

# ANTENNA GAIN-FACTOR EQUIVALENT FOR TEM CELLS

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**Abstract:** This paper derives a gain-factor equivalent for TEM cells. The gain factor is then used in simple transmission formulas to investigate emissions tests in TEM cells and correlation of TEM-cell emissions measurements to other methods. The correlation equations are not restricted to electrically small test objects, as is the case for dipole models. Thus, these results are of use as EMC test methods, and standards are extended to frequencies exceeding 1 GHz.

## I. INTRODUCTION

The equipment under test (EUT) in a TEM cell is typically modeled as a set of dipoles when one analyzes emission measurements [1-2]. Dipole models work well for electrically small emitters but may not accurately describe emissions from electrically large emitters. Thus, a more general emission model is needed. One approach is to account for further terms (e.g., quadrapole) in a multipole model of the EUT [3-4]; however, higher-order multipole models quickly become cumbersome and may not lead to simple, usable measurement procedures. This paper develops a simple alternative for the general emissions case, namely, an antenna gain-factor equivalent for TEM cells. Having a gain factor allows TEM cells to be used for emissions testing in the same manner as unguided wave test environments, such as fully anechoic rooms (FAR), semi-anechoic room (SAR), and open area test sites (OATS).

## II. TEM CELL IN TRANSMITTING MODE

In free space, the far-field (subscript FF) average power density  $S_{av,FF}$  at a distance  $r$  from a source is

$$S_{av,FF} = P_t g_t \frac{1}{4\pi r^2}, \quad (1)$$

where  $P_t$  and  $g_t$  are the average power and gain of the source (transmitter).

The TEM mode in a TEM cell approximates an ideal plane wave (uniform, linearly polarized) over a limited test volume. Thus, the average power density  $S_{av,TEM}$  for the TEM mode is

$$S_{av,TEM} = \frac{1}{2} \frac{E^2}{\eta_0}, \quad (2)$$

where  $E$  is the peak electric-field amplitude and  $\eta_0$  is the free-space wave impedance. Neglecting non-TEM modes, the peak electric field amplitude near the center of the test volume in a TEM cell is approximately the voltage amplitude  $V$  between the inner and outer conductor divided by their separation  $h$ , or  $E \approx V/h$ . Thus,

$$S_{av,TEM} \approx \frac{1}{2} \frac{1}{\eta_0} \left( \frac{V}{h} \right)^2. \quad (3)$$

The average input power to a TEM cell is related to the input voltage and characteristic impedance  $Z_0$  of the cell:

$$P_t = \frac{1}{2} \frac{V^2}{Z_0}. \quad (4)$$

Combining these results yields

$$S_{av,TEM} \approx P_t \frac{Z_0}{\eta_0} \frac{1}{h^2} \quad (5)$$

for the average power density in a TEM cell. This is similar in form to the free-field expression (2) and can be reformulated in equivalent parameters. For the case of a TEM cell with a constant flare of angle  $\theta$  in the test volume (e.g., a gigahertz TEM (GTEM) cell where typically  $\theta \approx 15^\circ$ ),  $h \approx r \sin \theta$ , where  $r$  is the distance from the apex (input/output port) to the receiving antenna location. For a TEM cell with uniform test section (tapers at each end),  $h \approx r \sin \theta$ , where  $r$  is the distance from the apex to the receiving antenna location projected back along the uniform section to the end of the taper. Substituting this approximation and rewriting yields

$$S_{av,TEM} \approx P_t \left( \frac{4\pi Z_0}{\eta_0 \sin^2 \theta} \right) \frac{1}{4\pi r^2}. \quad (6)$$

This expression is for the average power density near the center of the test volume in a TEM cell. Comparing equations (1) and (6) we see that

$$g_{t,TEM} \approx \left( \frac{4\pi Z_0}{\eta_0 \sin^2 \theta} \right) \quad (7)$$

defines an equivalent gain factor for the TEM cell. Alternately,  $g_{t,TEM}$  can be derived directly from the definition of gain,

$$g = 4\pi r^2 \frac{S_r}{P_t}, \quad (8)$$

by substituting (3) for the power density in the direction of the test volume, (4) for the transmitted power, and again using the approximation  $h \approx r \sin \theta$ . The advantage of the first derivation is that it emphasizes the geometrical similarities between a TEM cell and an antenna.

