Investigations of Noise in Measurements of Electronic Voltage Standards

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***Abstract*—We have investigated noise in measurements of the 10 V outputs of electronic voltage standards based on Zener diode references (Zeners). Zener outputs were compared to NIST Josephson standards using a digital voltmeter (DVM) to measure voltage differences. Because of the presence of serially correlated noise, the data were analyzed by calculating estimated Allan variances which were then used to determine the parameters of a power law model including white and $1/f$ noise. In many cases, the modeled Allan variances agree well with the estimated values over a wide range of sampling times. In all, we have estimated the $1/f$ noise floor for 25 Zeners of three types. We examined the impact on noise measurements of changes of the range of the DVM and of quantization of the recorded voltages by the DVM. We conclude that there is strong evidence of the presence of a high level of white noise in Zeners.**

***Index Terms*—$1/f$ noise, noise measurement, quantization, semiconductor noise, spectral analysis, white noise, Zener diodes.**

I. Introduction

THE uncertainty in voltage measurements of Zener standards is limited by $1/f$ noise that is characterized quantitatively by the Allan variance. This was demonstrated at the BIPM by using an analog voltmeter to compare 10-V Zener outputs to the output of a 10-V array of Josephson junctions [1]. Apart from the actual data acquisition, the BIPM system is operated manually.

The NIST has an active program to characterize its Zeners, particularly those used as traveling standards in its Measurement Assurance Program [2]. Here, we report on a collaborative project designed to estimate the uncertainty limits imposed by $1/f$ noise in the NIST Zeners and in the four Zener traveling standards used in the North American comparison of Josephson standards [3]. In contrast to the methods used in [1], for this work DVM's were used to compare Zener voltages with array voltages using the NIST Josephson voltage standard systems operating entirely under computer control. In these respects, the NIST methods are similar to those used in most laboratories operating Josephson standards and should therefore be more generally applicable.

II. Methods of Measurement and Analysis

To determine the voltage noise of a Zener, its output voltage is compared to that of an array of Josephson junctions. The voltage difference is adjusted to within 1 mV or less and repeatedly measured using a DVM at time intervals, τ_0, ranging from 0.6 s to 1 s or more. To reject undesired common-mode voltages related to the power mains, the number of power line cycles (NPLC) was set to 1 or 10. Measurement points (time-voltage pairs) were typically collected in frames of 8192 points. In good conditions, ten to twenty frames are usually sufficient to characterize the noise of a Zener.

The NIST data acquisition software is designed to accommodate unintended changes in the order number of the Josephson voltage step by readjusting to a new stable step. With an operating frequency near 75 GHz, a single step voltage corresponds to about 155 μV so that if the step order changes by more than six, the DVM must measure a voltage difference >1 mV. For the DVM used in this work, Agilent model 34420A,[1] the two highest-sensitivity ranges are 1 and 10 mV. To avoid the obvious disadvantages of range changes, many of the measurements were made on the 10-mV range. This led us to investigate the decreased precision on the 10-mV range with respect to that of the 1-mV range.

Voltmeter readings include contributions from the DVM's own noise as well as that of the instrument under test. Array noise is white and negligible compared to Zener noise. DVM noise is usually estimated from complete series of measurements made with a short-circuited input. If, within a given range, DVM noise is independent of the value of the voltage being measured, then the short-circuit results can be used to estimate voltmeter noise when measuring inputs of several hundred microvolts or more. One of the secondary objectives of this work was to demonstrate this. To do so, we examined the variation of voltage noise with input voltage by using the DVM to directly measure the output of an array operating on steps having voltages between 0.156 and 1.564 mV.

We investigated 25 Zeners; five Fluke 732B instruments fitted with Motorola reference/amplifiers (Type M), twelve 732B instruments fitted with Linear Technology reference/amplifiers (Type L) and eight Wavetek 7000 series instruments (Type W).

Measurement results were analyzed using the Allan variance [4]. For an infinite number of voltages, y_i, read at regular time intervals τ_0, the Allan variance is based on averages of y_i over successive groups of n measurements, corresponding to the sampling time $\tau = n\tau_0$. Beginning with $n = 1$, the value of

Manuscript received July 2, 2004; revised October 20, 2004.

