Impedance of Transfer Standards Used in the Calibration of High-Speed Samplers and Pulse Generators

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***Abstract*—The impedance of the calibration artifact or transfer standard (TS; pulse generator or sampler) is often ignored in high-speed pulse metrology. Using an air line as an integral part of the TS for providing a known impedance for calibration of other instruments is described. The utility of this method is compared to that of using a TS for which its impedance is determined by some other instrument.**

***Index Terms*—Air line (AL), calibration artifact, high-speed, impedance, pulse generator, sampler, transfer standard (TS).**

I. INTRODUCTION

THE INPUT impedance of high-speed samplers and the output impedance of high-speed pulse and high-frequency sources is nominally 50-Ω but can vary between 25 Ω and 100 Ω and is frequency-dependent. These high-speed devices are used as calibration artifacts or transfer standards (TSs) in the calibration of other high-speed devices [1]. For example, a sampler calibrated at a commercial or military calibration or standards laboratory is used as a basis to calibrate the output of high-speed pulse generators; these are the secondary devices. Similarly, a pulse generator calibrated in one of these standards laboratories is used to calibrate the step response of high-speed samplers; these are also the secondary devices. The variation in impedance of these TSs as a function of frequency may introduce errors in the calibration of the secondary devices, which typically will be instruments from which many other instruments will be calibrated. Therefore, it is important that the impedance uncertainty of the TS be minimized.

There are two fundamental ways of achieving a TS that has a known primary function (step response for a sampler, output pulse for a generator) and impedance. Both of these methods (see Fig. 1) can provide the required information. The issue is which method is more practical to use and which can provide the lowest uncertainties in the measurand. It will be assumed that methods are available for measuring the primary function (sampling or pulse generation) of the TS so that the issue considered here is its impedance. The complex impedance of the TS can be determined with a calibrated vector network analyzer (VNA) and, with knowledge of the complex impedance of the customer's device, can be used to determine the properties of the customer's device. Alternatively, the complex impedance of the TS can be fixed by connecting it to a 50-Ω air line (AL). In this case, the complex impedance of the TS does not have to be measured. The advantages and disadvantages of both methods will be discussed. It will be discussed later in more detail, but it is worth stating here that the AL–TS method (see Section II), as shown in the top of Fig. 1, works equally with the pulse generator or sampler acting as the device under test (DUT), because the AL is the reference for impedance. This is not true for the impedance-calibrated TS (IC–TS) method (see Section III) depicted in the bottom part of the figure, because the impedance transfer is sequential. The arrows in the bottom of Fig. 1 denote the path of calibration: In step 1, the "Set of Z standards" are used to calibrate the impedance of the VNA; in step 2, the now-calibrated VNA is used to calibrate the impedance of the sampler; and in step 3, the now-calibrated sampler is used to calibrate the impedance of the pulse generator.

II. METHOD USING TS WITH AL (TS–AL)

To confer the impedance of the AL to the TS, it is necessary that the waveform epoch in all measurements with the AL–TS be less than the roundtrip propagation time t_{RT} of the AL. This requirement is necessary because use of the AL for this purpose implies that no reflections will be observed from the "far end" of the AL. The "far end" is the end that is connected to the secondary device's input or output port. In addition, the AL should support the frequency content of the signal without generating higher order modes. Higher order modes are generated at discontinuities in waveguides excited by signals with frequency content exceeding the cutoff frequency of the waveguide. Therefore, for example, a 2.4-mm AL can be used with a 3.5-mm connected TS, but a 3.5-mm AL should not be used with a 2.4-mm connected TS. How important this requirement is in pulse metrology has not been adequately quantified but it can, if ignored, introduce an error that can be easily avoided.

