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Finite coplanar waveguide width effects in pulsed inductive microwave magnetometry

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The effect of finite coplanar waveguide (CPW) width on the measurement of the resonance frequency in thin ferromagnetic films has been characterized for pulsed inductive microwave magnetometry. A shift in resonant frequency is a linear function of the ratio of sample thickness to CPW width. The proportionality constant is experimentally determined to be 0.74 ± 0.1 times the saturation magnetization of the film. The frequency shift may be modeled as arising from an effective magnetic-anisotropy field. [DOI: 10.1063/1.1769084]

The switching speed of fundamental magnetic components in disk drives will need to greatly increase if the current trend in hard disk data transfer rates continues.[1] This increase in required bandwidth has stimulated recent research on the high frequency susceptibility and magnetic anisotropy of thin magnetic films.[2–6] One promising method of investigating magnetic anisotropy of thin magnetic films is the use of a pulsed inductive microwave magnetometer (PIMM). This method has the advantage of yielding time-domain data on the precessional motion of the magnetization.[7] In combination with classic magnetostatic measurements, a PIMM can provide information on the uniaxial anisotropy, the effective magnetization, the Landé g factor, and the Gilbert damping parameter.[3,7,8]

One complication in using the PIMM is the influence of the finite width of the coplanar waveguide (CPW)[9]. In this letter, we experimentally determine the influence of the finite CPW width on the resonance frequency of magnetic precession. A frequency shift is observed that may be explained by the excitation of a distribution of Damon-Eshbach magnetostatic modes. A model for the correction factor agrees within 6% of our experimentally measured value.

The samples used in this study were polycrystalline $Ni_{81}Fe_{19}$ Permalloy films with thicknesses of 25, 50, 75, and 100 nm. The Permalloy was deposited via dc magnetron sputtering in a 0.533 Pa pressure Ar atmosphere. The sputtering system had a base pressure of 10^{-6} Pa. The substrates used were $1\ \mathrm{cm} \times 1\ \mathrm{cm} \times 100\ \mu\mathrm{m}$ coupons of (0001)-oriented sapphire and $1\ \mathrm{cm} \times 1\ \mathrm{cm} \times 150\ \mu\mathrm{m}$ glass coupons, and were prepared with *in situ* Ar/O_2 ion milling prior to deposition. A 5 nm seed layer of Ta was deposited onto the cleaned substrates to enhance the adhesion of the Permalloy films. An external magnetic field of 20 kA/m was applied to the Permalloy films during growth to induce a uniaxial anisotropy. The samples were characterized using the hard-axis and easy-axis hysteresis loops as measured by a low-frequency inductive magnetometer. The quasistatic anisotropy field was 407 A/m for the 100 nm sample, 332 A/m for the 75 nm sample, 341 A/m for the 50 nm sample, and 299 A/m for the 25 nm sample.

The dynamic measurements were performed using a PIMM.[7] Three different CPWs, each of 50 Ω impedance, and CPW widths of 220, 525, and 990 μm used to perform the measurements. The easy axes of the Permalloy films were aligned parallel to the center conductor of the CPW in all measurements. A floating ground plane was positioned above the Permalloy sample in order to enhance the magnetic field pulse amplitude.[10] Using the Karlqvist equation[11] for fields produced by a current strip, and assuming a factor of 2 enhancement of the field amplitude by the floating ground plane, we estimated that a 10 V pulse applied to the 1 mm wide CPW yielded our highest field of 175 A/m. A detailed description on the measurement technique used with the PIMM can be found in Ref. 7. During the measurements, a longitudinal saturation field of 5570 A/m was applied parallel to the pulse field, and one set of data was acquired. After this, a bias field ranging from 1592 to 7958 A/m was applied parallel to the easy axis of the sample, and another set of data was acquired. The two data sets were subtracted, and the resonance frequency, as determined from the zero crossing of the real part of a fast Fourier transform of the acquired data[12] was found. The magnetic resonance frequency ranged from 1.5–3 GHz and was well within the measurable bandwidth of the system.[13]

