the fifth harmonic, values of Δ as large as $3°$ still show a considerable contribution of the first decay term to the total error.

Figs. 7 and 8 suggest that for any given harmonic and small enough values of Δ, a closed-form approximation to the solution for $\hat{t}$ of (9) will work well for the drift data produced by our HPS. That is, there is a sufficiently small $\Delta \leq \hat{\alpha}_2$ for which the waiting time estimate

$$\widetilde{t} = \frac{-1}{\hat{\beta}_2} \ln\left(\frac{\Delta}{\hat{\alpha}_2}\right) \tag{10}$$

is a good approximation to the value of $\hat{t}$. The POE formula for the variance of $\widetilde{t}$ is

$$\mathrm{var}(\widetilde{t}) = \left(\alpha_2^{-2}\beta_2^{-2}\right)\mathrm{var}(\hat{\alpha}_2) + \beta_2^{-4}\left(\ln\left(\frac{\Delta}{\alpha_2}\right)\right)^2 \times \mathrm{var}(\hat{\beta}_2) + 2\alpha_2^{-1}\beta_2^{-3}\ln\left(\frac{\Delta}{\alpha_2}\right)\mathrm{cov}(\hat{\alpha}_2, \hat{\beta}_2) \tag{11}$$

where the variances and covariance involving $\hat{\alpha}_2$ and $\hat{\beta}_2$ are theoretical quantities that are functions of the unknown decay parameters and error variance. As usual, in a POE uncertainty analysis, an estimate of $\mathrm{var}(\widetilde{t})$ would be computed by substituting appropriate estimates of the decay parameters and covariances appearing in (11) from the least square fits of the decay model.

The procedure outlined previously is useful for retrospective analysis of a single drift experiment or a combined analysis of two or more runs where the drift parameters are believed to be constant regardless of the measurement occasion. If we assume that the four decay parameters in the statistical model in (2) are fixed or unchanging between measurement occasions, then variations among fitted curves are the usual results of independent phase-measurement random errors arising in successive runs. Under the assumption of fixed values of the model parameters, it would be appropriate to pool the data sets and obtain the least square solution in Section II from the combined data, resulting in more precise estimates for model parameters and, subsequently, a better determination of the necessary waiting time.

Run-to-run variations among decay curves for our seven drift experiments, which spanned a period of several weeks, probably cannot be explained solely by the effects of random measurement errors in the statistical model of (2). A more realistic model allows for the possibility that the true values of the decay parameters may vary significantly from run to run, owing to changing environmental and other conditions, even though the same experimental protocol leading to the first measurement was followed for each run.

Run-to-run differences among true error curves can be modeled by treating the decay parameters as a vector of (possibly correlated) random coefficients. By this assumption, there is not just one true waiting time that is common to all runs; rather, there is a distribution of waiting times corresponding to the multivariate distribution of the four decay parameters. A sensible approach in this case is to combine the data from all available runs in order to estimate the average or expected time to stability and an associated uncertainty that could be used to specify a suitable HPS warm-up time for a future phase-calibration run.

TABLE II

WARM-UP TIME ESTIMATE $\hat{t}$ AND ITS SIMULATION-BASED STANDARD DEVIATION $s_{\hat{t}}$, ALONG WITH AN APPROXIMATE ESTIMATE $\widetilde{t}$ AND POE STANDARD DEVIATIONS $s_{\widetilde{t}}$, FOR SELECTED HARMONICS. MISSING VALUES ARE DUE TO INSTANCES WHERE THERE IS NO SUFFICIENTLY SMALL $\Delta \leq \hat{\alpha}_2$

Harmonic	Δ(deg.)	$\hat{t}$ (s)	$s_{\hat{t}}$ (s)	$\widetilde{t}$ (s)	$s_{\widetilde{t}}$ (s)
5	0.5	2001.4	83.9	2001.3	83.0
5	1	1347.6	52.6	1347.1	52.3
5	3	382.6	20.6	310.2	50.8
5	5	101.6	7.3	-	-
5	10	-	-	-	-
10	0.5	2794.7	115.2	2794.7	114.4
10	1	2129.2	80.3	2129.2	79.7
10	3	1076.6	41.6	1074.3	43.0
10	5	608.2	31.7	583.8	48.2
10	10	146.0	9.7	-	-
15	0.5	3234.5	152.2	3234.5	150.7
15	1	2572.0	114.2	2572.0	113.2
15	3	1522.1	60.3	1521.9	59.9
15	5	1036.3	43.3	1033.6	44.6
15	10	429.0	24.3	371.0	50.4
20	0.5	3442.9	294.5	3442.9	280.8
20	1	2790.5	225.5	2790.5	217.2
20	3	1756.6	123.8	1756.5	120.0
20	5	1276.4	82.3	1275.7	80.1
20	10	641.3	42.2	623.3	54.5
25	0.5	3809.2	102.5	3809.2	101.9
25	1	3129.0	78.1	3129.0	77.4
25	3	2050.9	49.0	2050.8	48.7
25	5	1549.9	43.9	1549.5	45.5
25	10	877.7	40.3	869.3	55.5
30	0.5	3955.0	329.7	3955.0	266.0
30	1	3285.8	229.7	3285.8	203.9
30	3	2225.1	130.1	2225.1	109.8
30	5	1731.9	96.1	1731.9	73.0
30	10	1064.7	70.6	1062.7	57.6

To illustrate one way that we can account for variation of the decay parameters from multiple drift experiments and still use waiting-time formulas discussed previously, we use the results of the seven warm-up experiments summarized in Table I. Table II lists the results of some combined-run analyses for each harmonic and selected values of the error bound Δ. An entry in column three of Table II is the solution for $\hat{t}$ in (9) but with the single-run vector $(\hat{\alpha}_1, \hat{\beta}_1, \hat{\alpha}_2, \hat{\beta}_2)$ replaced by the mean vector $(\overline{\alpha}_1, \overline{\beta}_1, \overline{\alpha}_2, \overline{\beta}_2)$, where the overbar notation signifies the averages of seven single-run estimates of parameters shown in columns 4–7 of Table I.