The TS, with an attached AL of appropriate length and type, is calibrated by the standards or calibration laboratory. The calibration of the AL–TS includes the transmission of the signal through the AL–TS interface. However, because

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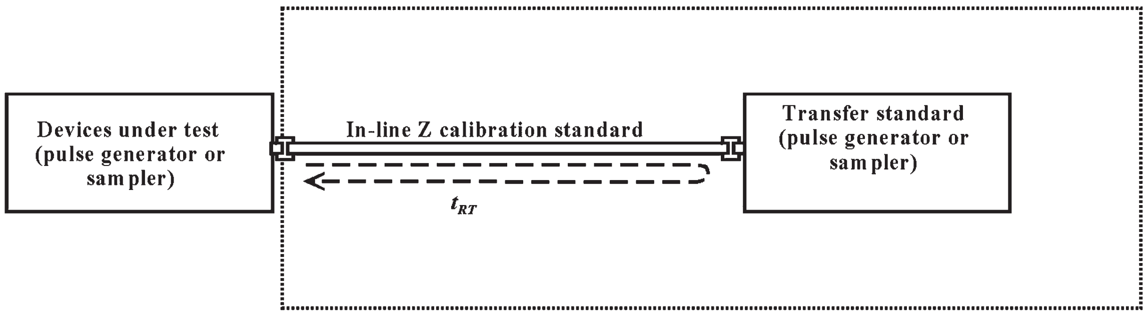
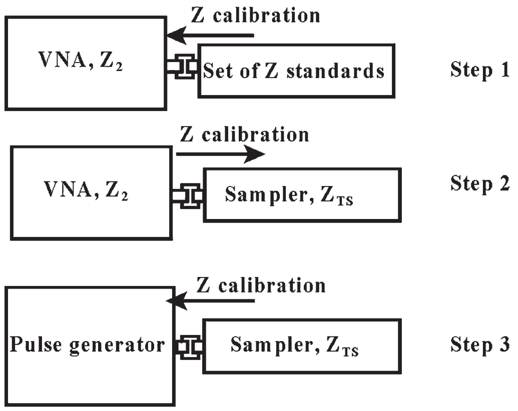


Transfer standard with air line



Impedance-calibrated transfer standard

Fig. 1. Two methods considered for determining the response or output of a DUT into a 50-Ω environment. The top picture depicts the method that uses an AL affixed to the TS, and the bottom depicts the steps required if the impedance of the TS must be determined. The arrows denote the path of calibration.

the roundtrip propagation time of the AL is greater than the waveform epoch, the reflection from this interface is not observed.

When the AL–TS is used in subsequent calibration of secondary instruments, these secondary instruments experience ("see") an electrical connection that is nominally 50-Ω. In the case of a sampler AL–TS, the secondary instrument is a pulse generator, which will be used to calibrate other samplers. The pulse generator launches a signal into the nominally 50-Ω load that is nominally constant over the operating bandwidth of the sampler. Because the pulse roundtrip time of the sampler AL–TS is longer than the waveform epoch, no reflections are observed. Similarly, a pulse-generator AL–TS is used to calibrate samplers. These samplers, when connected to the pulse generator AL–TS, are connected to a constant 50-Ω source over the operating bandwidth of the sampler.

The primary advantage of the AL method is that the target device actually "sees" a 50-Ω load or source. A second advantage is that the uncertainty in the impedance of the AL–TS is the impedance uncertainty of the mechanically calibrated AL, which is small [2]. For a standards-quality commercially available 3.5-mm AL, the impedance uncertainty for a 67% confidence interval is less than about 0.3 Ω. The impedance uncertainty for ALs increases with decreasing AL diameter. Another advantage is that the secondary device, when calibrated with the AL–TS, is calibrated in and traceable to a 50-Ω environment, which is the ideal situation. This is described by

