Progress on Johnson Noise Thermometry Using a Quantum Voltage Noise Source for Calibration

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***Abstract*—We describe our progress toward a high-precision measurement of temperature using Johnson noise. Using a quantized voltage noise source (QVNS) based on the Josephson effect as a calculable noise source, we have been able to measure the ratio of the gallium and water triple-point temperatures to within an accuracy better than 100 μK/K. We also describe the operation of our Johnson noise thermometry system that could be used as a primary thermometer and possible sources of error that limit our absolute temperature measurements to ~ 150 μK/K.**

***Index Terms*—Correlation, digital-analog conversion, frequency control, frequency synthesizers, Josephson junction arrays, noise, quantization, signal synthesis, standards, superconducting microwave devices, superconductor-normal-superconductor devices, temperature, temperature measurement, voltage control.**

I. Introduction

JOHNSON noise thermometry (JNT) involves the measurement of the statistical variance of a fluctuating voltage across a resistor in thermal equilibrium. The spectral density S^{R} of the voltage noise power across a resistor $R(T)$ is given by the Nyquist formula [1]

$$S^{\mathrm{R}} = 4kTR(T) \tag{1}$$

where k is Boltzmann's constant and T is the temperature of the resistor. We have built a system that uses a digitally-synthesized quantized voltage noise source (QVNS). The QVNS uses an array of Josephson junctions as an ac voltage reference to correct for the nonideal performance of the electronic systems used to measure the noise. The QVNS is programmed to emulate resistor noise with a constant spectral density that is stable, programmable, and calculable [2]. With the QVNS, we have operated our system in two distinct modes, "absolute" and "relative."

A. Absolute Mode

In the "absolute" mode of operation, the temperature of the thermometer (resistor) is directly determined by comparing the noise power from the resistor with the pseudonoise power from the QVNS [3]. Using (1), the thermodynamic temperature of the resistor can be determined from the following equation:

$$T = \left(\frac{S^{\mathrm{R}}_{\mathrm{measured}}}{S^{\mathrm{QVNS}}_{\mathrm{measured}}}\right)\frac{S^{\mathrm{QVNS}}_{\mathrm{calculated}}}{4kR(T)} \tag{2}$$

where $S^{\mathrm{R}}_{\mathrm{measured}}$ is the measured resistor noise power density, $S^{\mathrm{QVNS}}_{\mathrm{measured}}$ is the measured power density of the QVNS-synthesized pseudonoise waveform, and $S^{\mathrm{QVNS}}_{\mathrm{calculated}}$ is the calculated power density of the pseudonoise waveform.

B. Relative Mode

In the "relative" mode, the Johnson noise power of a resistor at a known temperature is first measured against a balanced, synthesized noise power from the QVNS. The process is repeated with the same resistor at an unknown temperature with a second QVNS waveform that is comparable to the noise power of the resistor at this temperature. The ratio of the unknown temperature T_0 to the reference temperature T_0 can be determined with the following formula:

$$T = \frac{S^{\mathrm{R@T}}_{\mathrm{measured}}}{S^{\mathrm{QVNS@T}}_{\mathrm{measured}}}\frac{S^{\mathrm{QVNS@T_0}}_{\mathrm{measured}}}{S^{\mathrm{R@T_0}}_{\mathrm{measured}}}\frac{S^{\mathrm{QVNS@T}}_{\mathrm{calculated}}}{S^{\mathrm{QVNS@T_0}}_{\mathrm{calculated}}}\frac{R(T_0)}{R(T)}T_0 \tag{3}$$

where $S^{\mathrm{R@T}}_{\mathrm{measured}}, S^{\mathrm{R@T_0}}_{\mathrm{measured}}, S^{\mathrm{QVNS@T}}_{\mathrm{measured}}$, and $S^{\mathrm{QVNS@T_0}}_{\mathrm{measured}}$ are the measured noise power density of the resistor and QVNS at both temperatures/power levels and $S^{\mathrm{QVNS@T}}_{\mathrm{calculated}}$ and $S^{\mathrm{QVNS@T_0}}_{\mathrm{calculated}}$ are the calculated power densities for the signal from the QVNS. This mode of operation will be less susceptible to systematic errors that occur in the absolute mode from differences in the transmitted noise power from the QVNS to the preamplifiers compared to that between the resistor and preamplifiers (e.g., different transmission lines).

