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Impulse spectrum amplitude uncertainty analysis

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Abstract
A detailed uncertainty analysis for an impulse spectrum amplitude (ISA) measurement system is presented. This analysis includes consideration of effects such as temperature, computation algorithms, history of instrument performance, equipment limitations and estimates of the response characteristics of the instrument. With the completion of this work, our published uncertainties have been reduced from ±0.5 dB to less than ±0.1 dB for the parameter of ISA.

Glossary of terms

a_i	dummy variable
c_i	partial derivative of the measurement formula with respect to the ith input variable
f_s	frequency used in timebase calibration
k	frequency counter
m	number of coefficients to fit data
n	discrete time index
k_{eff}	statistical weight
t_c	instant at which peak of jitter occurs in the waveform
t_n	discrete time
u_j	standard uncertainty of jth variable
B	3 dB attenuation bandwidth
H	transfer function of measurement instrument
J	jitter spectrum
M_1	number of waveforms in DUT measurement set
M_2	number of jitter measurements
M_3	number of temperature measurements taken during DUT measurement process
M_4	number of temperature measurements performed during sampler characterization
M_5	number of amplitude–temperature data pairs used to determine temperature effects on sampler's amplitude response
M_6	number of measurement instrument measurements used in calibration
M_7	number of power sensor measurements used in calibration
M_8	number of frequencies used to calibrate timebase
N	number of samples in waveform epoch
P_{ps}	power at power sensor during measurement system calibration
P_{src}	power provided by source
R_1	resistance value of the resistors in the power divider
S	impulse spectrum amplitude
S_R	impulse spectrum amplitude, in dB
S_0	reference for S_R, $S_0 = 1\ \mu$V/MHz
$S_{\Delta V/\Delta T}$	the temperature dependent change in pulse amplitude
$S_{\delta V/\delta T}$	the slope of the curve fit to the pulse amplitude versus temperature data
$SWR_{\max}$	maximum value of standing wave reflection coefficient
T	temperature
$\bar{T}_{\mathrm{meas}}$	average temperature at which a particular waveform was recorded
$\bar{T}_{\mathrm{ref}}$	average temperature during calibration of sampler

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V_k	spectrum of deconvolved spectrum
$V_{m,k}$	magnitude spectrum of measured impulse
$V_{m,T}$	V_m uncorrected for temperature effects
V_p	peak value of the impulse
V_{ps}	amplitude of signal measured by power sensor during measurement system calibration
V_{src}	amplitude of the signal provided by source
V_{sys}	amplitude of signal measured by measurement system during calibration
$W_{f,i}$	discrete spectrum of ith waveform
X	number of cycles of f_s observed in waveform epoch
Z_{dvdr}	impedance of the power divider
Z_{hi}	upper limit for impedance
Z_{lo}	lower limit for impedance
Z_{ps}	input impedance of power sensor
Z_{src}	source (synthesizer) output impedance
Z_{sys}	input impedance of measurement system
Z_{DUT}	output impedance of impulse generator (the DUT)
Z_0	impedance of terminations
Z_1	intermediate impedance value
α_{ps}	efficiency and responsivity of power sensor
Δf	frequency interval
Δt	sampling interval
Γ_{DUT}	transmission coefficient from DUT to the measurement system
Γ_{ps}	transmission coefficient from source to the power sensor as connected per figure 1
Γ_{sys}	transmission coefficient from source to the measurement system as connected per figure 1
ρ	reflection coefficient
ρ_{max}	maximum value of ρ
σ	jitter
σ_j	standard deviation in jth variable
τ	duration of waveform epoch
ν_{eff}	effective degrees of freedom
ν_i	degrees of freedom

1. Introduction

The National Institute of Standards and Technology (NIST) supports a service [1] for measurement of the parameter of impulse spectrum amplitude (ISA) [2] for the output of high-speed (pulse durations <1 ns) impulse generators. Other names for ISA that have been used are spectrum amplitude, voltage spectrum, impulse strength, spectral intensity, impulse spectral intensity, impulse area and spectral density. Heretofore, the primary application of this service has been in the measurement of ISA of signals generated by impulse generators used to characterize electromagnetic interference.

Impulse spectrum amplitude, or one of its synonyms or equivalents, is specified in several international and national standards. For example, one of the response characteristics of receivers (quasi-peak, peak, rms and average measuring) is its response to pulses of a given impulse area (units of µV s or dB(µV s)) [3, 4]. In [5], several related terms (peak power density, spectral power density) are defined and limits

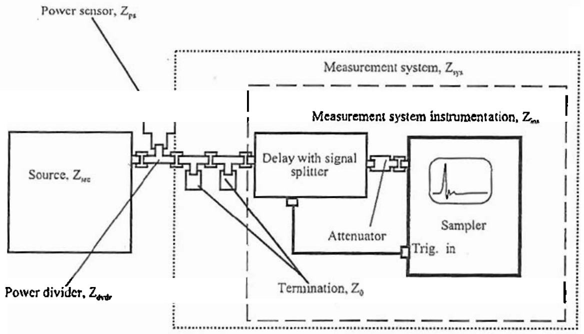

Figure 1. Diagram of instrument setup for measurement system calibration.

(in units of dBm/Hz or in dBm if a 100 kHz bandwidth is assumed) for spurious emissions from transmitters used in data transmission are given. The IEEE guide C62-41 [6] describes spectral density (in units of dB(µV s)) as a parameter for characterizing ac power circuits, and a standard for measuring impulse strength is given in [7]. For certain low-frequency applications, the US Federal Communication Commission (FCC) puts a limit on the power emitted, in units of mV/m or µV/m for a given frequency range, for intentional radiators and in units of µV/m/Hz for certain automotive applications (Subpart C—Intentional Radiators of [8]). For personal communication devices and national information infrastructure devices, the FCC expresses limits in terms of power spectral density (given either in units of mW for a specified bandwidth or dBm/MHz) (Subpart D—Unlicensed Personal Communications Service Devices of [8] and Subpart E—Unlicensed National Information Infrastructure Devices of [8]). More recently, the FCC has defined emission limits for ultra-wideband devices and expresses this limit in terms of EIRP (equivalent isotropically radiated power—defined in [8]) in units of dBm for a given resolution bandwidth (Subpart F—Ultra-Wideband Operation of [8]).