$$M_{2,k,50\,\Omega} = S_{2,k,50\,\Omega} F_{\mathrm{AL-TS},k} \tag{1}$$

where $S_{2,k,50\,\Omega}$ is the transfer function (spectrum of impulse response) or pulse spectrum of the secondary device in a 50-Ω environment, which is the desired quantity, $F_{\mathrm{AL-TS},k}$ is the spectrum of the appropriate function of the AL–TS, $M_{2,k,50\,\Omega}$ is the spectrum of the measured signal in a 50-Ω environment, and k is the frequency index of the discrete spectrum. If $S_{2,k,50\,\Omega}$ is the transfer function of a sampler, then $F_{\mathrm{AL-TS},k}$ for the AL–TS method is the measured spectrum of the output of the reference pulse generator with AL; if $S_{2,k,50\,\Omega}$ is the spectrum of the pulse-generator output, then $F_{\mathrm{AL-TS},k}$ for the AL–TS method is the measured transfer function of the sampler with the AL. The desired spectrum is then computed using

$$S_{2,k,50\,\Omega} = \frac{M_{2,k,50\,\Omega}}{F_{\mathrm{AL-TS},k}}. \tag{2}$$

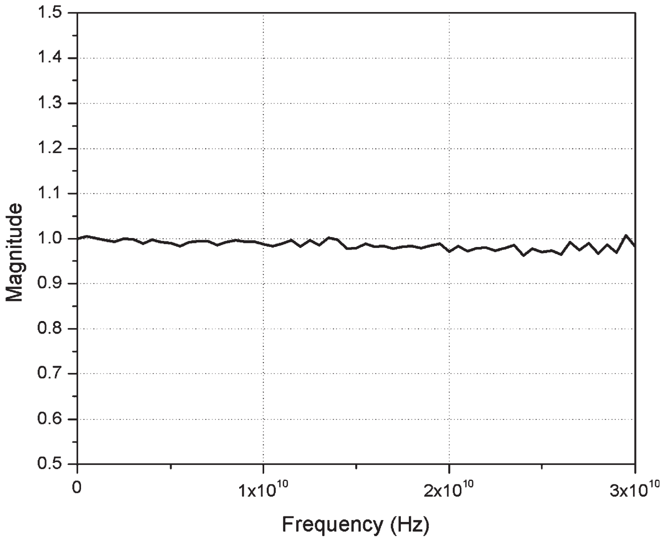


Fig. 2. Transfer function magnitude for a typical 15-cm-long 50-Ω AL.

The calibration of bench-level instruments with the secondary instruments is then described by

$$M_{3,k} = S_{3,k} T_{3-2,k} S_{2,k,50\,\Omega} \tag{3}$$

where the subscript "3" denotes the bench level or tertiary instrument, and $T_{3-2,k}$ is the transmission function between the tertiary and secondary instruments. $S_{3,k}$, which would be the desired quantity, is given by

$$S_{3,k} = \frac{M_{3,k}}{T_{3-2,k} S_{2,k,50\,\Omega}} = \frac{Z_{3,k} + Z_{2,k}}{2Z_{3,k}} \frac{M_{3,k}}{S_{2,k,50\,\Omega}} \tag{4}$$

where $Z_{2,k}$ and $Z_{3,k}$ are the impedances of the secondary and tertiary instruments, which are necessary to estimate $S_{3,k}$. If the secondary instrument also included an AL, then impedance measurements would not be necessary, and $S_{3,k}$ would be that for an instrument operating in a 50-Ω environment.

The effect of the AL is not significant, as shown from the plot in Fig. 2 showing the magnitude of the transfer function of a 15-cm-long AL. The data used in this figure was obtained by dividing the spectrum of a measured pulse with an AL inserted between the pulse generator and the sampler by that of the spectrum of a measurement taken without the AL. The AL has a 3.5-mm connector and is approximately 15 cm long. The approximate −3-dB bandwidth of the generated impulse-like signals was approximately 24 GHz, and that of the sampler was about 50 GHz.

III. METHOD USING CASCADED IMPEDANCE–CALIBRATED TSs (IC–TS)

In this method, the input impedance of a sampler or the output impedance of a pulse generator are measured using a VNA. This process is separate from measuring the primary function of the TS.