II. Experimental Setup

The Johnson noise thermometry system consists of three major components: the temperature cell and resistor probe, the QVNS subsystem, and cross-correlation electronics.

A. Temperature Cell and Resistor

The Johnson noise resistor is a metal-foil surface-mount device with a resistance of ~ 100 Ω. The resistor is mounted in a electrically shielded temperature probe that is inserted into an immersion-type, fixed-point cell. We report data from our experiments with both a gallium triple-point (GATP) cell and a water triple-point (WTP) cell.

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B. QVNS

The QVNS is used to generate a constant-amplitude comb of harmonic tones with random relative phases to match the noise power density from a resistor for frequencies less than 2 MHz. The signals from the QVNS originate from an array of microwave pulse-biased Josephson junctions and are calculable from fundamental constants and digital synthesis algorithms [2].

C. Cross-Correlation Electronics

The most successful JNT measurement technique developed to date is switched-input noise-correlation pioneered by Brixy *et al.* [4]. In a noise-correlation measurement, a cross-correlation technique is used to reduce the systematic error from the electronics that measure the noise. In our cross-correlation electronics, a signal from the resistor is amplified and digitized through two parallel electronics chains. The digitized signals are Fourier transformed and cross-correlated with a computer. After a short interval of time (typically 100 s), the inputs to the cross-correlation electronics are switched to the signal from the QVNS through the use of relays. The QVNS signal is also amplified and cross-correlated through the same electronics chain. After accumulation of a comparable number of spectra, the inputs to the cross-correlation electronics are switched back to the resistor.

III. Results

The biggest improvement to our JNT measurements [3] has resulted from the use of new circuit designs for the QVNS that have dramatically reduced the common mode voltage error measured by the Johnson noise measurement electronics. The design changes included directly grounding the two Josephson junction arrays to the code generator ground and removing inductive filters on the ground lines between the two arrays. These changes have enabled a significant improvement in the demonstrated accuracy of both the relative and absolute mode measurements.

Using the GaTP cell, we obtained the cross-correlated spectrum from a 100 Ω resistor shown in Fig. 1. The spectrum is an average of 30 000 spectra with frequency bins spaced at ~1 Hz. The cross-correlated spectrum from a QVNS is also shown in Fig. 1. The output from the QVNS is a comb of tones starting at ~1.589 kHz and continuing at that frequency spacing up to 2 MHz. Each tone has an rms value of approximately 51.55 nV to match the noise power density of the resistor. To directly determine the temperature of the resistor, the Johnson noise spectra from the resistor must be re-binned at each QVNS tone over a bandwidth equal to the spacing of the QVNS tones. The resulting re-binned data can then be divided by the power in each QVNS tone, producing a noise power ratio that is directly proportional to temperature and is shown in the lower scatter plot in Fig. 2. An analysis of data taken over 16 h over the first 100-kHz band results in $T_{\mathrm{GaTP}} = 302.872$ K with a type-A, 1-σ uncertainty of 10 mK which differs by 150 μK/K from the temperature 302.916 K of the GaTP cell that is derived from the ITS-90 melting point of gallium. Using the WTP cell and a properly balanced noise power from the QVNS, we found that $T_{\mathrm{WTP}} = 273.115$ K with a type-A, 1-σ uncertainty of 8 mK after 21 h of integration time. Our measured WTP temperature is smaller by 165 μK/K than the 273.160 K WTP ITS-90 value.