The fourth column in Table II shows estimated standard deviations $s_{\hat{t}}$ of the warm-up times in column 3. Monte Carlo methods were used to estimate the variance of $\hat{t}$ from a pseudo-random sample $\{\hat{t}_j, j = 1, \ldots, N\}$ of the solutions of (9), calculated from $N = 100\,000$ simulated vectors from a four-variate Gaussian distribution with mean vector $(\overline{\alpha}_1, \overline{\beta}_1, \overline{\alpha}_2, \overline{\beta}_2)$ and with covariance matrix formed from the sample variances and covariances of the seven vectors of decay parameters in Table I for a given harmonic.

The final two columns in Table II are included for comparison of the approximate warm-up time estimate $\widetilde{t}$ in (10) and POE standard deviation $s_{\widetilde{t}}$ from (11) to the estimate and simulation-based uncertainty in columns 3 and 4. We see that for values of $\Delta \leq 1^\circ$, the two solutions are essentially the same for practical purposes, at least for harmonics 5 or greater. A more detailed view of the adequacy of the approximate

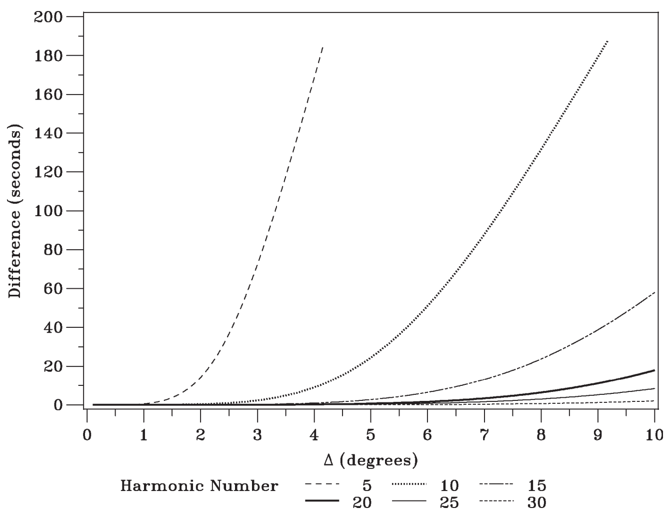

Fig. 9. Difference $\hat{t} - \widetilde{t}$ between exact and approximate warm-up time estimates versus Δ for harmonic numbers 5, 10, 15, 20, 25, and 30 clockwise or from the left.

warm-up time solution (11), which is always less than the exact solution for our data, is shown in Fig. 9, which shows the actual error $\hat{t} - \tilde{t}$ and is based on the mean vectors of our seven warm-up experiments.

IV. Estimate of the Warm-Up Time

Finally, there is the practical question of how our data can be used to specify a warm-up time for the next calibration occasion using the HPS. Since we have assumed that there is a distribution of warm-up times, any of the estimated times $\hat{t}$ in Table II probably have about 50% chance of meeting a specified error bound Δ in a future run. To obtain a higher degree of confidence that a predetermined warm-up time achieves the tolerance Δ, we may calculate a one-sided prediction interval for the next (unknown) warm-up time. An approximate one-sided $100(1-\alpha)\%$ upper prediction limit for the warm-up time of a future run

$$\widetilde{t} + c'_{(1-\alpha,n)} s_{\widetilde{t}} \tag{12}$$

where $c'_{(1-\alpha,n)}$ is a table value based on n previously sampled observations from a normal distribution [6, p. 61]. For example, we estimated $\widetilde{t} = 1347.1$ s and $s_{\widetilde{t}} = 52.3$ s for the fifth harmonic using the one-term approximation ($\Delta = 1.0^\circ$ and $n = 7$); therefore, the resulting upper bound for a 95% prediction interval using $c'_{(0.95,7)} = 2.08$ is

$$1347.1 + 2.08(52.3) = 1455.9 \text{ s}.$$

Thus, we are 95% confident that a future warm-up time will be less than or equal to 1455.9 s. In other words, if we wait at least 1455.9 s before taking measurements, we will be 95% confident that the phase angle is stable.

V. Conclusion

An empirical nonlinear model with two first-order decay terms was found to describe the behavior of phase-angle drift over time in a commercial HPS. The model was used to develop a statistical procedure for estimating the warm-up time required to assure, with acceptable uncertainty, that the expected phase-angle error of a subsequent HPS measurement is not greater than a stated error bound Δ°. We performed simulation studies to evaluate the performance of the procedure and found that it works well depending on the harmonic number and the chosen value Δ°. An example based on seven runs on an HPS illustrated how to calculate an approximate upper prediction limit for the warm-up time required to ensure that phase-angle stability is achieved with a stated level of 95% confidence.

The procedure to determine phase angle stability described in this paper was developed for a specific HPS device; other devices have different estimated parameter values and waiting times. We consider this procedure to be a first step in the process of determining a real-time general methodology for determining phase-angle stability.

We measured the time evolution of the port impedance during these experiments and did not observe a significant drift. Furthermore, we have not observed a drift of this magnitude in other long-term repeatability studies conducting on our system with other types of devices. We submit these data as evidence of a long-term time-constant thermal effect in our HPSs.

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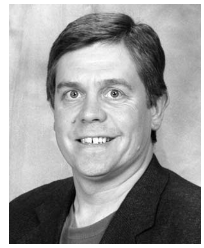

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