The measurement system described herein presently uses commercially available, high-bandwidth sampling oscilloscopes (3 dB attenuation bandwidths of approximately 80 GHz) to acquire waveforms of pulses generated by high-speed impulse generators. The purpose of this paper is to present the uncertainty analysis for the revised measurement system and system calibration processes used for determining the parameter of ISA. For brevity, not all variables may be described at the point of first use. A list of variables and their description is provided in the glossary.

2. Background

The ISA measurement system is used to acquire and analyse impulse-like signals. If the customer's device (the device under test or DUT) is a baseband impulse generator, such as those used in electromagnetic interference testing, the DUT is connected directly or through other coaxial components (see figure 1) to the measurement system. The ISA measurement system can also measure the response of a receiver to a known impulse. In this case, the parameter measured would not be ISA but the corresponding transfer function.

The signal produced by the generator, $V(t)$, is acquired by the measurement instrument to yield a discretized replica, $V_m[t_n]$ ($n = 1, 2, \ldots, N$, where N is the number of samples and t_n are the sample instants), of $V(t)$. A typical instrument setup used to acquire $V_m[t_n]$ is similar to that shown in figure 1 where the source, power sensor and interconnecting power divider are replaced by the DUT. The measurement process consists of a set of measurements of the DUT and a set of calibration measurements. The calibration measurements include timebase errors, trigger jitter and system magnitude frequency response. The measurement system magnitude response is calibrated using a swept frequency technique (see section 3.2.3). Measurement of the system frequency response is done periodically and a control chart of that response, with uncertainties, is maintained. The frequency intervals are calibrated using sinewave curve fitting methods (see section 3.3). Trigger jitter is measured using the internal software of the sampler. The arrangement for performing the system frequency response calibration is shown in figure 1.

Several sets of data are acquired for the customer's DUT. A set of data consists of M_1 sampler-acquired DUT waveforms and one measurement of the timebase error [9–12]. The DUT measurement sequence is as follows.

1. Measure timebase error: one independent measurement.
2. Acquire waveforms: M_1 independent measurements of DUT output.

The spectra of the acquired waveforms are obtained using Fourier transforms. Corrections are applied to these spectra. Only the magnitudes of the spectra, appropriately scaled to yield units in dB µV/MHz, are reported to the customer.

3. Uncertainty analysis

The reported spectrum is the result of the average of M_1 spectra, one spectrum from each of the M_1 acquired waveforms. Therefore, each frequency component, S_k, of the reported ISA is the average of M_1 values, one value from each of the M_1 spectra:

$$\bar{S}_k = \frac{1}{M_1}\sum_{i=1}^{M_1} S_{k,i}(\alpha_1, \alpha_2, \ldots, \alpha_j), \tag{1}$$

where the subscript k denotes discrete frequency and the a_j are the variables upon which S_k is dependent or is affected; the a_j are dummy variables that will be developed throughout this paper. The expanded uncertainty for $\bar{S}_k$ is given by

$$U_{\bar{S}_k} = k_{\text{eff}}\sqrt{\sum_{i=1}^{M_1}\left[\left(\frac{\partial \bar{S}_k}{\partial S_{k,i}}\right)^2\left\{\sum_j\left(\frac{\partial S_{k,i}(a_j)}{\partial a_j}\right)^2 u_j^2\right\}\right] + \frac{\sigma_{\bar{S}}^2}{M_1}}$$

$$= k_{\text{eff}}\sqrt{\sum_{i=1}\left[\frac{1}{M_1^2}\sum_j\left\{\left(\frac{\partial S_{k,i}(a_j)}{\partial a_j}\right)^2 u_j^2\right\}\right] + \frac{\sigma_{\bar{S}}^2}{M_1}} \tag{2a}$$

$$= k_{\text{eff}}\sqrt{\frac{1}{M_1}\sum_j\left(\frac{\partial S_k(a_j)}{\partial a_j}\right)^2 u_j^2 + \frac{\sigma_{\bar{S}}^2}{M_1}}, \tag{2b}$$

where u_j are the uncertainties in each of the a_j, $\sigma_{\bar{S}}$ is the standard deviation in the set of M_1 measurements of S_k and it is assumed in (2a) that the a_j are uncorrelated, which is the reason there are no cross terms in the partial derivatives with respect to the a_j. The a_j are assumed to be uncorrelated because they correspond to effects from temperature, the calibration process, impedance mismatches, etc. In (2b) it is further assumed that the u_j are the same for every S_i; that is, the uncertainties in the variables for a given parameter are the same for every waveform. The k_{eff} is the statistical weight [13] applied to the uncertainties of variables obtained from a limited number of trials. For a number of variables with different degrees of freedom, k_{eff} is found by first calculating the effective degrees of freedom using [13]:

$$\nu_{\text{eff}} = \frac{\left[\sum_{i=1}^{M}\left(\frac{\partial S_k(a_1, a_2, \ldots, a_M)}{\partial a_i}\right)^2 u_i^2\right]^2}{\sum_{i=1}^{M}\frac{c_i^4 u_i^4}{\nu_i}} \tag{3}$$

where ν_i is the number of degrees of freedom for the parameters and c_i are partial derivatives of the formula for the measurand with respect to each parameter (see tables 1–5). The k_{eff} is then found from ν_{eff} using the Student t-distribution [13].

The uncertainties in these variables, u_i (where the 'i' subscript refers to a parameter), are obtained from independent measurements or from other means that provide values for those particular variables. The tables list the source of u_i for the appropriate variables. To calculate the uncertainty of the different parameters, the partial derivatives of these parameters with respect to the independent variables must be calculated. These partial derivatives are also shown in the tables. The tables for each variable include its type of uncertainty [13] and its degrees of freedom, ν. For measured data, the degree of freedom is given by $\nu = M_q - 1$, where M_q is the number of data elements used to compute the value of the qth variable. For fits to data, ν is given by $\nu = M_q - m_q$, where m_q is the number of coefficients used to fit the data. The degree of freedom for certain variables is equal to infinity ($\nu = \infty$) because the calculation of the value of those variables is based on manufacturer-supplied information.