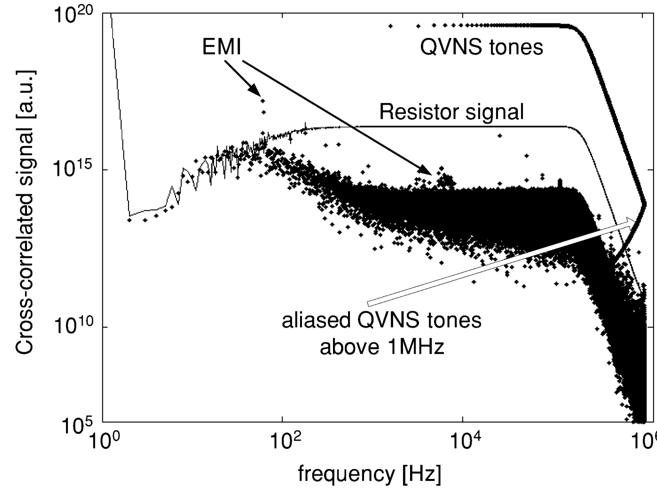

Fig. 1. Cross-correlated noise signal from a 100 Ω resistor at the gallium triple-point averaged for 30 000 s (line). The dots represent the signal from a QVNS matched in power density to the resistor noise. Each dot represents the power in a 1 Hz frequency bin. The upper dots represent signal generated by the QVNS. The dots below the line represent measurements of the noise floor.

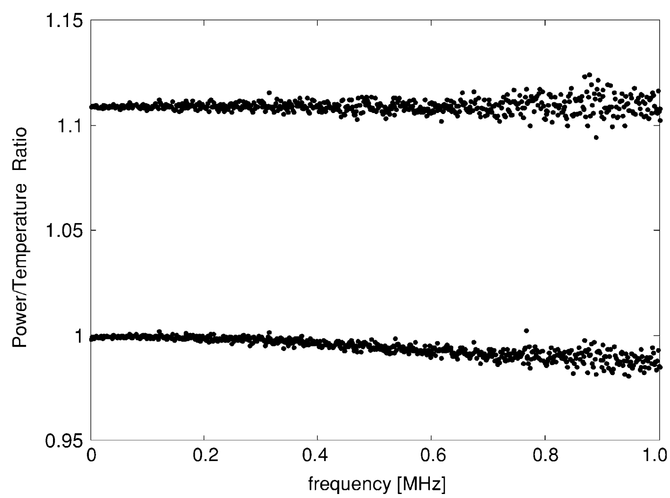

Fig. 2. Upper scatter plot is the measured noise power ratio of the resistor at the GaTP and at the WTP. The accepted ratio of the temperatures based on ITS-90 is 1.108 935. The lower scatter plot is the ratio of the noise power from the resistor to the noise power from the QVNS. The accepted ratio should be 0.999 771.

Using the two absolute measurements of the temperature, it is possible to determine the ratio of the GaTP to the WTP (relative mode of operation). The temperature ratio as a function of frequency is also shown in the upper plot in Fig. 2. Using the two absolute ratios below 100 kHz, we find the relative-mode, temperature ratio to be 1.108 955 with a type-A, 1-σ relative uncertainty of 4×10^{-5}. The accepted ratio derived from ITS-90 is 1.108 935. The ratio using a new measurement of the gallium melting point that was recently determined by acoustic gas thermometry is 1.108 952 [5].

IV. Discussion

For absolute determination of the thermodynamic temperature of the resistor, the power-ratio between the QVNS and the

