JOHNSON NOISE THERMOMETRY USING A QUANTUM VOLTAGE NOISE SOURCE FOR CALIBRATION[†]

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Abstract

We describe our progress towards a high-precision measurement of temperature using Johnson noise. Using a Quantized Voltage Noise Source (QVNS) based on the Josephson effect as a calibrated noise source, we have been able to measure the gallium and water triple point temperatures with an accuracy approaching 100 ppm.

Introduction

Johnson Noise Thermometry (JNT) involves the measurement of the statistical variance of a fluctuating voltage across a resistor in thermal equilibrium. The spectral density S^R of the voltage noise power across a resistor $R(T)$ is given by the Nyquist formula [1],

$$S^R = 4kTR(T), \qquad (1)$$

where k is Boltzmann's constant. We have built a system that uses a digitally synthesized Quantized Voltage Noise Source (QVNS) which uses an array of Joesphson junctions as an ac-voltage reference to correct for the non-ideal performance of the electronic systems used to measure the noise. The QVNS is used to emulate resistor noise with a constant spectral density that is stable, programmable, and calculable [2]. With the QVNS, we have operated our system in two distinct modes, "absolute" and "relative".

Absolute Mode. In the "absolute" mode of operation, the temperature of the thermometer (resistor) is directly determined by comparing the noise power from the resistor with the noise power from the QVNS [3]. Using Eq. 1, the thermodynamic temperature of the resistor can be determined using the following equation:

$$T = \left(\frac{S^R_{measured}}{S^{QVNS}_{measured}} \right) \frac{S^{QVNS}_{calculated}}{4kR(T)} \qquad (2)$$

where $S^R_{measured}$ is a measurement of the resistor noise power density, $S^{QVNS}_{measured}$ is a measurement of the QVNS power density, and $S^{QVNS}_{calculated}$ is the calculated QVNS power density.

Relative Mode. In the "relative" mode, the Johnson noise power of a resistor at a known temperature is first measured against a balanced, synthesized noise power from the QVNS. The process is repeated with the same resistor at an unknown temperature with a second QVNS signal which is comparable to the noise power of the resistor at this temperature. The ratio of the unknown temperature T to the reference temperature T_0 can be determined with the following formula:

$$T = \frac{S^{R@T}_{measured}}{S^{QVNS@T}_{measured}} \frac{S^{QVNS@T_0}_{measured}}{S^{R@T_0}_{measured}} \frac{S^{QVNS@T}_{calculated}}{S^{QVNS@T_0}_{calculated}} \frac{R(T_0)}{R(T)} T_0 , \qquad (3)$$

where $S^{R@T}_{measured}$, $S^{R@T_0}_{measured}$, $S^{QVNS@T}_{measured}$, and $S^{QVNS@T_0}_{measured}$ are measurements of the noise power density of the resistor and QVNS at both temperatures/power levels and $S^{QVNS@T}_{calculated}$ and $S^{QVNS@T_0}_{calculated}$ are the calculated power densities for the signal from the QVNS. This mode of operation will be less susceptible to systematic errors that occur in the absolute mode from intrinsic differences in the measurement of the power from the QVNS and resistor (e.g. different transmission lines).

Experimental Setup

The Johnson noise thermometry (JNT) system consists of three major components: the temperature cell and resistor probe, the QVNS sub-system, and cross-correlation electronics.

Temperature cell and resistor. The Johnson noise resistor is a metal-foil surface-mount device with a resistance of ~100 Ω. The resistor is mounted in a shielded temperature probe that is inserted into an immersion-type, fixed-point cell. We report data from our experiments with both a gallium triple point cell (GaTP) and a water triple point cell (WTP).

QVNS. The QVNS is used to generate a constant-amplitude comb of harmonic tones with random relative phases to match the noise power density from a resistor for frequencies less than 2 MHz. The signals from the QVNS originate from an array of microwave pulse-biased Josephson junctions and are calculable from fundamental constants [2].

Cross-correlation electronics. The most successful JNT measurement technique developed to date is switched-input noise-correlation pioneered by Brixy, *et al.* [4]. In a noise-correlation measurement, a cross-correlation technique is used to reduce the systematic error from the electronics used to measure the noise. In our cross-correlation electronics, a signal from either the resistor or QVNS is amplified and digitized through two parallel electronics chains. The digitized signals are Fourier