The gain factor can now be used in transmission formulas to consider simple coupling problems. In particular, we will use the Friis formula,

$$P_r = P_t g_t g_r \left( \frac{\lambda}{4\pi r} \right)^2, \quad (9)$$

that relates received  $P_r$  and transmitted  $P_t$  power between a receiving antenna with gain  $g_r$  and a transmitting antenna with gain  $g_t$ .

### III. TEM CELL IN RECEIVING MODE

For a general antenna (EUT) in the transmitting mode, the received power at the output port of a TEM cell may now be written from eqs. (7) and (9) as

$$P_r = P_t g_t \left( \frac{4\pi Z_0}{\eta_0 \sin^2 \theta} \right) \left( \frac{\lambda}{4\pi r} \right)^2. \quad (10)$$

This expression is next applied to some specific examples.

#### III.1 Short Electric Dipole

Consider a short electric dipole with  $P_t = P_0$  and  $g_t = 3/2$  (maximum gain). By use of (10), the received power at the output of a TEM cell, for the case of a dipole oriented for maximum coupling, is given by

$$P_r = P_0 \frac{3}{2} \left( \frac{4\pi Z_0}{\eta_0 \sin^2 \theta} \right) \left( \frac{\lambda}{4\pi r} \right)^2. \quad (11)$$

This result may be compared to previously derived expressions based on a waveguide analysis. For example, in IEC 61000-4-20 [2],  $P_0$  for an electric dipole is given as

$$P_0 = \frac{\eta_0}{3\pi} \frac{k_0^2}{e_{0y}^2 Z_0} S_V^2, \quad (12)$$

where  $k_0$  is the wavenumber,  $e_{0y}$  is a normalized field factor ( $e_{0y}^2 \approx Z_0 / h^2$ , [2]), and  $S_V$  is related to measured output port voltage ( $S_V^2 \approx V^2$  for the electric dipole as emitter [2]; a subscript V has been added here to avoid confusion with power density).

Substituting these results into (12) yields

$$P_0 = \frac{\eta_0}{3\pi} \frac{k_0^2 h^2}{Z_0^2} V^2. \quad (13)$$

Substituting in  $V^2$  from (4), with  $h \approx r \sin \theta$  and  $k_0 = 2\pi/\lambda$ , gives

$$P_0 = \frac{\eta_0}{3\pi} \left( \frac{2\pi}{\lambda} \right)^2 \frac{r^2 \sin^2 \theta}{Z_0^2} 2Z_0 P_r. \quad (14)$$

Solving for  $P_r$  yields

$$P_r = P_0 \frac{3}{2} \left( \frac{4\pi Z_0}{\eta_0 \sin^2 \theta} \right) \left( \frac{\lambda}{4\pi r} \right)^2, \quad (15)$$

which is the same as (11). Thus, (11) is consistent with previously derived dipole equations.

#### III.2 Electrically Large EUT

For electrically large unintentional emitters, the expected value ( $\langle \rangle$ ) of the directivity (maximum gain) can be estimated as [5]

$$\langle D \rangle \approx \frac{1}{2} \left( 0.577 + \ln N_s + \frac{1}{2N_s} \right), \quad (16)$$

where  $N_s = 4(k_0 a)^2 + 8(k_0 a)$ , and  $a$  is the radius of the minimum sphere enclosing the EUT. Electrically large is here defined by  $k_0 a > 1$ .

Inserting this expression in (10) yields an expected value for the received power, when the EUT is oriented for maximum emissions, given by

$$\langle P_r \rangle = P_t \langle D \rangle \left( \frac{4\pi Z_0}{\eta_0 \sin^2 \theta} \right) \left( \frac{\lambda}{4\pi r} \right)^2. \quad (17)$$

This can be used as a means of estimating  $P_t$  if the orientation for maximum coupling is known. Alternately,  $P_t$  can be estimated by averaging over multiple orientations [6].