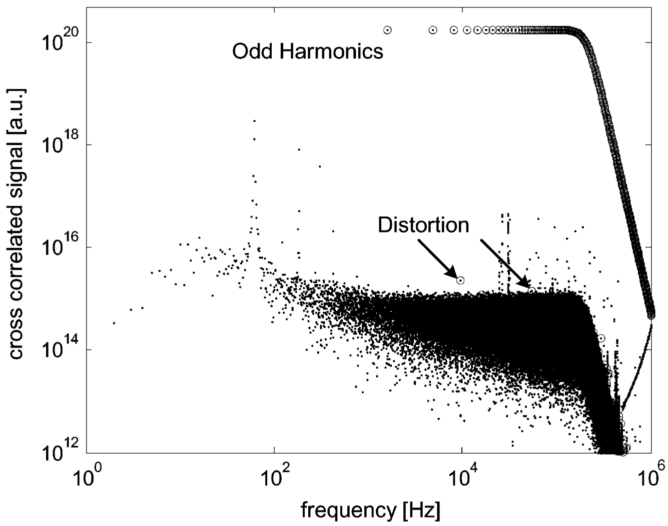

Fig. 3. Measurement of the power from the QVNS that is generating a frequency comb with the same power at each peak at the odd harmonics of the fundamental (1.589 kHz). Each harmonics is denoted with a "o". Undesired signal is apparent in a few of the even tones just above the cross-correlation noise floor. The power at the even harmonics is a product of unwanted mixing of high-frequency signals.

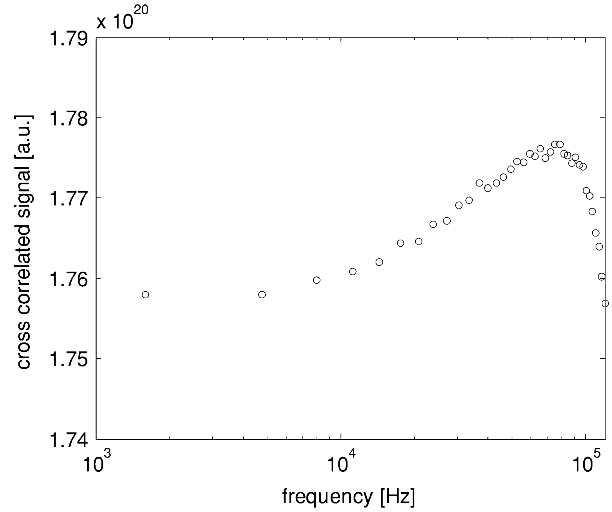

Fig. 4. Measurement of the frequency response of the cross-correlation electronics by looking at the odd harmonics of the fundamental tone at 1.589 kHz. Ideally, the response would be constant. With the QVNS it is possible to measure the deviations from a flat response.

Johnson noise spectra shown in Fig. 2 exhibits a frequency dependence that deviates from a constant ratio. The systematic difference between the QVNS and resistor noise at higher frequencies is probably due to transmission-line effects and common-mode errors from accidental coupling of the digital code generator signals used to drive the Josephson array circuit in the QVNS. It is also likely that this is the dominant source of error (systematic) for both the absolute and relative measurements of temperature. For the relative mode determination of the temperature ratio between the GaTP and WTP, the frequency dependence of the ratio is more constant. This indicates that there is a systematic error in the absolute mode of operation that is mostly canceled in the ratio.

In order to understand the systematic error in the absolute-mode measurement of the temperature, we have begun to test the performance of the JNT system including the QVNS chip packaging and probe, the transmission lines between the QVNS and the electronics, and the cross correlation electronics. In [3], we found that signals at high frequencies (>1 MHz) from the bit stream generator that are used by the QVNS to generate the noise waveforms via a sigma-delta modulation technique could mix at the field effect transistors (FETs) in the preamplifier in the cross-correlation electronics and introduce spurious signals in the band of interest. By using a low-pass filter, we were able to reduce these errors [3]. However, it is difficult to quantify the magnitude of the improvement because of the low signal levels and integration time ($\sim$ at least 1 h) needed to decrease the cross-correlation noise floor below the amplitude of the mixed-down signals.

An example of a measurement to look for spurious signals is shown in Fig. 3. In this figure, we have measured a spectrum from a QVNS that is generating only the odd harmonics of a waveform that would be used to simulate the noise of a resistor at the GaTP. We specifically synthesized an odd-harmonic pseudonoise waveform because any measured even harmonics would indicate the presence of undesirable signals detected by the measurement system. For the most part, the signals are clean (absence of power at the even harmonics). However, we have found that reproducibility of the "cleanliness" of the spectra can be challenging.