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Digital Object Identifier 10.1109/TIM.2004.843096

[1] Commercial equipment and materials are identified in order to adequately specify certain procedures. In no case does such identification imply recommendation or endorsement by the National Institute of Standards and Technology, nor does it imply that the materials or equipment identified are necessarily the best available for the purpose.

n is increased in some regular series. The Allan variance is defined as

$$\sigma_y^2(\tau) = \frac{\left\langle \left(\overline{y}_{j+1}(\tau) - \overline{y}_j(\tau)\right)^2 \right\rangle}{2} \quad (1)$$

where $\overline{y}_j(\tau)$is the average voltage of the j-th group of n successive readings and the angular brackets indicate an infinite time average. The Allan variance is not constant but varies with sampling time.

In low-frequency electrical measurements we observe only $1/f$ noise and white noise. The spectral density is $S_y(f) = h_{-1}f^{-1} + h_0 f^0$ where the intensity coefficients h_{-1} and h_0 are constants. The Allan variance is related to the spectral density by the integral expression

$$\sigma_y^2(\tau) = 2\int_0^\infty S_y(f)\,|H(f)|^2 \left[\frac{\sin^4(\pi f\tau)}{(\pi f\tau)^2}\right] df \quad (2)$$

where $H(f)$ is the transfer function relating the Fourier transform of the output of the detector to that of the input. (A detailed discussion of this expression is given in [5], section 5.) When using a DVM having a sharp cut-off filter, the Allan variance for the mixed process is

$$\sigma_y^2(\tau) = 2h_{-1}\ln 2 + \frac{h_0}{2\tau} \quad (3)$$

and this is the model we use to interpret our results. The constant term $2h_{-1}\ln 2$ is the value of the Allan variance in a $1/f$ noise process. In the absence of drift or external perturbations such as those due to temperature fluctuations, its experimental value is estimated from the asymptotic value of the sample Allan variance for long sampling times. This experimental value is then used as a constraint in estimating h_0 from a least-squares regression. To reduce the uncertainty of the estimated Allan variance we use mean values of the estimated Allan variances taken over from 10 to 100 frames.

III. Results and Discussion

In Fig. 1 different graph symbols show Allan deviations (square root of the Allan variance) as a function of sampling time estimated from DVM measurements of: (a) a Type L Zener, (b) a Type M Zener, and (c) the 625-μV output of an NIST array. Solid lines show the results of regressions of the estimated Allan deviations to the model in (3). For short sampling times, (a) and (b) asymptotically approach slopes of $-1/2$, indicating the predominance of white noise in those regions. For long sampling times, (a) and (b) take on nearly constant values, termed the $1/f$-noise floors, indicated by the dashed lines. These are important characteristics of the individual Zeners because they represent the lower limit of the random uncertainty achievable with each instrument. Comparing (a) and (b), we see that the $1/f$ noise of the Type M Zener is 2.5 times smaller than that of the Type L Zener. This is especially interesting since this particular Type M Zener is one of the four travelling standards used in the National Conference of Standard Laboratories International (NCSLI) comparisons of

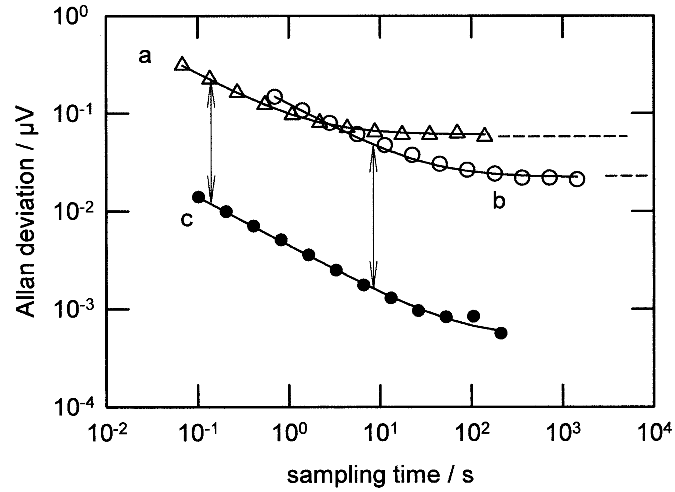

Fig. 1. Estimated Allan deviation versus sampling time for: (a) Type L Zener, triangles; (b) a Type M Zener, open circles; and (c) a Josephson voltage step at 625 μV, filled circles. Arrows indicate the excess white noise of the Zeners over that of the DVM and the Josephson array. Solid lines joining points result from regressions to (3); dashed lines indicate the Zener $1/f$ noise floors.