#### IV. CORRELATION OF MAXIMUM EMISSIONS BETWEEN VARIOUS TEST SITES

Correlation between test sites can be used to set equivalent test limits and to compare test data. The transmission formulas above allow us to write simple expressions for the received power at various test sites when the EUT is oriented for maximum emissions. We have:

$$\begin{aligned}\frac{P_r}{P_t g_t} &\approx \left( \frac{4\pi Z_0}{\eta_0} \right) \left( \frac{\lambda}{4\pi h} \right)^2 \quad (\text{TEM cell}), \\ \frac{P_r}{P_t g_t} &= g_r \left( \frac{\lambda}{4\pi r} \right)^2 \quad (\text{FAR, free space}), \\ \frac{P_r}{P_t g_t} &\approx g_r 4 \left( \frac{\lambda}{4\pi r} \right)^2 \quad (\text{OATS, SAR, ground plane}), \\ \frac{P_r}{P_t g_t} &\approx 1 \quad (\text{reverberation chamber}).\end{aligned}\quad (18)$$

In the above OATS expression, we make the approximation that the ideal ground plane (infinite size, perfectly conducting) doubles the electric field (quadruple received power) compared to the free-space case when the EUT is oriented for maximum emissions and constructive interference (via a height scan). In the expression for the reverberation chamber, we make the approximation that averaging over multiple paddle positions negates transmitter gain and only the total transmitted power is measured. Correlation between measurement sites is achieved by forming the ratios of the above expressions:

$$\begin{aligned}\frac{P_{r,TEM}}{P_{r,FAR}} &= \frac{\left( \frac{4\pi Z_0}{\eta_0} \right) \left( \frac{r^2}{h^2} \right)}{g_r}, \\ \frac{P_{r,TEM}}{P_{r,OATS}} &= \frac{\left( \frac{4\pi Z_0}{\eta_0} \right) \left( \frac{4r^2}{h^2} \right)}{g_r}, \\ \frac{P_{r,TEM}}{P_{r,REVERB}} &= \left( \frac{4\pi Z_0}{\eta_0} \right) \left( \frac{\lambda}{4\pi h} \right)^2, \\ \frac{P_{r,FAR}}{P_{r,OATS}} &= \frac{1}{4}, \\ \frac{P_{r,FAR}}{P_{r,REVERB}} &= g_r \left( \frac{\lambda}{4\pi r} \right)^2, \text{ and} \\ \frac{P_{r,OATS}}{P_{r,REVERB}} &= g_r 4 \left( \frac{\lambda}{4\pi r} \right)^2.\end{aligned}\quad (19)$$

In the author's opinion, the best use of these expressions is to establish emission limits for various site types that "equivalently" test the EUT. Correlating actual measured data from differing site

types is difficult due to site imperfections, set-up variability, operator procedures, and other factors. There is the unfortunate tendency when comparing data between EMC emission sites to assume that one site yields "correct" results and correlated data from other sites must match this "reference data" or be judged to be in "error." Differences may be caused more by each site type accentuating differing aspects of the overall EUT emissions.

A simple example of this is the spatial sampling used at various sites: planar cuts are typical for a FAR, sectoral wedges typical for an OATS, and three-axis rotations are typical for a TEM cell. The true maximum emissions may or may not be detected by any one of these differing schemes. Correlated results may differ simply because the full 3D emission pattern is sampled in very different ways and not because data or correlation algorithms are "in error." Despite these reservations, an example of measured data is presented next.

#### V. MEASURED DATA

As part of a site comparison, we obtained data for a self-contained comb generator with an attached loop antenna, 30 cm square, as described in [7]. The overall diameter of the minimum sphere containing both the loop and comb generator box is approximately 60 cm. Thus,  $a \approx 0.3$  m, and the emitter becomes electrically large ( $k_0 a > 1$ ) above 160 MHz. Data were taken at various NIST facilities over the frequency range 500 MHz to 2 GHz. Only data to 1 GHz will be used here, as signal strengths were relatively weak for frequencies above 1 GHz. No uncertainties were developed for these data; however, for a discussion of uncertainties for various EMC test sites see [8].