We have made several subtle improvements to the QVNS to improve the reproducibility. We have found that the connections and wiring between the Josephson junctions which are at the heart of the QVNS and FETs in the preamplifier of the cross-correlation electronics are a potential source of problems. Initially, pressure contacts on a printed circuit board were used to make electrical contact to the Josephson junctions. We found that these pressure contacts eventually degraded with time and thermal cycling, probably due to growth of an oxide film between the contacts. We have replaced the pressure contacts with low-temperature soldering contacts in which the Josephson junction chip is flip-chip mounted onto a flexible substrate. In addition, we have found that room-temperature gold-to-gold connectors can also be a source of mixing and distortion. Whenever possible, we have replaced connectors with solder joints. If a connector is needed, a connector that also applies pressure to the joint is used. In addition to replacing poor connectors, we found that the wiring of the probe which carries the signals from liquid helium (QVNS) to room temperature can introduce errors. We have found that mono-filament magnet wire is more reliable and introduces less distortion than fine stranded wire.

Another test for the distortion added into our signal band is to look at the shape of the transfer function of the cross-correlation electronics. An example is shown in Fig. 4. In principle, the QVNS is generating a perfectly "flat" or constant-amplitude comb of tones. The coarse-scale (i.e., gradual) deviation from a flat spectrum is a result of the transfer function of the electronics while the fine-scale deviations are presumably due to corruption of the tones by signals (distortion) that exceed the

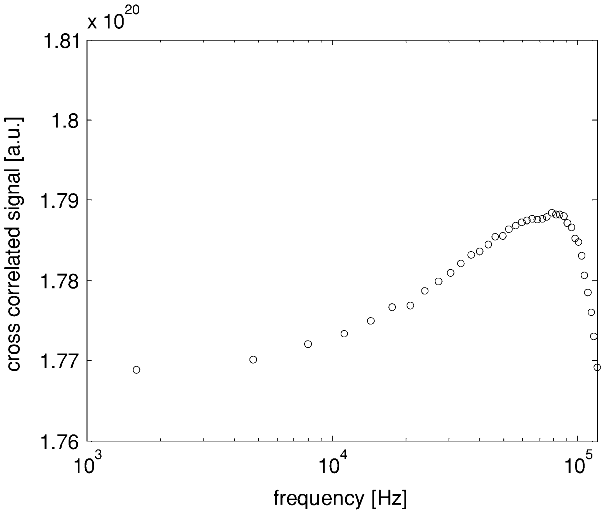

Fig. 5. Measurement of the frequency response of the cross-correlation electronics after fixing problems with poor connectors.

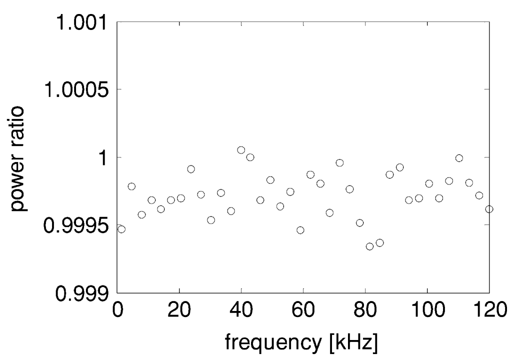

Fig. 6. Measurement of noise power ratio from two different QVNS systems. Ideally, the ratio should be scattered around one.

measurement noise. In the absence of any tone distortions, we would expect the real transfer function to vary slowly as a function of frequency. After improving the wiring and contacts between the QVNS and inputs to the cross-correlation electronics, there is marked improvement in the smoothness of the measured transfer function as shown in Fig. 5.

