Alignment Uncertainties of the NIST Watt Experiment

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Abstract—**The effects of alignment uncertainties of the NIST watt balance with respect to local gravity and the magnetic flux density of the balance have been analyzed. Techniques for measuring all quantities relevant to misalignment have been developed. The components of the relative combined standard uncertainty of the measured value of the watt due to alignment uncertainties have been reduced to 20 nW/W, and potential improvements in the balance design have been identified which could ultimately lead to a reduction of that uncertainty to below 10 nW/W.**

I. Introduction

The NIST WATT balance [1], [2] has been developed to compare the practical realizations of the ohm and the volt derived from the quantum Hall effect and the Josephson effect to the meter, kilogram, and second by measuring the watt in both electrical and mechanical units, following the technique proposed by Kibble [3]. First, the force F on an induction coil carrying a current I in a magnetic flux density is measured. Then the coil is moved at a velocity ν and the induced voltage ε is measured. By comparing the force times the velocity to the voltage times the current, the same power is measured in both electrical and mechanical units:

$$\frac{F\nu}{I\varepsilon} = \frac{\text{mechanical power}}{\text{electrical power}}. \tag{1}$$

A schematic of the watt balance is shown in Fig 1. One side has a pan for holding a standard of mass and an induction coil in a radial magnetic flux density produced by a superconducting solenoid (not shown). A vertical force is applied by adding a 1 kg mass standard to the pan, and a current through the induction coil supplies a force to balance the weight of the mass standard. The induction coil is then open-circuited, moved with a known velocity in the vertical direction (measured with a set of 3 laser interferometers oriented at spacings of 120° about the induction coil) by rotating the balance wheel about a knife edge, and the induced voltage in the coil is measured.

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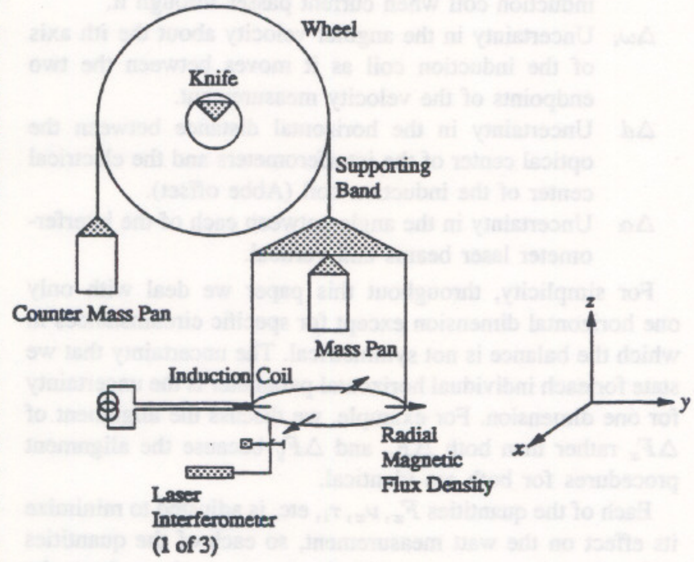

Fig. 1. Schematic view of the NIST watt balance. The radial magnetic flux density is generated by a superconducting solenoid (not shown).

Force and velocity are vector quantities, so both their magnitudes and directions are important in determining the power. Additional mechanical power will also be generated if there is any torque and angular velocity. The total mechanical power is thus

$$P = \vec{F}\cdot\vec{\nu} + \vec{\tau}\cdot\vec{\omega} = |\vec{F}||\vec{\nu}|\cos\psi + \vec{\tau}\cdot\vec{\omega} \tag{2}$$

where $P, \vec{F}, \vec{\nu}, \psi, \vec{\tau}$, and $\vec{\omega}$ are the power, force, velocity, angle between the force and velocity, torque, and angular velocity, respectively. The quantities $\psi, \vec{\tau}$, and $\vec{\omega}$ are adjusted to be as close to zero as possible, so that the power is approximated simply as the magnitude of the force times the magnitude of the velocity, as in (1). Errors in the measured watt value can then occur due to horizontal forces and velocities, or to torques and angular velocities. Even with perfect alignment of the balance, measurement error can result from misalignment with respect to vertical of the laser interferometers which make up the velocity measurement system. These effects, which can cause an error in the measured watt value, are collectively referred to as alignment errors. This paper discusses how we align the force and velocity, minimize the torque on the balance, and identify the corresponding alignment uncertainty components of the watt measurement.

II. The Alignment Uncertainties and Their Contribution to the Watt Measurement

The specific alignment uncertainties that have been identified are as follows:

- ΔF_x Uncertainty in the horizontal component of the force on the induction coil when current passes through it.
- $\Delta \nu_x$ Uncertainty in the horizontal component of the velocity.
- $\Delta \tau_i$ Uncertainty in the torque about the ith axis of the induction coil when current passes through it.
- $\Delta \omega_i$ Uncertainty in the angular velocity about the ith axis of the induction coil as it moves between the two endpoints of the velocity measurement.
- Δd Uncertainty in the horizontal distance between the optical center of the interferometers and the electrical center of the induction coil (Abbe offset).
- $\Delta \alpha$ Uncertainty in the angle between each of the interferometer laser beams and vertical.

For simplicity, throughout this paper we deal with only one horizontal dimension except for specific circumstances in which the balance is not symmetrical. The uncertainty that we state for each individual horizontal parameter is the uncertainty for one dimension. For example, we discuss the alignment of ΔF_x rather than both ΔF_x and ΔF_y because the alignment procedures for both are identical.