Josephson standards [3]. Plot (c) shows the Allan deviation estimated from DVM measurements of a nonzero array voltage. The arrows indicate that it is 18 times smaller than the white noise in (a) and 29 times smaller than the white noise of (b).

Two related questions arise: (1) does the residual white noise in Fig. 1(c) come from the array or the DVM itself, and (2) does the noise vary significantly with the level of the voltage measured? To answer the first question we measured the noise voltage of the DVM on the 1-mV range and found Allan deviations indistinguishable from those in Fig. 1(c), leading us to conclude that the noise represented in that plot is dominated by the noise of the DVM itself. To answer question (2) we examined the noise in direct DVM measurements of array voltage steps both below and above 1 mV using both the 1-and 10-mV ranges of the DVM. Fig. 2 summarizes the results of the Allan deviations for DVM voltage measurements of four voltages on the 1-mV range (open symbols) and five voltages on the 10-mV range (solid symbols). Dashed lines join values corresponding to short-circuit measurements on the 1-mV range and heavy solid lines join values corresponding to short-circuit measurements on the 10-mV range. The vertical dashed line indicates the difference between the Allan deviations for short-circuit measurements on the two ranges; it corresponds to a factor of 1.78 within the white noise regimes. The adjacent horizontal dashed line shows that on the 10-mV range a sampling time of about 1 s is required to achieve the same Allan deviation as that obtained on the 1-mV range with a sampling time of 0.31 s. The main conclusions from Fig. 2 are the following: (1) for the same sampling time, the Allan deviations for the 10-mV range are about 1.8 times greater (not 10 times greater) than those obtained on the 1-mV range; (2) for a given sampling time, there is no significant variation of the Allan deviations with applied voltage within the 1-mV range; (3) for input voltages ranging from 0 to 1.6 mV, measured on the 10-mV range, the Allan deviation is constant to within about 10%. In addition to conclusion (1), another surprising observation is the unexpectedly good resolution on the 10-mV range. The DVM output is quantized into

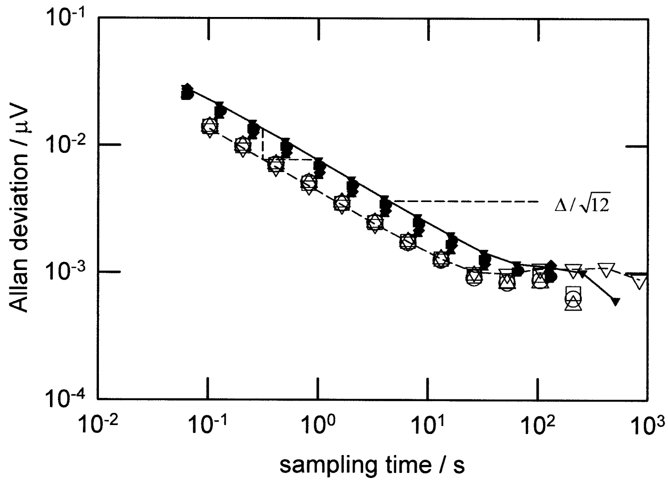

Fig. 2. Estimated Allan deviation versus sampling time for Josephson voltage steps at 156 μV, 625 μV and 782 μV and a short circuit measured with a DVM on the 1 mV range (open circle, upward triangle, square and downward triangle, respectively) and for Josephson voltage steps at 1.09 mV, 1.25 mV, 1.41 mV, 1.56 mV and a short circuit measured with a DVM on the 10 mV range (filled circle, upward triangle, square, diamond and downward triangle, respectively). The long dashed horizontal line indicates the expected size of the DVM quantization error for the 10 mV range.

steps by the analog-to-digital conversion effected by the instrument. On the 10-mV range the step size, Δ, was observed to be 12.7 nV in this case. Assuming that the input voltages are uniformly distributed about the quantized levels, and recalling that the standard deviation for a uniform distribution of width w is $w/\sqrt{12}$, one would expect [6] the standard uncertainty due to the limited resolution to be $\Delta/\sqrt{12} = 3.7$ nV, as indicated by the dashed horizontal line in Fig. 2. On the other hand, the Allan deviation for the 10 mV range (solid black line in Fig. 2) continues to decrease, as if the noise were white, taking on a value of 1.4 nV at a sampling time of about 32 s. In other words, the resolution as measured by the Allan deviation is considerably better than what one expects given the level of quantization in the DVM electronics.