The primary advantage of the IC–TS method is that the impedance of the TS will be known. However, if the impedance of the secondary device is not known, knowledge of the TS impedance is not useful in determining the function (sampler response, generator output) of the secondary device into 50-Ω. This is apparent from the following formula that describes the spectrum $M_{2,k}$ of the measured signal as a function of the TS spectrum $F_{\mathrm{IC-TS},k}$ and the transmission through their connection

$$M_{2,k} = \frac{2Z_{2,k}}{Z_{2,k} + Z_{\mathrm{TS},k}} S_{2,k} F_{\mathrm{IC-TS},k} \tag{5}$$

where $S_{2,k}$ is the transfer function (spectrum of impulse response) or pulse spectrum of the secondary device, $F_{\mathrm{IC-TS},k}$ is the spectrum of the appropriate function of the TS (per the bottom of Fig. 1, $F_{\mathrm{IC-TS},k}$ for this method will be that of the IC VNA for sampler calibration or that of the IC sampler or pulse-generator calibration), $Z_{2,k}$ is the impedance of the secondary device, $Z_{\mathrm{TS},k}$ is the impedance of the TS, and k is the discrete frequency index. To estimate the measured spectrum $M_{2,k,50\,\Omega}$ of the secondary device in a 50-Ω environment, the following must be used:

$$M_{2,k,50\,\Omega} = \frac{2Z_0}{Z_0 + Z_{2,k}} M_{2,k} \tag{6}$$

where Z_0 is 50-Ω. To compute the value of $S_{2,k}$ in a 50-Ω environment $S_{2,k,50\,\Omega}$, the following is necessary:

$$\begin{aligned} S_{2,k,50\,\Omega} &= \frac{Z_{2,k} + Z_{\mathrm{TS},k}}{2Z_{2,k}} \frac{M_{2,k,50}}{F_{\mathrm{IC-TS},k}} \\ &= \frac{Z_{2,k} + Z_{\mathrm{TS},k}}{2Z_{2,k}} \frac{2Z_0}{Z_0 + Z_{2,k}} \frac{M_{2,k}}{F_{\mathrm{IC-TS},k}}. \end{aligned} \tag{7}$$

Without knowing $Z_{2,k}$, the estimation of the operation of the secondary device in a 50-Ω environment, which is given by $M_{2,k,50\,\Omega}$, cannot be made; this is a disadvantage. To compare, $M_{2,k,50\,\Omega}$ is measured directly with the AL–TS method. Another disadvantage of the IC–TS method is that the impedances of the TS and secondary device while in their nonactive state are measured with the VNA. What should be measured are the dynamic impedances of these devices, i.e., the impedances during the transients of the pulse generation or sampling process. Otherwise, errors may be introduced in the estimate of the impedance of the TS and secondary device. Furthermore, the VNA will likely be calibrated using a 50-Ω AL. Consequently, the uncertainty in the impedance calculation of the TS and secondary device using the AL-calibrated VNA will likely be two to three times greater than the uncertainty in the AL impedance. The uncertainties in $S_{2,k,50\,\Omega}$ will also be greater for the IC–TS method than for the AL–TS method [compare (2) and (7)]. Another disadvantage is that two separate measurements and instruments are required in the IC–TS method to determine the response of the TS: one for its impedance $(Z_{2,k})$ and the other for its primary function $(F_{\mathrm{IC-TS},k})$.

Calibration of tertiary instruments by this method would not require knowing the secondary instrument's response in a 50-Ω environment. However, determining the response of the tertiary

TABLE I

PARAMETERS AND THEIR UNCERTAINTY CONTRIBUTIONS. THE VALUES SHOWN ARE NOMINAL VALUES; ACTUAL VALUES ARE A FUNCTION OF FREQUENCY, TYPICALLY INCREASING WITH INCREASING FREQUENCY. THE UNCERTAINTIES AND NOMINAL VALUES OF $M_{2,k,50\,\Omega}$ AND $F_{\mathrm{TS},k}$ ARE BASED ON MEASUREMENTS PERFORMED AT NIST, AND THOSE FOR Z_0, Z_2, AND Z_{TS} ARE BASED ON MANUFACTURER DATA. THE UNCERTAINTIES IN Z_2 AND Z_{TS} WILL LIKELY BE LARGER THAN ASSUMED HERE. IMPEDANCE UNCERTAINTIES ARE BEST CASE AND WILL TYPICALLY BE LARGER. ALL FIVE UNCERTAINTIES ARE USED FOR THE IC–TS METHOD, WHEREAS ONLY THE TOP TWO ARE USED FOR THE AL–TS METHOD