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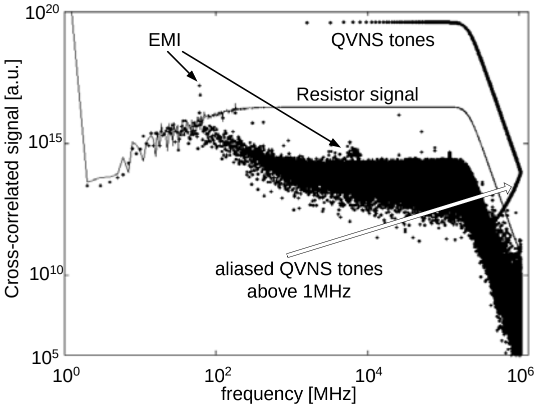

Fig. 1. The cross-correlated noise signal from a 100 Ω resistor at the gallium triple-point averaged for 30000 sec (line). The dots represent the signal from a QVNS matched in power density to the resistor noise. Each dot represents a 1Hz frequency bin.

transformed and cross-correlated with a computer.

Results

Using the GaTP cell, we obtained the cross-correlated spectrum from a 100 Ω resistor shown in Fig. 1. The spectrum is an average of 30000 traces with frequency bins spaced at ~1 Hz. The cross-correlated spectrum from a QVNS is also shown in Fig. 1. The signal from the QVNS are a comb of tones spaced by ~1.589 kHz from dc to 2 MHz. Each tone is approximately 51.55 nVrms to match the noise power density of the resistor. To directly determine the temperature of the resistor, the Johnson noise spectra from the resistor must be re-binned at each QVNS tone over a bandwidth equal to the spacing of the QVNS tones. The resulting re-binned data can be divided by the power in each QVNS tone producing a noise power ratio which is directly proportional to temperature and is shown in the lower scatter plot in Fig. 2. A preliminary analysis of data taken over 16 hours over the first 100 kHz band results in a T = 302.870 K with a type-A, 1σ-uncertainty of 7 mK which differs from the ITS-90 value of 302.916 K for the GaTP by 150 ppm. With the WTP cell and adjusting the power from the QVNS, we found that WTP to be 273.078 K with a type-A, 1σ-uncertainty of 4 mK. The ITS-90 value for the WTP is 273.160 K.

Using the two absolute measurements of the temperature, it is possible to determine the ratio of the GaTP to the WTP. The temperature ratio as a function of frequency is also shown in Fig. 2. Using the two absolute ratios below 100 kHz, we find the relative-mode, two temperature ratio to be 1.10911 with a type-A 1σ-uncertainty of 3e-5. The accepted ITS-90 ratio is 1.10893.

Discussion

For absolute determination of the thermodynamic temperature of the resistor, the power-ratio between the QVNS and the Johnson noise spectra shown in Fig. 2 exhibits a frequency dependence that deviates from a constant ratio. The systematic difference between the QVNS and resistor noise at higher frequencies is probably due to transmission-line effects and common-mode errors from accidental coupling of the digital code generator signals used to drive the Josesphson array circuit in the QVNS. It is also likely that this is the dominant source of error for both the absolute and relative measurements of temperature. For the relative mode determination of the temperature ratio between the GaTP and WTP, the frequency dependence of the ratio is more constant. This indicates that there is a systematic error in the absolute-mode of operation which is reduced by looking at the ratio.

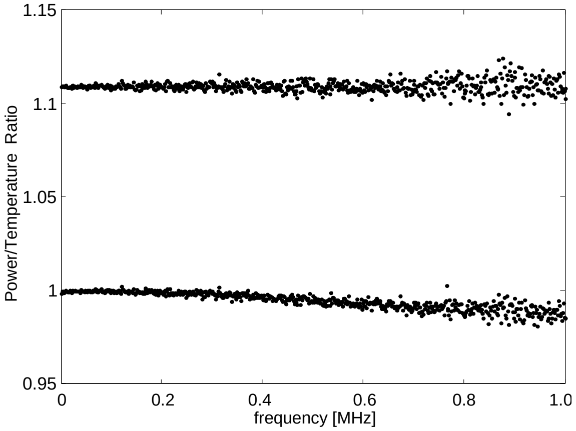

Fig. 2. The upper scatter plot is the measured noise power ratio of the resistor at the GaTP and at the WTP. The ratio should be the ratio of the temperatures, 1.10893. The lower scatter plot is the ratio of the noise power from the resistor to the noise power from the QVNS. The ratio should be 1.

Conclusions

We have made absolute measurements of the temperature of a resistor at the gallium and water triple points. We believe that the errors in the measurements are dominated by transmission-line effects and common-mode signals in the QVNS that should be correctable. We plan to test new Josesphson junction circuit designs to reduce common-mode errors and characterize our transmission lines. We also plan to perform more absolute and relative mode measurements at higher temperatures (273 K to 1000 K) through comparisons with a calibrated platinum resistance thermometer.

References

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