AMPLIFIER NOISE-PARAMETER MEASUREMENT CHECKS AND VERIFICATION[*]

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***Abstract* — We propose two verification methods for measurements of noise parameters of amplifiers, particularly low-noise amplifiers (LNAs). One method is a direct measurement of the parameter T_{rev}, the noise temperature from the amplifier input, and the comparison of that to the value derived from the noise-parameter measurement. The other check involves the measurement of the noise parameters for the amplifier with an isolator connected to the input and comparison to the noise parameters of the amplifier alone. Relationships between the noise parameters with and without the isolator are given. We demonstrate both verification methods with measurements on a sample LNA in the 8 – 12 GHz range. Uncertainties in the noise parameters are evaluated using a previously developed Monte Carlo method, and both checks are found to be satisfied within the uncertainties.**

***Index Terms* — amplifier; noise; noise measurement; noise parameters**

I. INTRODUCTION

Accurately measuring the noise parameters of a low-noise amplifier (LNA) can be a difficult task, replete with opportunities for error. Redundancy of measurements [1],[2] provides some degree of checking, in that a good fit (*i.e.*, small residuals) indicates consistency of the measurements for the different terminations. Measurements of a passive device [3] can also provide a check. Nonetheless, it would be helpful to have a way of checking that exercises the same aspects of the measurement method that enter into the measurement of active devices. This paper suggests and demonstrates two such tests.

We refer to one method as the “T-reverse” (T_{rev}) method. It consists of measuring the noise temperature at the *input* of the amplifier when the output of the amplifier is terminated in a matched load [4]. This input noise temperature, denoted T_{rev}, can be expressed in terms of the amplifier’s noise parameters and scattering parameters, and so the value from the direct measurement can be compared to the value computed from the measured noise parameters and S-parameters. Since T_{rev} is not normally used in the measurement of the noise parameters, this comparison constitutes an independent check of the measured noise parameters. The second method will be called the isolator method. It consists of connecting an isolator to the input of the amplifier and measuring the noise parameters of the isolator-amplifier combination. These noise parameters can be computed in terms of the noise parameters and S-parameters of the amplifier alone plus the S-parameters and physical temperature of the isolator. Thus comparison of the computed and measured values for the noise parameters of the isolator-amplifier combination constitutes a test of the measured values of the amplifier’s noise parameters.

The next section presents the relationships between T_{rev} and the amplifier’s noise parameters and between the amplifier’s noise parameters and those of the amplifier-isolator combination. Section 3 presents measurement results implementing the two checks on a sample LNA from 8 – 12 GHz. Conclusions are presented in Section 4.

II. THEORY

Both tests are most conveniently expressed in terms of the noise parameters in the wave formulation of the noise matrix [5] referred to the input port [6,7]. Referring to Fig. 1, we define

$$\begin{aligned} X_1 &\equiv \overline{|c_1|^2}, \\ X_2 &\equiv \overline{|c_2 / S_{21}|^2}, \\ X_{12} &\equiv \overline{c_1 c_2^*} / S_{21}^*, \end{aligned} \tag{1}$$

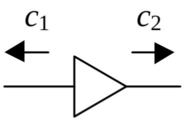

Fig. 1. Noise waves from amplifier.

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where the bar indicates a time average, the asterisk indicates complex conjugate, S_{21} is a scattering parameter of the amplifier, and c_1 and c_2 are the intrinsic noise waves emerging from the two ports of the amplifier. The X parameters or noise matrix elements can be related to the traditional IEEE noise parameters [8], so that if one set has been measured the other can be readily computed. The relationship between the two sets of parameters is easily obtained from the relationship between the noise matrix and the IEEE parameters [5]; the equations are given in [6], and we do not reproduce them here. We assume that the noise parameters X_1, X_2, and X_{12} have been measured or determined from the measured IEEE parameters. The particular form of the IEEE parameters that we use is defined by

$$T_e = T_{\min} + t\frac{\left|G_{opt} - G_1\right|^2}{\left|1 + G_{opt}\right|^2 (1 - \left|G_1\right|^2)} , \tag{2}$$

where T_e is the effective input noise temperature, G_1 is the reflection coefficient of the input termination, and the four noise parameters are $T_{\min}$, t, and the real and imaginary parts of the optimal reflection coefficient G_{opt}.