The variation in measurements represented by the symbol 'σ' in the tables and in the text is, unless otherwise indicated, the standard deviation of the mean of a set of measurement values of a given parameter. For an example, $\sigma_{\sigma_{\text{jit}}}$ described in section 3.2.1 is the standard deviation of M_2 values, one value taken from each of the M_2 jitter measurements. The values shown in curly brackets in the leftmost column of the tables represent typical values of uncertainty or standard deviation. These typical values can be used to approximate the expanded uncertainties expected with this measurement system.

3.1. Computation of ISA

The parameter reported to customers for their DUT is the ISA, $S_{R,k}$, in logarithmic units referenced to 1 µV/MHz. $S_{R,k}$ is given by

$$S_{R,k} = 20\log\left(\frac{S_k}{S_0}\right), \tag{4}$$

where k is the discrete frequency index, $S_0 = 1$ µV/MHz and

$$S_k = 2\frac{|V_k|}{\Delta f}; \tag{5}$$

Table 1. $V_{\delta T}$ uncertainty contribution.

Variable, a_i	Uncertainty, u_{a_i} {typical value[a]}	Partial derivative, $\left\lvert\frac{\partial V_{\delta T}}{\partial a_i}\right\rvert$ {typical value[a]}	Type	Degrees of freedom, ν_i
$\overline{T}_{\mathrm{meas}}$	$\sigma_{T_{\mathrm{meas}}}\{0.5\ \mathrm{K}\}$	$S_{\delta V/\delta T}\{1\ \mathrm{mV\,K^{-1}}\}$	A	M_3-1
$\overline{T}_{\mathrm{ref}}$	$\sigma_{T_{\mathrm{ref}}}\{0.5\ \mathrm{K}\}$	$S_{\delta V/\delta T}\{1\ \mathrm{mV\,K^{-1}}\}$	A	M_4-1
$S_{\delta V/\delta T}$	$u_{S_{\delta V/\delta T}}\{10^{-5}\ \mathrm{V\,K^{-1}}\}$	$\overline{T}_{\mathrm{meas}}-\overline{T}_{\mathrm{ref}}\{1\ \mathrm{K}\}$	A	M_5-2

[a] See section 3.2.2.1 for an explanation.

The uncertainty in the estimated slope from the fit of a straight-line model using least-squares regression is

$$u_{S_{\delta V/\delta T}}=\frac{\sqrt{\dfrac{\sum_{i=1}^{M_5}\left[V_i-\left(V_0+S_{\delta V/\delta T}T_i\right)\right]^2}{M_5-2}}}{\sqrt{\sum_{i=1}^{M_5}\left[T_i-\bar{T}_{\mathrm{meas}}\right]^2}}$$

where V_0 is the voltage intercept.

Table 2. $Z_{\mathrm{src+ps}}$ uncertainty contributions.

Variable, a_i	Uncertainty, u_{a_i} {typical value[a]}	Partial derivative, $\left\lvert\frac{\partial Z_{\mathrm{src+ps}}}{\partial a_i}\right\rvert$ {typical value[a]}	Type	Degrees of freedom, ν_i
Z_{ps}	$u_{Z_{\mathrm{ps}}}\{0.05\}$	$\frac{(R_1+Z_{\mathrm{src}})^2}{(2R_1+Z_{\mathrm{ps}}+Z_{\mathrm{src}})^2}\{0.25\}$	B	∞
R_1	$u_{R_1}\{0.67\}$	$2-2\frac{R_1^2+R_1Z_{\mathrm{ps}}+R_1Z_{\mathrm{src}}+Z_{\mathrm{ps}}Z_{\mathrm{src}}}{(2R_1+Z_{\mathrm{ps}}+Z_{\mathrm{src}})^2}\{1.5\}$	B	∞
Z_{src}	$u_{Z_{\mathrm{src}}}\{0.67\}$	$\frac{(R_1+Z_{\mathrm{ps}})^2}{(2R_1+Z_{\mathrm{ps}}+Z_{\mathrm{src}})^2}\{0.25\}$	B	∞

[a] See section 3.2.3 for an explanation.

Table 3. Z_1 uncertainty contributions.

Variable, a_i	Uncertainty, u_{a_i} {typical value[a]}	Partial derivative, $\left\lvert\frac{\partial Z_1}{\partial a_i}\right\rvert$ {typical value[a]}	Type	Degrees of freedom, ν_i
Z_0	$u_{Z_0}\{0.2\}$	$\frac{(R_1+Z_{\mathrm{ins}})^2}{(2R_1+Z_0+Z_{\mathrm{ins}})^2}\{0.25\}$	B	∞
R_1	$u_{R_1}\{0.67\}$	$2-2\frac{R_1^2+R_1Z_0+R_1Z_{\mathrm{ins}}+Z_0Z_{\mathrm{ins}}}{(2R_1+Z_0+Z_{\mathrm{ins}})^2}\{1.5\}$	B	∞
Z_{ins}	$u_{Z_{\mathrm{ins}}}\{0.25\}$	$\frac{(R_1+Z_0)^2}{(2R_1+Z_0+Z_{\mathrm{ins}})^2}\{0.25\}$	B	∞

[a] See section 3.2.3 for an explanation.

Table 4. Z_{sys} uncertainty contributions.

Variable, a_i	Uncertainty, u_{a_i} {typical value[a]}	Partial derivative, $\left\lvert\frac{\partial Z_{\mathrm{sys}}}{\partial a_i}\right\rvert$ {typical value[a]}	Type	Degrees of freedom, ν_i
Z_0	$u_{Z_0}\{0.2\}$	$\frac{(R_1+Z_1)^2}{(2R_1+Z_0+Z_1)^2}\{0.25\}$	B	∞
R_1	$u_{R_1}\{0.67\}$	$2-2\frac{R_1^2+R_1Z_0+R_1Z_1+Z_0Z_1}{(2R_1+Z_0+Z_1)^2}\{1.5\}$	B	∞
Z_1	u_{Z_1}	$\frac{(R_1+Z_0)^2}{(2R_1+Z_0+Z_1)^2}\{0.25\}$	B	∞

[a] See section 3.2.3 for an explanation.

Table 5. $Z_{\mathrm{src+sys}}$ uncertainty contributions.