Result (1) is of practical importance when using a DVM to compare voltages of sources dominated by white noise such as the direct comparison of voltages of two arrays of Josephson junctions. If, as is the case for the instrument used in these measurements, the DVM can operate on a range of either 1 or 10 mV and if the array tends to be unstable, there is an advantage in measuring on the 10-mV range. Since, as noted in Section II, a change of one in the order number of the voltage step corresponds to a change in the measured voltage of 155 μV, a change exceeding six step numbers would require the DVM to measure a voltage exceeding 1 mV. If the measurement began on the 1-mV range, before the step number changed, the measurement would have to be repeated. On the other hand, we might anticipate that if the measurements were all made on the 10-mV range, the resolution would decrease by a factor of ten. The plots in Fig. 2 show that this is not so and that for a given sampling time the Allan deviation for measurements on the 10-mV range is increased by a factor of about 1.8 with respect to that for the 1-mV range.

To investigate the effects of DVM range changes on measurements dominated by $1/f$ noise processes, we used an Agilent

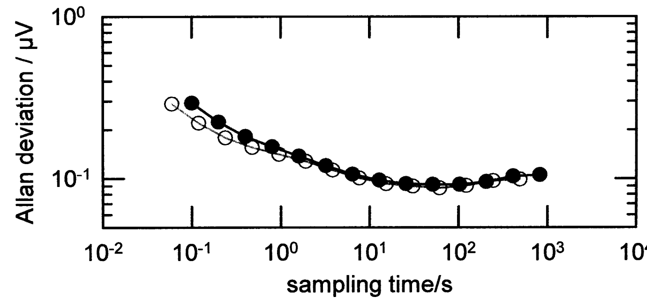

Fig. 3. Estimated Allan deviation versus sampling time for DVM measurements of the voltage difference between two Zeners on the 1-mV range (open circles) and the 10-mV range (solid circles).

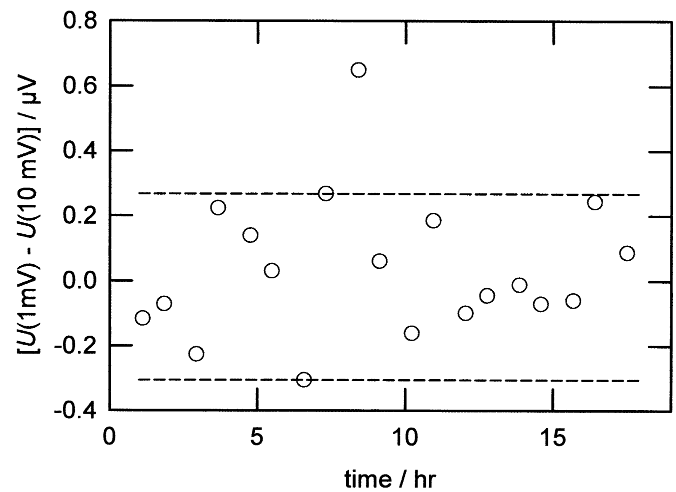

Fig. 4. Mean values versus time for the voltage difference between two Zeners measured successively on the 1-mV range, U_{H}, and the 10-mV range, U_{L}; dashed lines indicate the limits containing 95% of the results.

34420A DVM to measure the difference between the 10-V outputs of two Type L Zeners following this pattern: five frames of 8192 measurements each on the 1-mV range followed by five frames of 8192 measurements each on the 10-mV range. This pattern was repeated 20 times resulting in a total of 100 frames on each range. The Allan deviations for each range are quite similar as shown in Fig. 3. The Allan deviation for the $1/f$ floor for measurements on the 1-mV range is about 0.095 μV and as nearly the same, 0.091 μV for the 10-mV range. We note incidentally that for these measurements we observed a quantized step size on the 10-mV range of approximately 0.0104 μV. Since this is much less than the noise floor, we would not expect the quantization error to have a noticeable effect in this case.