The first check is a measurement of the reverse noise temperature, T_{rev} [4]. This is the noise temperature measured at the amplifier input when the output is terminated in a (nearly) reflectionless load, as indicated in Fig. 2. It is related to X_1 by

$$k_B T_{rev} = \frac{X_1}{\left(1 - \left|G_1\right|^2\right)} , \tag{3}$$

where G_1 is the reflection coefficient at the input of the amplifier in Fig. 2, and k_B is Boltzmann's constant. Thus a measurement of T_{rev} constitutes a direct check of X_1. In terms of the IEEE parameters, T_{rev} is given by

$$T_{rev} = \left[T_{e,\min}\left(\left|S_{11}\right|^2 - 1\right) + \frac{t\left|1 - S_{11}G_{opt}\right|^2}{\left|1 + G_{opt}\right|^2} \right] \left(1 - \left|G_1\right|^2\right)^{-1} . \tag{4}$$

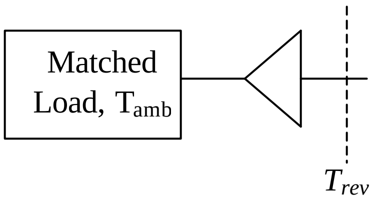

Fig. 2. Configuration for T_{rev}.

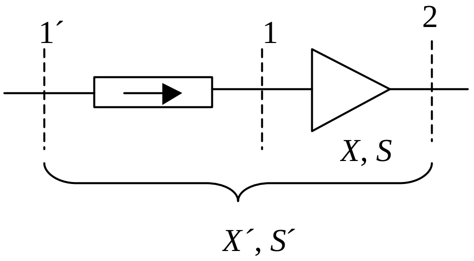

Fig. 3. Configuration for the isolator test.

The second test is to connect an isolator to the input of the amplifier under test and to measure the noise parameters of the amplifier-isolator combination. If we denote the noise parameters of the amplifier-isolator combination by X_1', X_2', and X_{12}', as in Fig. 3, they can be expressed in terms of the X parameters of the amplifier alone by

$$X_1' = \left|\frac{S_{12}^I}{1 - S_{11}S_{22}^I}\right|^2 X_1 + k_B T_I (A_1 - A_2),$$

$$X_2' = \frac{1}{\left|S_{21}^I\right|^2}\left\{B_1 X_2 + B_2 X_1 + 2\,\mathrm{Re}\left[B_3 X_{12}\right] + k_B T_I B_4\right\}, \tag{5}$$

$$X_{12}' = \frac{S_{12}^I\left(1 - S_{11}S_{22}^I\right)^*}{S_{21}^{I*}\left(1 - S_{11}S_{22}^I\right)} X_{12} + \frac{S_{12}^I S_{22}^{I*}}{S_{21}^{I*}\left(1 - S_{11}S_{22}^I\right)} X_1 - k_B T_I C_1 ,$$

where

$$A_1 = \left\{\left(1 - \left|S_{11}^I\right|^2 - \left|S_{12}^I\right|^2\right) + \left|\frac{S_{11}S_{12}^I}{1 - S_{11}S_{22}^I}\right|^2 \left(1 - \left|S_{21}^I\right|^2 - \left|S_{22}^I\right|^2\right)\right\},$$

$$A_2 = 2\,\mathrm{Re}\left[\frac{S_{12}^I S_{11}}{(1 - S_{11}S_{22}^I)}\left(S_{21}^I S_{11}^{I*} + S_{12}^{I*} S_{22}^I\right)\right],$$

$$B_1 = \left|1 - S_{11}S_{22}^I\right|^2,$$

$$B_2 = \left|S_{22}^I\right|^2,$$

$$B_3 = S_{22}^I\left(1 - S_{11}S_{22}^I\right)^*,$$

$$B_4 = \left(1 - \left|S_{22}^I\right|^2 - \left|S_{21}^I\right|^2\right),$$

$$C_1 = \left[\left(\frac{S_{21}^{I*}S_{11}^I + S_{12}^I S_{22}^{I*}}{S_{21}^{I*}}\right) - \frac{S_{12}^I S_{11}}{S_{21}^{I*}\left(1 - S_{11}S_{22}^I\right)}\left(1 - \left|S_{22}^I\right|^2 - \left|S_{21}^I\right|^2\right)\right],$$

where S_{ij} refers to an amplifier S-parameter, S_{ij}^I refers to an isolator S-parameter, and T_I is the noise temperature of the isolator. For a good isolator ($\left|S_{12}^I\right|$, $\left|S_{11}^I\right|$, $\left|S_{22}^I\right|$ all small) and small amplifier $|S_{11}|$, the expressions reduce to the more manageable form,

$$\begin{aligned} X_1{}' &\approx k_B T_I, \\ X_2{}' &\approx \frac{\left(X_2 + k_B T_I\left(1-\left|S_{21}^I\right|^2\right)\right)}{\left|S_{21}^I\right|^2}, \qquad (6) \\ X_{12}{}' &\approx \frac{S_{12}^I}{S_{21}^{I*}} X_{12} - k_B T_I S_{11}^I. \end{aligned}$$