Variable, a_i	Uncertainty, u_{a_i} {typical value[a]}	Partial derivative, $\left\lvert\frac{\partial Z_{\mathrm{sys}}}{\partial a_i}\right\rvert$ {typical value[a]}	Type	Degrees of freedom, ν_i
Z_{src}	$u_{Z_{\mathrm{src}}}\{0.67\}$	$\frac{(R_1+Z_{\mathrm{sys}})^2}{(2R_1+Z_{\mathrm{src}}+Z_{\mathrm{sys}})^2}\{0.25\}$	B	∞
R_1	$u_{R_1}\{0.67\}$	$2-2\frac{R_1^2+R_1Z_{\mathrm{src}}+R_1Z_{\mathrm{sys}}+Z_{\mathrm{src}}Z_{\mathrm{sys}}}{(2R_1+Z_{\mathrm{src}}+Z_{\mathrm{sys}})^2}\{1.5\}$	B	∞
Z_{sys}	$u_{Z_{\mathrm{sys}}}$	$\frac{(R_1+Z_{\mathrm{src}})^2}{(2R_1+Z_{\mathrm{src}}+Z_{\mathrm{sys}})^2}\{0.25\}$	B	∞

[a] See section 3.2.4 for an explanation.

where V_k is the kth spectral component of the Fourier transform of the output of the impulse generator and Δf is the frequency interval of the spectrum, which is equal to $1/\tau$, where τ is the waveform epoch. The uncertainty, $u_{S_{R,k}}$ in $S_{R,k}$ is

$$u_{S_{R,k}} = \frac{20}{\ln(10)}\sqrt{\left(\frac{\partial S_{R,k}}{\partial S_k}\right)^2 u_{S_k}^2 + \left(\frac{\partial S_{R,k}}{\partial S_0}\right)^2 u_{S_0}^2}$$
$$= \frac{20}{\ln(10)}\sqrt{\frac{u_{S_k}^2}{S_k^2} + \frac{u_{S_0}^2}{S_0^2}} = \frac{20}{\ln(10)}\frac{u_{S_k}}{S_k}, \quad (6)$$

where $\log(x) = \ln(x)/\ln(10)$, $u_{S_0} = 0$ and the uncertainty in S_k is given by

$$u_{s,k} = \frac{2}{\Delta f}\sqrt{u_{V,k}^2 + \left(\frac{V_k}{\Delta f}\right)^2 u_{\Delta f}^2}, \quad (7)$$

where $u_{V,k}$ is the uncertainty in the voltage spectrum (see section 3.2) and $u_{\Delta f}$ is the uncertainty in the frequency interval (see section 3.3).

3.2. Uncertainty in voltage spectrum, $u_{V,k}$

V_k is obtained from the spectrum, $V_{m,k}$, of the measured signal by deconvolving from $V_{m,k}$ the effect of the measurement instrument, correcting $V_{m,k}$ for temperature effects on the signal and correcting $V_{m,k}$ based on instrument calibration.

Since the ISA is a frequency domain representation, the following discussions will be based on frequency domain data. Time-domain deconvolution is a division of appropriate spectra in the frequency domain. For V_k, this is

$$V_k = \frac{V_{m,k}}{H_{\mathrm{sys},k} J_k T_k}, \quad (8)$$

where $V_{m,k}$ is the discrete spectrum of the measured output of the DUT, J_k is the discrete spectrum of the trigger jitter (this will be discussed in section 3.2.1), $H_{\mathrm{sys},k}$ is the transfer function (Fourier transform of the impulse response) of the measurement system and T_k is the transmission coefficient from the DUT to the measurement system. $H_{\mathrm{sys},k}$ includes both the sampling instrument transfer function, $H_{s,k}$, and that of the auxiliary electronics, $H_{\mathrm{aux},k}$, required to measure the output of the impulse generator:

$$H_{\mathrm{sys},k} = H_{\mathrm{aux},k} H_{s,k}. \quad (9)$$

In the calibration process, $H_{\mathrm{aux},k}$ and $H_{s,k}$ are obtained with the same measurement and are not separable, nor do they need to be.

The uncertainty in V_k, u_{V_k} is

$$u_{V,k} = \left\{\left(\frac{\partial V_k}{\partial V_{m,k}}\right)^2 u_{V_{m,k}}^2 + \left(\frac{\partial V_k}{\partial H_{\mathrm{sys},k}}\right)^2 u_{H_{\mathrm{sys},k}}^2 + \left(\frac{\partial V_k}{\partial J_k}\right)^2 u_{J_k}^2 + \left(\frac{\partial V_k}{\partial T_k}\right)^2 u_{T_k}^2\right\}^{1/2}$$
$$= \frac{V_{m,k}}{H_{\mathrm{sys},k} J_k T_k}\sqrt{\frac{u_{V_{m,k}}^2}{V_{m,k}^2} + \frac{u_{H_{\mathrm{sys},k}}^2}{H_{\mathrm{sys},k}^2} + \frac{u_{J_k}^2}{J_k^2} + \frac{u_{T_k}^2}{T_k^2}}. \quad (10)$$

The V_k-related uncertainties shown in (10) are discussed in the following sections: u_{J_k} is discussed in section 3.2.1, $u_{V_{m,k}}$ is discussed in section 3.2.2, $u_{H_{\mathrm{sys},k}}$ is discussed in section 3.2.3 and u_{T_k} is discussed in section 3.2.4.

3.2.1. Uncertainty from trigger jitter.

Trigger jitter acts as a low pass filter [14] and can be removed by deconvolution if necessary. To compute the effect of the jitter on the ISA, the mean value of the jitter and its uncertainty must be obtained. This information is obtained by measuring the jitter M_2 times, yielding M_2 rms jitter values, $\sigma_{\mathrm{jit},m}$, $m = 1, 2, \ldots, M_2$. The standard deviation, $\sigma_{\sigma_{\mathrm{jit}}}$, in the mean of the set of M_2 rms jitter values is

$$\sigma_{\sigma_{\mathrm{jit}}} = \sqrt{\frac{1}{M_2}\frac{1}{M_2 - 1}}\sqrt{\sum_m \left(\bar{\sigma}_{\mathrm{jit}} - \sigma_{\mathrm{jit},m}\right)^2}, \quad (11)$$

where

$$\bar{\sigma}_{\mathrm{jit}} = \frac{1}{M_2}\sum_{m=1}^{M_2} \sigma_{\mathrm{jit},m}. \quad (12)$$