Another technique that we have used to understand the performance of the JNT system has been to replace the noise signal from the resistor with a second QVNS system having a different number of junctions and using a different waveform that produces an output power equivalent to the other QVNS system. With this setup, it is possible to compare equivalent pseudonoise power spectra from two different QVNS system designs. An example of a comparison between two QVNS signals is shown in Fig. 6. We have plotted the power ratio of the tones generated by a QVNS system that uses 512 Josephson junctions and a QVNS that uses 64 junctions. In theory, we expect the ratio to be 1. The fact that there is a difference indicates that there are further systematic errors to be resolved. One possible source of the error is noise coupling from the arbitrary bit-stream generator to the output of the QVNS due to inadequate high-pass filtering of the pulses from the bit-stream generator. In order to test this theory, we used every high-pass filter in the lab (eleven

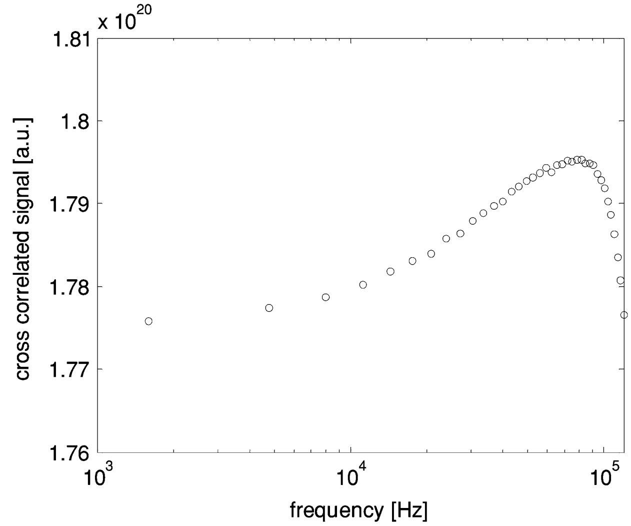

Fig. 7. Measurement of the frequency response of the cross-correlation electronics after using additional high-pass filters on the output of the bit-stream generator.

250-MHz dc blocks on both pulse inputs) to remove unwanted bitstream generator signals and improve the smoothness of the transfer function of the cross-correlation electronics. The results shown in Fig. 7 show the smoothest transfer function curve we have observed to date.

V. Conclusion

We have made absolute measurements of the temperature of a resistor at the gallium and water triple-points. We believe that the errors in these measurements are dominated by nonlinear defects in the transmission line, the frequency response of the transmission-line, mixing products from the amplifiers FETs, and common-mode signals from the bit-stream generator in the QVNS. These errors should be correctable or able to be reduced significantly. We plan to test new designs of Josephson junction circuits to further reduce common-mode errors and further characterize our transmission lines. We also plan to perform more absolute and relative mode measurements at higher temperatures (273–1000 K) through comparisons with an ITS-90 calibrated platinum resistance thermometer.

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References

[1] H. Nyquist, "Thermal agitation of electric charge in conductors," *Phys. Rev.*, vol. 32, pp. 110–113, 1928.

[2] S. P. Benz, J. M. Martinis, P. D. Dresselhaus, and S. Nam, "An AC Josephson source for Johnson noise thermometry," *IEEE Trans. Instrum. Meas.*, vol. 52, no. 2, pp. 545–549, Apr. 2003.

[3] S. Nam, S. Benz, P. Dresselhaus, W. L. Tew, D. R. White, and J. M. Martinis, "Johnson noise thermometry using a quantum voltage noise source for calibration," *IEEE Trans. Instrum. Meas.*, vol. 52, no. 2, pp. 550–553, Apr. 2003.

[4] H. Brixy *et al.*, *Temperature, Its Measurement and Control in Science and Industry*, J. Schooley *et al.*, Eds. New York: Amer. Inst. Phys., 1992, vol. 6, pp. 993–996.

[5] G. F. Strouse, D. R. Defibaugh, A. R. Moldover, and D. C. Ripple, "Progress in primary acoustic thermometry at NIST: 273 K to 505 K," in *Temperature, Its Measurement and Control in Science and Industry*, D. Ripple, Ed. New York: Amer. Inst. Phys., 2003, pt. 1, vol. 7, pp. 31–36.