The effective resonance frequency ω_{eff} of the magnetic excitation is the result of a distribution of surface magnetostatic modes for a slab of thickness δ.[14] In the long wavelength limit $k\delta \ll 1$ and weak magnetic fields $H_b + H_k \ll M_s$, the dispersion relation for the characteristic frequency ω_0 of such modes is given by[15]

$$\omega_0(k) \approx \gamma\mu_0 \sqrt{M_s\left(H_b + H_k + \frac{M_s}{2}k\delta\right)}, \tag{1}$$

where $|\gamma| = (g\mu_B)/\hbar$ is the gyromagnetic ratio, g is the spectroscopic splitting factor, μ_B is the Bohr magneton, $\mu_0 = 4\pi \times 10^{-7}(H/m)$ is the permeability of free space, M_s is the saturation magnetization, H_b is the applied field, and H_k is the in-plane anisotropy field.

Figure 1 shows resonance frequency squared, $f_0^2 = (\omega_0/2\pi)^2$, versus applied longitudinal bias field H_b measured on a 100 nm thick Permalloy film for the three different CPW widths w. We estimate the error in our data to be 5 MHz, which, on the graph, is smaller than the size of the data points. We extrapolated the linear fit of the data to $f_0^2 = 0$ to find H_k^{eff} for the Permalloy thin films.

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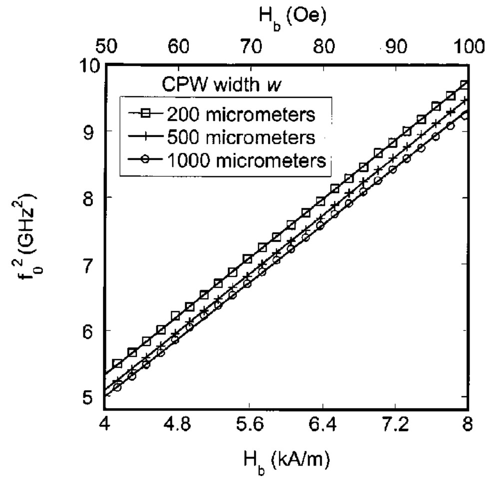

FIG. 1. The measured values of squared resonance frequency $f_0^2 = (\omega_0/2\pi)^2$ vs applied field H_b for a 100 nm thick Permalloy film for three different CPW widths.

Figure 2 shows the measured values for H_k^{eff} for four different samples, each measured on the three different CPWs. Note that the value of H_k^{eff} increases as the ratio of sample thickness δ to CPW width w is increased. The change in the value of H_k^{eff}, when using a narrow CPW with a wide sample, arises because the excitation fields generated by the CPW are inhomogeneous.

It is important to quantify the effect of field inhomogeneity in the limit of a purely linear response. Assuming M is initially oriented perpendicular to the direction of the excitation field, and assuming the small angle motion of M, the magnetic excitation precession frequency is given by Eq. (1).

The susceptibility spectrum for each mode is found by linearizing the Landau-Lifshitz equation for damped gyromagnetic precession in a ferromagnetic film.[16] In the limit of small-angle motion of M and large M_s such that $M_s \gg H + H_k$, the resultant equations of motion are those of a harmonic oscillator. Using Fourier analysis, the spectrum for a given mode with characteristic frequency ω_0 is

$$\tilde{\chi}(\omega_0,\omega) = \tilde{\chi}_0(k)\frac{\omega_0^2}{\omega_0^2 - \omega^2 - i(2\omega/\tau)}, \tag{2}$$

where $\tau \approx 1/(2\alpha\gamma\mu_0 M_s)$, α is the damping parameter where $\alpha \leqslant 0.01$ for all samples, and $\chi_0(k)$ is the dc susceptibility for