However, (12) gives a time-domain value and what is needed in the uncertainty analysis is a frequency domain value. To get that frequency domain value, the effective discrete time jitter impulse response is first computed, and this is given by

$$j[t_n] = \frac{1}{\bar{\sigma}_{\mathrm{jit}}\sqrt{2\pi}} \mathrm{e}^{-\frac{1}{2}\left(\frac{t_n - t_c}{\bar{\sigma}_{\mathrm{jit}}}\right)^2}, \quad (13)$$

where t_c is an arbitrary time when the peak of $j[t_n]$ occurs. The discrete spectrum of $j[t_n]$, J_k is

$$J_k = \mathrm{e}^{-2(\pi\bar{\sigma}_{\mathrm{jit}} k\Delta f)^2}. \quad (14)$$

This can be approximated accurately for typical waveforms[15] by keeping the first two terms of a Taylor series expansion:

$$J_k \approx 1 - 2(\pi\bar{\sigma}_{\mathrm{jit}} k\Delta f)^2. \quad (15)$$

The uncertainty u_{J_k}, for J_k shown in (14), is

$$u_{J_k} = 4\pi^2 k^2 \bar{\sigma}_{\mathrm{jit}} \Delta f \sqrt{\bar{\sigma}_{\mathrm{jit}}^2 u_{\Delta f}^2 + \Delta f^2 \sigma_{\sigma_{\mathrm{jit}}}^2}\, \mathrm{e}^{-2(\pi\bar{\sigma}_{\mathrm{jit}} k\Delta f)^2}. \quad (16)$$

A typical value that u_{J_k} will not exceed is 2.47×10^{-5}, which is for $k = 100$ (hundredth frequency element), $\Delta f = 12.5$ MHz, $\bar{\sigma}_{\mathrm{jit}} \approx 10^{-12}$ s, $u_{\Delta f} = 1$ kHz (see section 3.3) and $\sigma_{\sigma_{\mathrm{jit}}} = 10^{-13}$ s.

3.2.2. Uncertainty in $V_{m,k}$

There are several possible contributors to the uncertainty in $V_{m,k}$, which include sampler impulse response estimate uncertainty, sampler amplitude gain uncertainty, measurement temperature variations, measurement noise, aliasing and connector repeatability.

Measurement noise and connector repeatability are manifested as Type A uncertainties and, as such, are estimated from the statistics of repeated measurements. Therefore, their effects on total uncertainty do not need to be computed independently. Sampler amplitude gain and impulse response uncertainties are determined in the measurement system calibration process. Consequently, the uncertainty in $V_{m,k}$, $u_{V_{m,k}}$, can be described by

$$u_{V_{m,k}} = \sqrt{\frac{\sigma_{V_{m,k}}^2}{M_1} + u_{\delta V_T}^2 + u_{\mathrm{alias},k}^2}, \quad (17)$$

where $\sigma_{V_{m,k}}$ is the standard deviation of the mean of the set of M_1 measurements of $V_{m,k}$. The other two contributions to $u_{V_{m,k}}$, temperature and aliasing, are discussed in the following sections.

Measurement temperature. Measurement temperature variations can affect the electrical characteristics and behaviour of the components of the ISA measurement system. For example, temperature will affect resistance values, diode capacitances, etc, which are integral components in power dividers, terminations, delay lines, power sensor and samplers. Other than for the sampler, the uncertainties in the values of the applicable performance characteristics of the measurement system components include temperature effects. For example, the values of impedance uncertainties for the power dividers and terminations used herein are very conservative, thus accommodating effects caused by any minor temperature changes (less than 1 K) encountered during a typical measurement. The change in pulse amplitude and transition duration caused by the sampler temperature effects has been documented [16]. The change in transition duration with temperature is small and, for the sampling interval presently used (40 ps) for the ISA measurement system, will not be observed. The effect of temperature on the amplitude of the measured impulse is to add an additional component, $V_{\delta T}$, to the signal, where

$$V_{\delta T} = S_{\delta V/\delta T}(\overline{T}_{\text{meas}} - \overline{T}_{\text{ref}}) \tag{18}$$

and $\overline{T}_{\text{meas}}$ is the average of M_3 sampler temperature values taken during the measurement process, S is the slope of a straight line fit through a set of previously-acquired amplitude-versus-temperature data and $\overline{T}_{\text{ref}}$ is the average reference sampler temperature that is taken to be the mean of M_4 temperature values of the sampling head taken when the sampler impulse response was determined. The amplitude-versus-temperature data consist of a set of M_5 data pairs and are taken over a temperature range between T_2 and T_1; the difference between these two temperatures is δT. $V_{\delta T}$ will cause the amplitude of the impulse to change. This change can be approximated as a change in the amplitude gain of the sampler that will result in a uniform change in the amplitude of each component of the spectrum of the impulse, to give

$$V_{m,k} = V_{m,T,k}\left(1 - \frac{V_{\delta T}}{V_{\text{p}}}\right), \tag{19}$$

where V_{p} is the peak amplitude of the impulse and $V_{m,T,k}$ is $V_{m,k}$ uncorrected for temperature effects. The uncertainty in the spectrum amplitude due to temperature effects can be described by

$$u_{\delta V_T,k} = \frac{V_{m,T,k}}{V_{\text{p}}}\sqrt{\left(\frac{V_{\delta T}}{V_{\text{p}}}\right)^2 \sigma_{V_{\text{p}}}^2 + u_{V_{\delta T}}^2}, \tag{20}$$

where $u_{V_{\delta T}} = 7.14 \times 10^{-4}$ V (see table 1), $V_{\delta T} \approx 10^{-5}$ V [10], $V_{\text{p}} \approx 500$ mV (after attenuator), $\sigma_{V_{\text{p}}} \approx 10^{-4}$ V and $V_{m,T,k}$ is the spectrum magnitude uncorrected for temperature. These give an estimate of the value of $u_{\delta V_T,k}$ of $1.4 \times 10^{-3} V_{m,T,k}$.