For the amplifier and isolator that we used in the measurements, S_{12}^I and S_{11}^I are comparable in magnitude, but T_I is about ten times larger than X_{12}, and consequently X_{12}' is given approximately by $X_{12}' \approx -k_B T_I S_{11}^I$.

Thus a measurement of the noise parameters of the amplifier with an isolator can serve as a check of our ability to correctly measure the amplifier noise parameters (and S-parameters). Note that T_{rev} for the isolator-amplifier combination should be approximately equal to the ambient noise temperature.

In principle, some other passive 2-port, such as an attenuator, could be used instead of the isolator. Equation (5) would still hold, although of course the approximations of eq. (6) would no longer be valid. An advantage of using an attenuator is that the X' parameters depend more on the X parameters of the amplifier alone, whereas with an isolator X_1' and X_{12}' are nearly independent of the amplifier. Thus, an attenuator provides a better test of the consistency of the measured noise parameters of the particular amplifier under test. On the other hand, the isolator-amplifier combination provides something akin to a calculable standard, a device whose noise parameters (except X_2') are known (approximately) *a priori* from the ambient temperature and the isolator S-parameters. The isolated amplifier therefore provides an absolute test of the ability to correctly measure the noise parameters. Furthermore, with the (well matched) isolator one obtains a small value of X_{12}', which constitutes a more demanding test. For these reasons, we chose an isolator rather than an attenuator for this first set of tests.

III. MEASUREMENTS

We measured the noise parameters of a sample LNA in the 8 – 12 GHz band and have applied the tests outlined above. For the noise measurements we used eight different input terminations on the amplifier, measuring the output noise temperature for each and determining the four noise parameters and the gain by a least-squares fit to the expression for the output noise temperature in terms of the X's and the gain [7]. All the input terminations were passive, at ambient temperature, except for a diode noise source with noise temperature of approximately 1120 K. Of the ambient terminations, one was a matched load, and the other six were reflective terminations with different phases. The reflection coefficients of all the input terminations, as well as the S-parameters of the amplifier, were measured on a vector network analyzer. The fits were all very good, with the largest c^2 per degree of freedom being 0.32.

Any meaningful comparison of measured and predicted values requires uncertainty estimates of the quantities to be compared. All uncertainties are expanded (k = 2) uncertainties, corresponding to a confidence level of approximately 95 %. All uncertainties are combined uncertainties, including both type-A and type-B uncertainties [9],[10]. The type-A uncertainties in the noise-parameter measurements are the square roots of the diagonal elements of the covariance matrix of the fitted parameters. The type-B uncertainties in the noise-parameter measurements were estimated using the Monte Carlo program of [7],[11]. Uncertainties in predicted values of the X' parameters were estimated in a straightforward manner from eq. (6). Uncertainties in measured noise temperatures were determined in the usual manner for NIST noise-temperature measurements [12[,[13]. The measured values for the noise parameters of the amplifier, in both the X representation and in the IEEE representation, are compiled in Table 1.

In all the results, the values of the X parameters have been converted to temperatures by dividing by Boltzmann's constant. The results of the T_{rev} test are shown in Fig. 4. The values determined from eq. (3) and the fitted noise parameter X_1 are referred to as the predicted values, and the measured values are obtained from a direct measurement of the configuration of Fig. 2. The results of the isolator tests are shown in Figs. 5 – 8. For the isolator tests, the measured values are those obtained from the measurements of the noise parameters of the amplifier with isolator, and the predicted values are obtained from the measured noise parameters of the amplifier alone by use of eq. (5). The agreement is good to very good, with all cases being consistent within the estimated uncertainties. Thus the comparisons test, and in this case confirm, not only the noise-parameter measurement capability, but also the associated uncertainties.