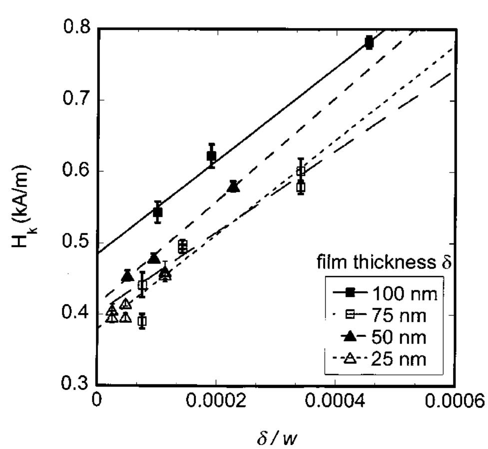

FIG. 2. The value of the in-plane anisotropy field H_k as measured by the PIMM vs the sample thickness divided by the CPW width. Four different samples were measured to determine the effect of sample thickness. While there is a consistent change in the offset of the data between samples, no consistent change of slope was found while changing sample thickness.

a given mode of wave number k. The susceptibility for a mode may be found by inspecting Eq. (1), where the dipole fields produced by the sinusoidal magnetization distribution act as an additional anisotropy term that increases the resonance frequency. Therefore,

$$\tilde{\chi}_0(k) = \frac{M_s}{H_b + H_k + (M_s/2)k\delta}. \tag{3}$$

The spacing between the sample and CPW surface is d. For a thin magnetic film ($\delta \ll d$) of dimension L that is large compared to the CPW width ($L \gg w$), the detected flux per mode is

$$d\tilde{\Phi}(\omega) = \frac{\mu_0 L\,\delta I}{4\pi}\,\mathrm{sinc}^2\left(\frac{kw}{2}\right)e^{-2kd}\tilde{\chi}(k,\omega)dk, \tag{4}$$

where we have used reciprocity and the Karlqvist equation[10] to determine the spectral sensitivity function of the CPW and $\sin c(x) = \sin(x)/x$. Using Faraday's law, the inductive voltage is $V(\omega) = i\omega\Phi(\omega)$.

So far, we have ignored the temporal frequency spectrum of the exciting field pulse. For excitation with a perfect step function, the spectral distribution of the excitation is weighted by a factor of $1/i\omega$. Thus, the final result for the measured inductive voltage in the case of a step excitation is

$$V(\omega) = \frac{\mu_0 L\,\delta I}{8\pi}\int_0^\infty \mathrm{sinc}^2\left(\frac{kw}{2}\right)e^{-2kd}\tilde{\chi}(k,\omega)dk. \tag{5}$$

Here, we have divided by an additional factor of 2 in order to account for the CPW symmetry, which divides the inductive voltage between two counterpropagating signals that radiate away from the sample.[7]

The ramifications of the multimode excitation with a narrow CPW are twofold. First, the peak in the measured susceptibility spectrum is shifted toward higher wavelength magnetostatic surface modes. Second, the resonance peak is broadened due to the superposition of a distribution of resonance lines. There is evidence that this line broadening due to the finite CPW width may, in general, have an effect on the damping of the resonance.[17] While the inhomogeneous line broadening is difficult to calculate, we can estimate the frequency shift by approximating the integral in Eq. (4), since $\mathrm{sinc}^2(x)$ falls off rapidly for $x > \pi/2$. Furthermore, we are operating in the long wavelength limit, where $kd \ll 1$. Thus we are left with

$$V(\omega) = \frac{\mu_0 L\,\delta I}{8\pi}\int_0^{\pi/w}\tilde{\chi}(k,\omega)dk. \tag{6}$$