Aliasing. Aliasing occurs when the sampling rate is not sufficient to capture the fast transients of a signal. This error is approximated by [17, 18]

$$e[k\Delta f] = 9.5(B\Delta t)^2 V_m, \tag{21}$$

where B is the 3 dB attenuation bandwidth of the spectrum of the measured impulse. To be conservative, this error is used as the aliasing contribution, u_{alias}, to the spectrum amplitude uncertainty. At 2 GHz and for typical measurement conditions ($B = 1$ GHz, $\Delta t = 20$ ps), $u_{\text{alias},k} \approx 4 \times 10^{-3} V_{m,k}$ (for $k\Delta f = 2$ GHz). For typical values, $u_{\text{alias},k} \approx 2 \times 10^{-5}$ V up to 2 GHz (or 5×10^{-6} V/GHz^2).

3.2.3. *Computation of uncertainty contribution from $H_{\text{sys},k}$.*

$H_{\text{sys},k}$ is obtained during the measurement instrument calibration process and is found from an independent set of measurements using a microwave synthesizer. In this calibration process, $H_{\text{sys},k}$ is found by comparing the signal measured using the ISA measurement system, $V_{\text{sys},k}$, to that measured using an rf power meter, $V_{\text{ps},k}$:

$$H_{\text{sys},k} = \frac{V_{\text{sys},k}}{V_{\text{ps},k}}. \tag{22}$$

The standard uncertainty, $u_{H_{\text{sys},k}}$, in $H_{\text{sys},k}$ is given by

$$\begin{aligned} u_{H_{\text{sys},k}} &= \sqrt{\left(\frac{\partial H_{\text{sys},k}}{\partial V_{\text{sys},k}}\right)^2 u_{V_{\text{sys},k}}^2 + \left(\frac{\partial H_{\text{sys},k}}{\partial V_{\text{ps},k}}\right)^2 u_{V_{\text{ps},k}}^2} \\ &= \frac{V_{\text{sys},k}}{V_{\text{ps},k}}\sqrt{\frac{u_{V_{\text{sys},k}}^2}{V_{\text{sys},k}^2} + \frac{u_{V_{\text{ps},k}}^2}{V_{\text{ps},k}^2}}. \end{aligned} \tag{23}$$

$V_{\text{sys},k}$ and $V_{\text{ps},k}$ can be expanded as

$$V_{\text{sys},k} = \Gamma_{\text{sys}} V_{\text{src},k}, \tag{24}$$

$$V_{\text{ps},k} = \Gamma_{\text{ps}}\sqrt{\alpha_{\text{ps}}} V_{\text{src},k}, \tag{25}$$

where Γ_{ps} is the transmission coefficient of the signal transmitted to the power sensor from the source, Γ_{sys} is the transmission coefficient of the signal transmitted to the measurement system from the source, α_{ps} is the conversion efficiency of the power sensor (that may be a function of frequency) and $V_{\text{src},k}$ is the voltage signal from the source, which is given by

$$V_{\text{src},k} = \sqrt{P_{\text{src},k} Z_{\text{src}}}. \tag{26}$$

Because the power and voltage from the source are a set quantity and are common to both the power sensor and the measurement system during calibration of the measurement system (which is a ratiometric method), there are no uncertainties to ISA from $V_{\text{src},k}$ and $P_{\text{src},k}$. However, noise and fluctuations in $V_{\text{src},k}$ and $P_{\text{src},k}$ will be manifest as noise in $V_{\text{sys},k}$ and $V_{\text{ps},k}$, which are already considered (see subsections below). The impedance for each system component is taken to be the same (50 Ω) and constant over the measurement frequency range. The impedance uncertainty for each system component is set to a value that accommodates any deviation of a component's actual impedance from 50 Ω. Therefore, Γ_{ps} and Γ_{sys} are not shown here as a function of frequency. However, all impedances and consequently, all impedance-dependent variables can be changed to being frequency dependent (by adding the 'k' subscript) in this uncertainty analysis without impacting this analysis.

Uncertainty in $V_{\mathrm{sys},k}$. The uncertainty in $V_{\mathrm{sys},k}$ can be computed using (24):

$$u_{V_{\mathrm{sys},k}} = \sqrt{V_{\mathrm{src},k}^2 u_{\Gamma_{\mathrm{sys}}}^2 + \frac{\sigma_{V_{\mathrm{sys},k}}^2}{M_6}}, \quad (27)$$

where $\sigma_{V_{\mathrm{sys},k}}$ is the standard deviation in the mean of the set of M_6 measurements of $V_{\mathrm{sys},k}$. Γ_{sys} describes the efficiency at which the signal is transmitted from the source to the measurement instrument. This efficiency is based on the impedance observed by the instrument and by the source. Γ_{sys} can be written as

$$\Gamma_{\mathrm{sys}} = \frac{2Z_{\mathrm{src+ps}}}{Z_{\mathrm{sys}} + Z_{\mathrm{src+ps}}}, \quad (28)$$

where Z_{sys} is the input impedance of the measurement system (see figure 1) and $Z_{\mathrm{src+ps}}$ is the impedance of the combination of the source with the power sensor connected by the power divider.

The uncertainty in Γ_{sys} is given by

$$u_{\Gamma_{\mathrm{sys}}} = \frac{2}{(Z_{\mathrm{sys}} + Z_{\mathrm{src+ps}})^2}\sqrt{Z_{\mathrm{sys}}^2 u_{Z_{\mathrm{src+ps}}}^2 + Z_{\mathrm{src+ps}}^2 u_{Z_{\mathrm{sys}}}^2}. \quad (29)$$

Using figure 1, $Z_{\mathrm{src+ps}}$ can be obtained, and it is

$$Z_{\mathrm{src+ps}} = R_1 + \frac{(R_1 + Z_{\mathrm{ps}})(R_1 + Z_{\mathrm{src}})}{2R_1 + Z_{\mathrm{ps}} + Z_{\mathrm{src}}}, \quad (30)$$

where R_1 is the resistance value of one arm of the power divider. (The power divider is a three-port device with the signal conductor of each port connected to a common node via a resistor of resistance R_1. R_1 is nominally the same for each of the three arms of the divider.) Table 2 gives the weights, degrees of freedom, type and uncertainty for the contributions to the uncertainty in $Z_{\mathrm{src+ps}}$.