Among the facilities used were a TEM cell (GTEM 750), a FAR, and a reverberation chamber (facility details are given in [7]). GTEM emissions were measured with the EUT in three orthogonal orientations. Maximum coupling (the data used here), for the orientations considered, occurred when the plane of the loop was perpendicular to the magnetic field in the GTEM cell (magnetic dipole moment aligned with the magnetic field). FAR measurements, at a distance of 3 m, were also made for three orthogonal orientations. For the TEM cell, only data for the orientation yielding maximum coupling are used here. The reverberation chamber data determined total radiated power based on the method.

Figures 1-3 show the correlated data based on eqs. (19). The measured power ratios are normalized using our derived analytical approximations (right-hand-side expressions in eq. (19)) so that exact agreement would yield zero decibels (dB). Figure 1 compares the FAR and reverberation-chamber data. Figure 2

compares the TEM cell and reverberation-chamber data. Figure 3 compares the FAR and TEM-cell data.

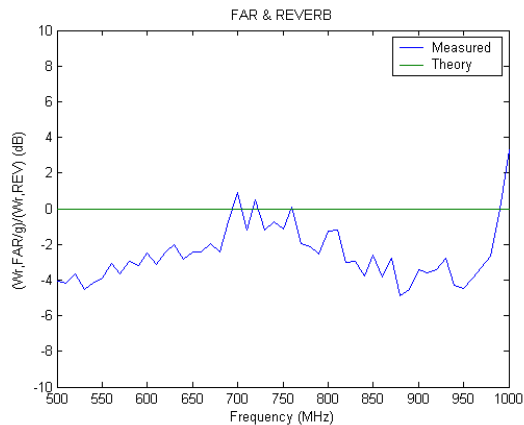


Figure 1. The ratio of the received powers in a FAR and reverberation chamber.

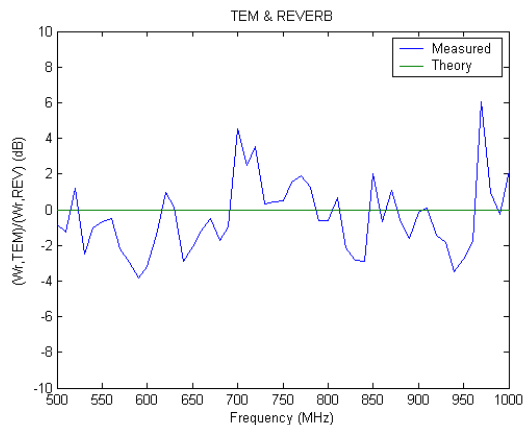


Figure 2. The ratio of the received powers in a TEM cell and reverberation chamber.

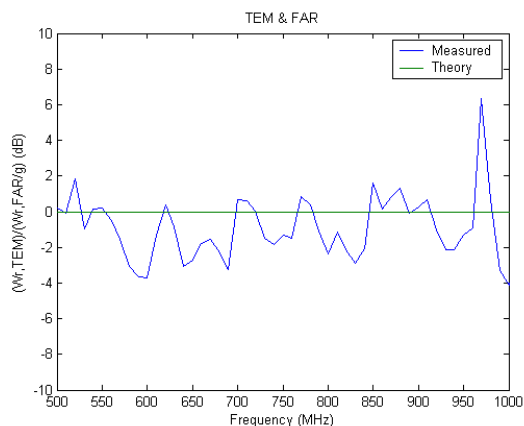


Figure 3. The ratio of the received powers in a TEM cell and a FAR.

In each case the agreement is on the order of  $\pm 4$  dB, except very near 1 GHz, where weak signal strength begins to make the data noisy. For the reasons outlined in the previous section, this agreement is reasonable. Clearly, more such data are needed to better explore the usefulness of the expressions presented here.

## VI. CONCLUSION

This paper derived a simple equivalent antenna gain factor for a TEM cell. The gain factor is then applied to EUT emissions in a TEM cell for both electrically large and electrically small EUTs and to emissions correlation between EMC test facilities. We believe that the best use of these correlation expressions is to set equivalent limits between facilities and not to compare measured data between sites.

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