Parameter p	uncertainty u_p	sensitivity coefficient, relative			
		air line		impedance calibrated	
		formula	value	formula	value
$M_{2,k,50\Omega}$	50x10^{-6}V	$\frac{1}{M_{2,k,50\Omega}}$	50 V^{-1}	$\frac{1}{M_{2,k}}$	50 V^{-1}
$F_{TS,k}$	0.01 V$_{\mathrm{out}}$/V$_{\mathrm{in}}$	$\frac{1}{F_{TS,k}}$	1 V$_{\mathrm{in}}$/V$_{\mathrm{out}}$	$\frac{1}{F_{TS,k}}$	1 V$_{\mathrm{in}}$/V$_{\mathrm{out}}$
Z_2	0.4 Ω	NA	NA	$\frac{Z_0^2 Z_{TS} + Z_2^2 Z_0 + 2 Z_0 Z_2 Z_{TS}}{\left(Z_0 Z_2 + Z_2^2\right)^2}$	0.02 Ω^{-1}
Z_0	0.3 Ω	NA	NA	$\frac{Z_2^3 + Z_2^2 Z_{TS}}{\left(Z_0 Z_2 + Z_2^2\right)^2}$	0.01 Ω^{-1}
Z_{TS}	0.6 Ω	NA	NA	$\frac{Z_0}{\left(Z_0 + Z_2\right) Z_2}$	0.01 Ω^{-1}

instrument in a 50-Ω environment would require a formula similar to that in (7). The uncertainty in $S_{3,k}$ would be greater for the IC–TS method than for the AL–TS method because of the larger number of uncertainty components in the IC–TS method [compare (4) and (7)].

IV. COMPARISON/CONCLUSION

The best way to see the benefits of the TS–AL calibration method compared to the IC–TS method is to compare the uncertainties. These uncertainties are computed and combined using standard methods [3]. However, it should first be stated that not all pulse generators can accept an input signal and, hence, have their input impedance determined. For these pulse generators (typical of high-speed pulse generators), the IC–TS method cannot be applied. Table I provides uncertainty contributions associated with measurements described by (2) and (7). To compute the uncertainty in the measured values, either $u_{S,\mathrm{AL-TS},k}$, for $S_{2,k,50\,\Omega}$ given in (2), or $u_{S,\mathrm{IC-TS},k}$, for $S_{2,k,50\,\Omega}$ given in (7), first compute the square root of the sum of the squares of the product of the uncertainty and its relative sensitivity coefficient [4]. The product of the uncertainty value and its relative sensitivity coefficient is unitless. Therefore, to obtain an uncertainty value (number with units), these products must be multiplied by the corresponding value of $S_{2,k,50\,\Omega}$ that was computed using either (2) or (7).

The first two uncertainty contributions are the same for both methods, which yields an uncertainty of about 0.01 $S_{2,k,50\,\Omega}$. The impedance introduces an additional uncertainty in the IC–TS method of at least 0.01 $S_{2,k,50\,\Omega}$ to give a total estimated uncertainty of at least 0.0144 $S_{2,k,50\,\Omega}$ in this method.

The use of an AL attached to a TS (either a pulse generator or a sampler) to confer the impedance of the AL to that TS is an effective way of measuring how a secondary device will function when attached to a 50-Ω environment. We further propose that the AL method (see Section II) is preferred to the impedance-calculation method (see Section III) because of the following advantages of the AL method over the impedance-calculation method.

1) DUT actually operates in a 50-Ω environment.
2) Uncertainties in measured values are smaller.
3) There are fewer measurement and analysis steps that can introduce biases and errors.
4) There are fewer test instruments and shorter test times.

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