Z_{sys} is the result of two power dividers, two 50 Ω terminations and the measurement instrument. The uncertainty in $Z_{\mathrm{sys},k}$ is more easily computed if Z_{sys} is computed in two steps, one using the first power divider to yield Z_1, and the second step using Z_1 and the second power divider. The value of Z_1 can be computed from

$$Z_1 = R_1 + \frac{(R_1 + Z_0)(R_1 + Z_{\mathrm{ins}})}{2R_1 + Z_0 + Z_{\mathrm{ins}}}, \quad (31)$$

where Z_0 is the impedance of the terminations (nominally 50 Ω) and Z_{ins} is the input impedance of the measurement instrument (see figure 1). Table 3 gives the weights, degrees of freedom, type, and uncertainty for the contributions to the uncertainty in Z_1.

Z_{sys} can then be computed using Z_1

$$Z_{\mathrm{sys}} = R_1 + \frac{(R_1 + Z_0)(R_1 + Z_1)}{2R_1 + Z_0 + Z_1}. \quad (32)$$

Table 4 gives the weights, degrees of freedom, type and uncertainty for the contributions to the uncertainty in Z_{sys}. The value of R_1 is given by

$$R_1 = \frac{Z_{\mathrm{dvdr}}}{3}, \quad (33)$$

where Z_{dvdr} is the impedance of the power divider. The uncertainty in R_1 is then

$$u_{R_1} = \frac{1}{3} u_{Z_{\mathrm{dvdr}}}. \quad (34)$$

The uncertainty in impedance is computed from the maximum standing wave reflection (SWR) values provided by the manufacturer (see section 3.2.5).

Uncertainty in $V_{\mathrm{ps},k}$. The uncertainty in $V_{\mathrm{ps},k}$, using (25), is

$$u_{V_{\mathrm{ps},k}} = \sqrt{\left(\Gamma_{\mathrm{ps}} V_{\mathrm{src},k} \alpha_{\mathrm{ps}}^{1/2}\right)^2 \left(\frac{u_{\Gamma_{\mathrm{ps}}}^2}{\Gamma_{\mathrm{ps}}^2} + \frac{1}{4}\frac{u_{\alpha_{\mathrm{ps}}}^2}{\alpha_{\mathrm{ps}}^2}\right) + \frac{\sigma_{P_{\mathrm{ps},k}}^2}{M_7} Z_{\mathrm{ps}}}, \quad (35)$$

where Γ_{ps} describes the efficiency at which the signal is transmitted from the source to the power sensor, $\sigma_{P_{\mathrm{ps},k}}$ is the standard deviation in the mean of the set of M_7 power measurement readings taken with the power sensor and the values for α_{ps} and $u_{\alpha_{\mathrm{ps}}}$ are provided by the manufacturer.

The uncertainty in Γ_{ps} is computed from Γ_{ps}, which is given by

$$\Gamma_{\mathrm{ps}} = \frac{2Z_{\mathrm{src+sys}}}{Z_{\mathrm{ps}} + Z_{\mathrm{src+sys}}}, \quad (36)$$

where $Z_{\mathrm{src+sys}}$ is the impedance of the combination of the source with the measurement system. The uncertainty in Γ_{ps} is given by

$$u_{\Gamma_{\mathrm{sys}}} = \frac{2}{\left(Z_{\mathrm{ps}} + Z_{\mathrm{src+sys}}\right)^2}\sqrt{Z_{\mathrm{sp}}^2 u_{Z_{\mathrm{src+sys}}}^2 + Z_{\mathrm{src+sys}}^2 u_{Z_{\mathrm{ps}}}^2}, \quad (37)$$

where $Z_{\mathrm{src+sys}}$ is

$$Z_{\mathrm{src+sys}} = R_1 + \frac{(R_1 + Z_{\mathrm{src}})\left(R_1 + Z_{\mathrm{sys}}\right)}{2R_1 + Z_{\mathrm{src}} + Z_{\mathrm{sys}}}. \quad (38)$$

Table 5 gives the weights, degrees of freedom, type and uncertainty for the contributions to the uncertainty in $Z_{\mathrm{src+sys}}$.

3.2.4. Uncertainty in Γ_{DUT}. The transmission coefficient of the interface between the DUT and the measurement instrument for a signal propagating from the DUT towards the measurement instrument is

$$\Gamma_{\mathrm{DUT}} = \frac{2Z_{\mathrm{sys}}}{Z_{\mathrm{sys}} + Z_{\mathrm{DUT}}}, \quad (39)$$

where Z_{DUT} is the impedance of the DUT. The uncertainty in Γ_{DUT} is

$$u_{\Gamma_{\mathrm{DUT}}} = \frac{2Z_{\mathrm{src}}}{(Z_{\mathrm{sys}} + Z_{\mathrm{DUT}})^2} u_{Z_{\mathrm{sys}}}, \quad (40)$$

where $u_{Z_{\mathrm{sys}}}$ is shown in table 4. There is no uncertainty associated with Z_{DUT} because this value is assumed to be known exactly. However, if a customer provides Z_{DUT} and uncertainties, then the uncertainty analysis can be adjusted to accommodate that information.

3.2.5. Impedance uncertainties from SWR values. The typical impedance uncertainties used in this measurement system, which are given in curly brackets in the second column of the tables, are based on the maximum value of SWR (SWR_{max}) coefficients provided by the manufacturer. From SWR_{max}, the maximum impedance uncertainty, $u_{Z\,max}$, in a 50 Ω environment is computed. Since the manufacturing tolerances of the terminations, power splitters and coaxial connectors exhibit a normal distribution, it is presumed herein that the maximum uncertainty corresponds to a uniform distribution. To obtain a 1-sigma confidence interval for computation of ISA uncertainty, $u_{Z\,max}$ is divided by $3^{1/2}$.

$u_{Z\,max}$ is computed by starting with the reflection coefficient, ρ, from an impedance discontinuity:

$$Z_{DUT} = \frac{1+\rho}{1-\rho} Z_0, \tag{41}$$

where Z_{DUT} is the input impedance of the device for which the SWR value is given and Z_0 is the impedance of the measurement system, typically 50 Ω. The absolute value of the maximum value of ρ can be determined from SWR_{max}:

$$|\rho_{max}| = \frac{SWR_{max} - 1}{SWR_{max} + 1}. \tag{42}$$

Then the following is used to determine $u_{Z\,max}$:

$$\begin{aligned} Z_{lo} &= \frac{1-|\rho_{max}|}{1+|\rho_{max}|} Z_0, \\ Z_{hi} &= \frac{1+|\rho_{max}|}{1-|\rho_{max}|} Z_0, \\ u_{Z\,max} &= \frac{Z_{hi} - Z_{lo}}{2}. \end{aligned} \tag{43}$$

The typical value for the partial derivatives, which are given in curly brackets in the third column of the tables, is based on using the nominal values for the formula parameters. For example, the nominal values of Z_0, Z_1, R_0, Z_{src} and Z_{sys} are 50 Ω.