TABLE I

Measured Values of the Noise Parameters for the Amplifier Alone.

	8 GHz	9 GHz	10 GHz	11 GHz	12 GHz
X_1(K)	64.5	67.7	68.6	70.4	80.3
X_2(K)	110.0	115.7	117.6	124.6	134.4
ReX_{12}(K)	9.45	-7.22	8.17	-10.23	14.3
ImX_{12}(K)	20.21	-14.29	10.73	-14.26	18.9
G_0	2031	2047	1987	2121	1649
T_{min}(K)	112.6	112.2	115.1	123.4	133.4
t(K)	128.3	234.2	145.8	223.9	209.8
ReΓ_{opt}	-0.172	0.130	-0.115	0.077	-0.006
ImΓ_{opt}	0.101	-0.046	-0.003	-0.004	-0.069

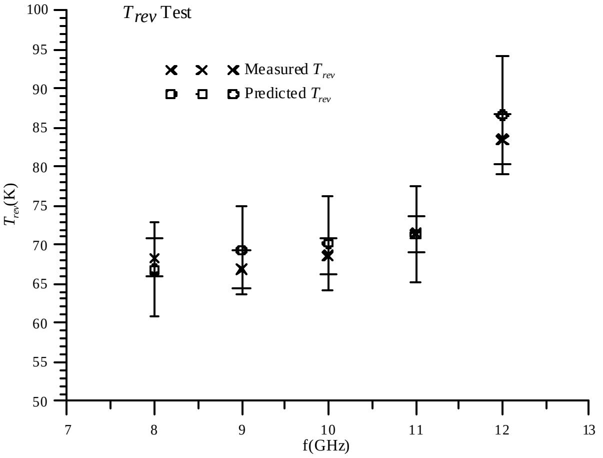

Fig. 4. Results of T_{rev} test.

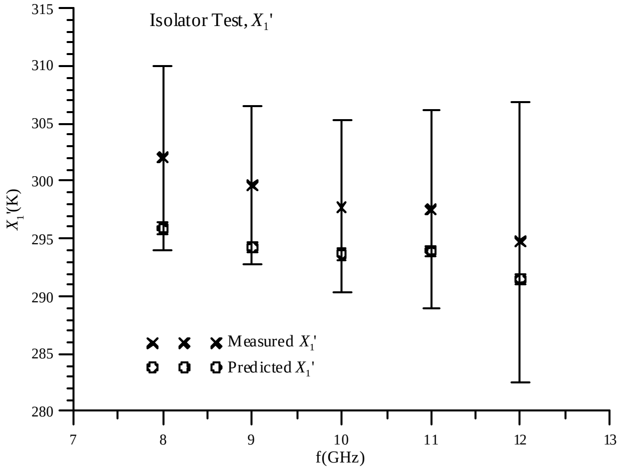

Fig. 5. Results of isolator test for X_1'.

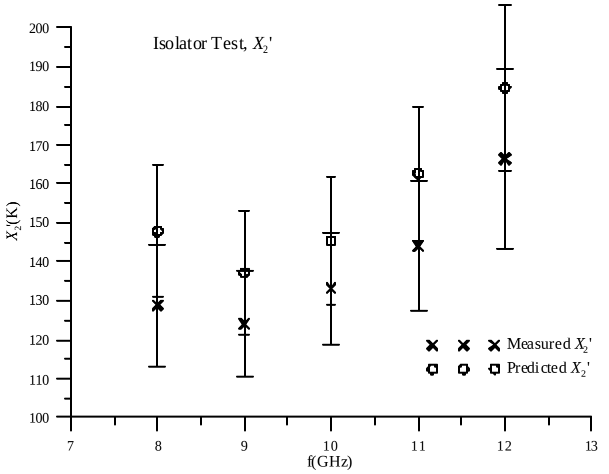

Fig. 6. Results for isolator test for X_2'.

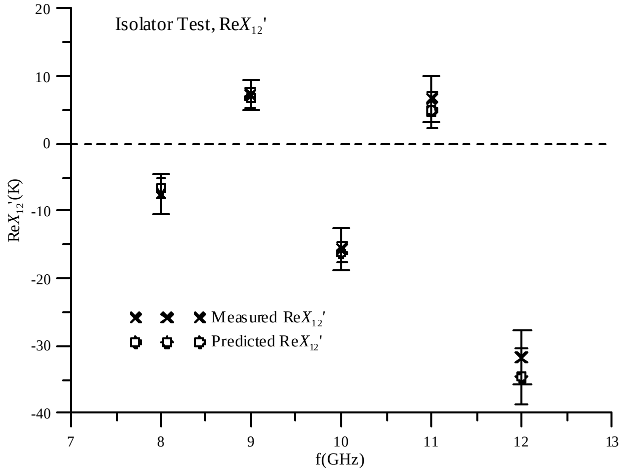

Fig. 7. Results of isolator test for ReX_{12}'.