To demonstrate the coherence of the Allan variance approach with an alternative statistical approach, these data were analyzed in the following way. The mean voltage, U_{H}, measured on the high-sensitivity (1 mV) range in the fifth frame was compared to the mean voltage, U_{L}, measured on the low-sensitivity (10 mV) range in the sixth frame by calculating the difference $D = U_{\mathrm{H}} - U_{\mathrm{L}}$. Values of D were calculated from the results of frames 10 and 11, 15 and 16 and so on, resulting in a total of 19 estimates. These results are plotted as a function of time in Fig. 4 where the dashed horizontal lines indicate the observed interval of 0.57 μV height containing 95% of the points. If we equate this to the 95% confidence interval $(\pm 2\sigma)$, the estimated standard deviation is

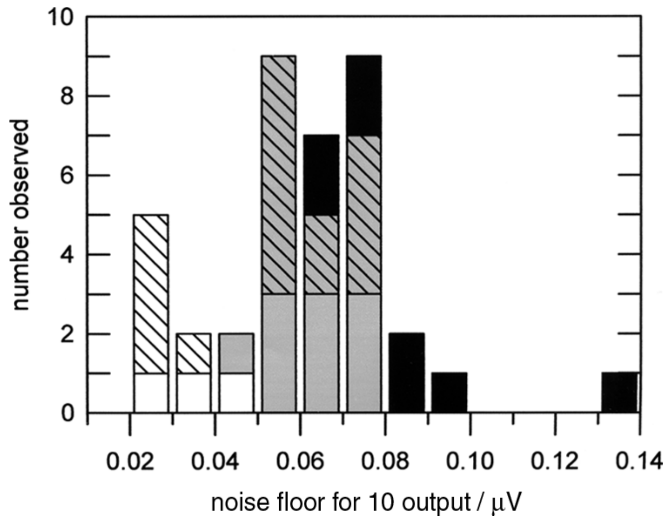

Fig. 5. Noise floors for the 10-V outputs of the 25 Zeners of 3 types (white: Type M; gray: Type L; black: Type W) determined in the present study and represented by diagonally hatched cell patterns. Also included are results from 13 other Zeners of 2 types (white: Type M; gray: Type L) previously studied by the BIPM and represented by nonhatched cell patterns.

0.14 μV and is compatible with the Allan deviations in Fig. 3. Of course, we are most interested in knowing if the average *values* issuing from measurements on the two ranges are compatible and the results in Fig. 4 show that they are. We conclude from this experiment involving high levels of $1/f$ noise that this type of DVM could be used to calibrate Zeners on the 10-mV range with an array without loss of accuracy.

Fig. 5 shows the results of our determinations of the noise floors of the 25 Zeners. They are presented along with the results of BIPM determinations [1] of the noise floors of 13 other Zeners. Our conclusions are that: 1) all Zeners examined in this way show $1/f$ noise that limits the precision with which the voltages are defined; 2) Type M Zeners systematically exhibit the lowest noise floors, ranging between 0.02 and 0.05 μV; 3) Type L Zener noise floors range between 0.04 and 0.08 μV; 4) Type W Zener noise floors range, with one exception, between 0.06 and 0.10 μV; and 5) the results of the noise floor measurements at the NIST using an automated array system and a DVM detector are in good agreement with the results obtained at the BIPM using a manually operated Josephson system and an analog detector.

IV. Conclusion

To generalize our results, we conclude from this work that the model of (3) expressing the noise in Zener measurements as the sum of two terms is justified by the results of regression analysis of the estimated Allan variance. The measured voltages exhibit a level of white noise that is greater by more than one order of magnitude than the intrinsic noise of the DVM used for the measurements. On the 1-mV range, the DVM noise does not depend on the level of the input signal and so there is no evidence that the high white noise level of the Zener measurements is due to the DVM. When comparing results from the 10-mV range with those from the 1-mV range for white noise processes, the change in the noise level as expressed by the Allan deviation is surprisingly small, only a factor of 1.8, and not a factor of 10. Furthermore, the Allan deviation is significantly smaller than the standard deviation expected from a simple model of the quantization error; this may be an interesting topic of further study. For $1/f$ noise, the noise level is nearly unaffected by a range change from 1 to 10 mV and the measured voltages agree within expected uncertainties. This information should be useful in designing procedures for array calibrations of Zeners. Finally, the results of our study of the noise characteristics of 25 Zeners agree well with the values and trends reported earlier by the BIPM work [1] and, by tripling the total number of instruments on which noise studies have been conducted, the ensemble of results is put on a more firm statistical footing.

Acknowledgment

The authors thank D. Deaver of the Fluke Corporation for lending the four Zener travelling standards used in the National Conference of Standard Laboratories International (NCSLI) comparisons of Josephson voltage standards. This work was performed in part at NIST in the Quantum Electrical Metrology Division, Electronics and Electrical Engineering Laboratory, Technology Administration, U.S. Department of Commerce.

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