3.3. Frequency interval uncertainty, $u_{\Delta f}$

The value of the frequency interval (Δf) is based on the duration of the waveform epoch (τ), which is in turn calibrated using sinewave curve fitting techniques. In this calibration process, a nominally single-frequency sinusoidal signal provided by a microwave synthesized frequency source is input into the measurement system and waveforms of these signals are acquired. Errors in the timebase are derived from sinewave curve fitting techniques [11]. The vertical gain of the samplers is calibrated using swept frequency techniques to ensure the sinewave curve fitting method is not affected by gain non-linearities. No statistically significant change in sinewave curve fitting parameters has been observed as a function of amplitude values in a range typically encountered in the ISA measurement system. The vertical offset is always set to 0 V and is not considered in this uncertainty analysis. The synthesizer is the artefact standard and the manufacturer-claimed uncertainty in frequency accuracy (10 ppm of set value) is used as the uncertainty in the frequencies used to calibrate the timebase. Δf is related to τ by

$$\Delta f = \frac{1}{M_8} \sum_{i=1}^{M_8} \frac{1}{\tau_i} = \frac{1}{M_8} \sum_{i=1}^{M_8} \frac{f_{s,i}}{X_i}, \tag{44}$$

where X_i is the number of cycles of $f_{s,i}$ observed in the waveform epoch, $f_{s,i}$ is the ith frequency used to calibrate the timebase and M_8 is the number of frequencies used in the sinewave curve fitting method. The uncertainty in Δf, $u_{\Delta f}$, is

$$\begin{aligned} u_{\Delta f} &= \frac{1}{M_8} \sqrt{\sum_{i=1}^{M_8} \left[\left(\frac{1}{X_i}\right)^2 u_{f_{s,i}}^2 + \left(\frac{f_{s,i}}{X_i^2}\right)^2 u_{X_i}^2 \right]} \\ &\leqslant \frac{1}{M_8} \sqrt{\sum_{i=1}^{M_8} \left[\left(\frac{1}{X}\right)^2 u_{f_s}^2 + \left(\frac{f_s}{X^2}\right)^2 u_X^2 \right]} \\ &= \Delta f_{max} \sqrt{M_8} \sqrt{\left(\frac{u_{f_s}^2}{f_s^2} + \frac{u_X^2}{X^2} \right)}, \end{aligned} \tag{45}$$

where X is the smallest of the X_i, u_X is the uncertainty in the sinewave curve fitting method and is independent of frequency, f_s is the largest of the frequencies used, u_f is the largest uncertainty in the frequencies produced by the frequency source and Δf_{max} is the largest value of Δf. The value of u_X is approximately

$$u_X \approx \frac{1}{2} \frac{\Delta t}{\tau} = \frac{1}{2N}, \tag{46}$$

where Δt is the sampling interval, τ is the duration of the waveform epoch and N is the number of samples in the waveform. Typical values for the parameters in (45) are: $M_8 \geqslant 4$, $N \geqslant 4096$, $X = 10$, $u_X \approx 10^{-4}$, $\Delta f_{max} = 10$ MHz and $u_{f_s} = 1$ kHz (for $f_s = 100$ MHz) and these values give a $u_{\Delta f} \leqslant 500$ Hz. We are in the process of reducing u_f by more than 100 times by using global-positioning-system-(GPS-) based traceability to NIST frequency standards.

4. Example measurement result

Figure 2 shows an example of measurement results obtained using the new ISA measurement system calibration method and associated uncertainty analysis. The pulse generator used produces pulses with a peak amplitude of approximately 5.5 V into 50 Ω at a pulse repetition rate of approximately 100 Hz. The values of ISA (curve labelled 'S' in figure 2) are nominally the same (within measurement reproducibility) as that obtained for the ISA measurement system using the previous calibration method. The published uncertainties for the measurement system and the 'old' calibration method are 0.5 dB, which is much greater than for the new calibration method and associated uncertainty analysis. The published uncertainties will be approximately 0.1 dB, much larger than the expanded uncertainties shown in figure 1, until an adequate number of tests have been performed to corroborate the significant reduction in expanded uncertainty for ISA.

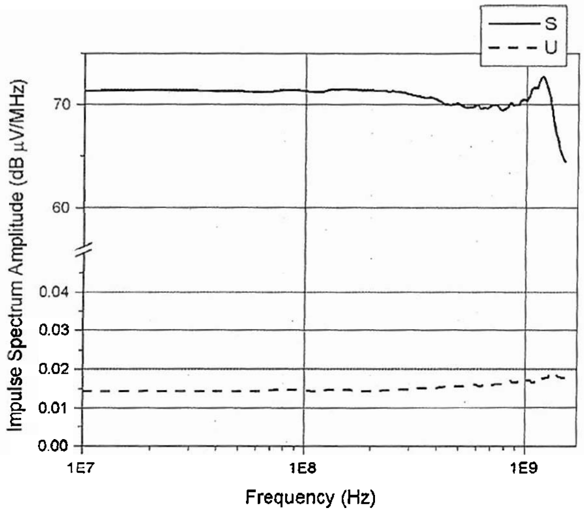

Figure 2. Impulse spectrum amplitude (S) and expanded uncertainty (U) obtained from an impulse generator using the new calibration method and associated uncertainty analysis.

5. Summary

A detailed uncertainty analysis of the parameter of spectrum amplitude of impulse-like waveforms was performed. This analysis included a consideration of effects that can impact the value of the reported parameters, such as temperature, computation algorithms, history of instrument performance, equipment limitations and estimates of the response characteristics of the instrument. New published uncertainties for the parameter of ISA for high-speed (pulse duration <1 ns) impulse generators and receivers are ±0.1 dB, which, to be conservative, is much larger than the expanded uncertainties. These new uncertainties are the result of the new calibration process and associated uncertainty analysis.

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