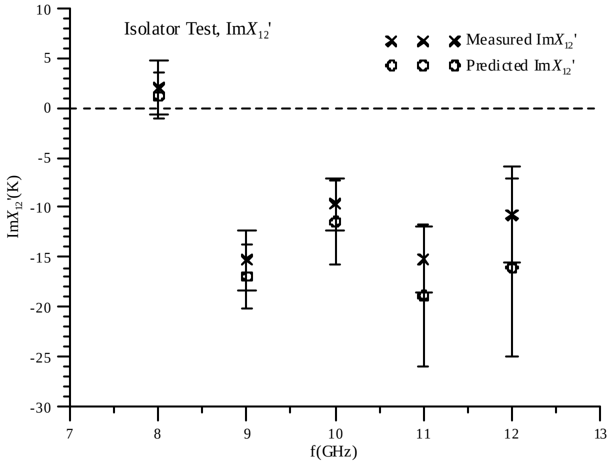

Fig. 8. Results of isolator test for $\mathrm{Im}X_{12}'$.

IV. CONCLUSIONS

In summary, we proposed and demonstrated verification tests for measurements of LNA noise parameters. Good agreement was found in the sample measurements. The tests require additional measurement effort, but they constitute a valuable tool if high accuracy is a primary concern. An important point to note is that not only do the tests provide assurance for the measurement results, but they also provide support for the uncertainty estimates.

REFERENCES

[1] R.Q. Lane, "The determination of device noise parameters," *Proc. IEEE*, vol. 57, pp. 1461-1462, August 1969.

[2] V. Adamian and A. Uhlir, “A novel procedure for receiver noise characterization,” *IEEE Trans. on Instrum. and Meas.*, vol. IM-22, no.2, pp. 181 – 182, June 1973.

[3] A. Boudiaf, C. Dubon-Chevallier, and D. Pasquet, “Verification of on-wafer noise parameter measurements,” *IEEE Trans. Instrum. Meas.*, vol. 44, no. 2, pp. 332 – 335, April 1995.

[4] D.F. Wait and G.F. Engen, “Application of radiometry to the accurate measurement of amplifier noise,” *IEEE Trans. Insrum. Meas.*, vol. 40, 433 – 437, 1991.

[5] S.W. Wedge and D.B. Rutledge, “Wave techniques for noise modeling and measurement, *IEEE Trans. Microwave Theory Tech.*, vol. 40, no. 11, pp. 2004 – 2012, November 1992.

[6] J. Randa and W. Wiatr, “Monte Carlo Estimation of noise-parameter uncertainties,” *IEE Proceedings—Science, Measurement and Technology*, vol. 149, pp. 333 – 337, 2002.

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[7] J. Randa, “Noise-parameter uncertainties: a Monte Carlo simulation,” *J. Res. Natl. Inst. Stand. Technol.*, vol. 107, pp. 431 – 444, 2002.

[8] H. Haus *et al.*, “IRE standards on methods of measuring noise in linear twoports 1959,” *Proc. IRE*, vol. 48, pp. 60 – 68, 1960.

[9] “ISO Guide to the Expression of Uncertainty in Measurement,” Int. Org. Standard., Geneva, Switzerland, 1993.

[10] B.N. Taylor and C.E. Kuyatt, “Guidelines for evaluating and expressing the uncertainty of NIST measurement results,” NIST Tech. Note 1297, 1994.

[11] J. Randa, “Simulations of noise-parameter uncertainties,” *2002 IEEE MTT-S International Microwave Symposium Digest*, Seattle, WA (2002) pp. 1845 – 1848.

[12] J. Randa, “Uncertainties in NIST noise-temperature measurements,” NIST Tech. Note 1502, March 1998.

[13] C.A. Grosvenor, J. Randa, and R.L. Billinger, “Design and testing of NFRad—a new noise measurement system,” NIST Tech. Note 1518, March 2000.

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