The most important factor in determining the effective peak in the resonance spectrum for the collection of modes is the distribution of the dc susceptibilities in Eq. (2). This results from the fact that the functional shape of the spectrum in Eq. (1) does not strongly depend on the resonance frequency, with the exception of the position of the resonance. Thus, the average resonance frequency $\langle\omega_0\rangle$ for an ensemble of oscillators with relative strength $\tilde{\chi}_0(k)$ is given by

$$\langle \omega_0 \rangle = \frac{\displaystyle\int_{\omega(k=0)}^{\omega(k=\pi/w)} \omega_0 \tilde{\chi}_0[k(\omega_0)] d\omega_0}{\displaystyle\int_{\omega(k=0)}^{\omega(k=\pi/w)} \tilde{\chi}_0[k(\omega_0)] d\omega_0}$$

$$= \frac{\omega_0(k=0)}{2}\left(\frac{\overline{\chi}}{\ln(1+\overline{\chi})}+1\right), \quad (7)$$

where

$$\overline{\chi} = \frac{\pi\delta\tilde{\chi}_0(k=0)}{2w}. \quad (8)$$

In the limit, $\overline{\chi} \ll 1$ and the frequency shift is equivalent to that arising from an additional anisotropy component H_k^{MSSW} in Eq. (1), where

$$H_k^{MSSW} = C_w M_s\left(\frac{\delta}{w}\right), \quad C_w = \frac{\pi}{4} \quad (9)$$

for the uniform mode $(k=0)$. This limiting behavior for the frequency shift has been verified to be valid up to $2\overline{\chi}/\pi \leqslant \pi$ by numerical integration of Eq. (4).

Equation (9) establishes a way to experimentally measure the additional anisotropy component. M_s, δ, and w are all known values; thus we define $C_w = \pi/4 \cong 0.785$ in Eq. (9). To check our model with experimental measurements, we used the plots of Fig. 2. In Fig. 2 we plot the measured H_k for each sample and each CPW width versus the ratio of sample thickness over CPW width. Equation (9) is an additional anisotropy term that is added to the H_k term in Eq. (1). Note that this additional anisotropy term is the only one that depends on the CPW width. We thus use the slope $C_w M_s$ to experimentally check the value of C_w.

Figure 3 shows our experimentally measured values for the constant C_w as a function of film thickness. The data are shown for three different excitation voltages. There is no apparent dependence of C_w on the excitation pulse voltage. This helps affirm that we are in the small angle limit in all of our measurements, as assumed in the model. There is no clear trend in C_w as a function of sample thickness, as expected. Averaging the data at all voltages, and for all values of sample thickness, yields a mean value for $C_w = 0.74 \pm 0.1$, which is 94% of the calculated value of $\pi/4$.

In summary, CPW width has an effect on the measurement of H_k when using a PIMM. This change in the measured H_k is due to the nonuniform field created by a thin CPW structure. The effect of the nonuniform field on a thin film has been modeled and it was found that the change in H_k is a linear function of the ratio of sample thickness to CPW width. We performed several measurements on thin Permalloy films using a PIMM in the small-angle regime, and experimentally determined the constant shift in the value of H_k as a function of sample thickness over CPW width. We found good agreement between our experimental results and a model based on a linear superposition of magnetostatic surface modes in the small-angle limit.

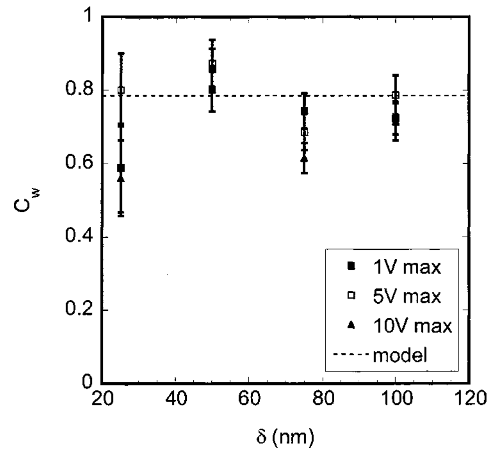

FIG. 3. The experimentally measured values of the constant C_w and a dotted line representing the theoretically predicted value of $\pi/4$ from Eq. (9). This constant represents a frequency shift equivalent to that arising from an additional anisotropy component H_k^{MSSW} in Eq. (1).

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