Each of the quantities F_x, ν_x, τ_i, etc. is adjusted to minimize its effect on the watt measurement, so each of the quantities $\Delta F_x, \Delta \nu_x, \Delta \tau_i$, etc. is implicitly the uncertainty about the nominal value of zero. The magnitudes of the stated uncertainties were estimated using scientific judgement (Type B evaluation) rather than by purely statistical means. An interval was estimated which contained essentially all of the probable values for a particular quantity. This interval was established by asking the question, "What misalignment can clearly be seen with our measurement systems?" Each individual quantity was varied in each direction in a controlled fashion until misalignments could be unambiguously identified; those points were defined as the endpoints of the interval, $-a$ and $+a$. The probability distribution within the interval is assumed to be uniform in nature. Under these assumptions, the standard uncertainty is simply $a/\sqrt{3}$, where a is the half width of the interval $-a$ to $+a$ for the quantity in question [4].

The uncertainties in these quantities combine to give the alignment uncertainty components in the watt measurement listed in the second column of Table I. In the table, r is the radius of the induction coil (about 350 mm), and the values for $\omega_i/|\vec{v}|$ are expressed as the total angle of rotation of the induction coil during a velocity measurement divided by the distance between the starting and stopping points of the induction coil during that measurement (about 70 mm). The first four terms are simply due to a force and velocity misalignment or from a residual torque and tilt, i.e., they are the ratios of the unwanted power terms from the dot product of (2) to the total power. The last five terms are associated with possible measurement errors due to misalignments in the velocity measurement system which lead directly to an uncertainty in the watt measurement. In particular, the fifth term is an offset between the optical center of the three interferometers and the electrical center of the coil coupled to a tilt (often called the Abbe error). Terms six through eight are due to a coupling between an angular misalignment of the interferometer laser beams to vertical and nonvertical motions of the laser interferometer's corner cubes attached to the induction coil. The last term is simply the uncertainty from the cosine error of the velocity measurement due to laser misalignment, which exists even if the coil velocity is perfectly vertical. It is a bias term since an error in α always decreases the measured value of the mechanical watt.

TABLE I
Summary of the Components of the Standard Uncertainty in the Watt Measurement due to Uncertainties in the Alignment of the Balance

Term no.	Alignment uncertainty components	Values	Dimension	Relative standard uncertainty contribution to $\frac{\Delta W}{W}$
1	$\left(\frac{\Delta F_x}{\lvert\vec{F}\rvert}\right)\left(\frac{\Delta v_x}{\lvert\vec{v}\rvert}\right)$	$(6\times10^{-5})(8\times10^{-5})$	1	5×10^{-9}
2	$\left(\frac{\Delta F_y}{\lvert\vec{F}\rvert}\right)\left(\frac{\Delta v_y}{\lvert\vec{v}\rvert}\right)$	$(6\times10^{-5})(1.7\times10^{-4})$	1	1.0×10^{-8}
3	$\left(\frac{\Delta \tau_x}{\lvert\vec{F}\rvert}\right)\left(\frac{\Delta \omega_x}{\lvert\vec{v}\rvert}\right)$	$(0.17\text{ mm})\left(\frac{3\times10^{-7}}{70\text{ mm}}\right)$	2	1.0×10^{-9}
4	$\left(\frac{\Delta \tau_z}{\lvert\vec{F}\rvert}\right)\left(\frac{\Delta \omega_z}{\lvert\vec{v}\rvert}\right)$	$(1.6\times10^{-4}\text{ mm})\left(\frac{1.7\times10^{-5}}{70\text{ mm}}\right)$	1	3.9×10^{-11}
5	$\Delta d\left(\frac{\Delta \omega_x}{\lvert\vec{v}\rvert}\right)$	$(0.2\text{ mm})\left(\frac{3\times10^{-7}}{70\text{ mm}}\right)$	2	1.2×10^{-9}
6	$\Delta \alpha r\left(\frac{\Delta \omega_z}{\lvert\vec{v}\rvert}\right)$	$(4\times10^{-5})(350\text{ mm})\left(\frac{1.7\times10^{-5}}{70\text{ mm}}\right)$	1	3×10^{-9}
7	$\Delta \alpha\left(\frac{\Delta v_x}{\lvert\vec{v}\rvert}\right)$	$(4\times10^{-5})(8\times10^{-5})$	1	3×10^{-9}
8	$\Delta \alpha\left(\frac{\Delta v_y}{\lvert\vec{v}\rvert}\right)$	$(4\times10^{-5})(1.7\times10^{-4})$	1	7×10^{-9}
9	$\frac{1}{2}\Delta\alpha^2$	$\frac{1}{2}(4\times10^{-5})^2$	2	1.1×10^{-9}
Relative combined standard uncertainty of $\frac{\Delta W}{W}$ due to alignment:				2×10^{-8}

The components listed in Table I describe only the lowest order contribution of the alignment errors to the watt measurement, and as such are only valid for small misalignments. A computer simulation of the watt balance which models interactions between the superconducting solenoid and the induction coil using a grid approximation of the magnetic flux densities was used to test the components [5]. The simulation and components were in agreement (differences were all less than 10%) for the size of misalignments that are likely, thus giving us confidence that the lowest order approximation is sufficient. With this simulation we are examining correlation effects between various quantities and will include these effects in future estimates of the combined uncertainties.

III. Evaluating the Uncertainties

The horizontal velocity uncertainty $\Delta \nu_x$ is evaluated by monitoring the position of vertical laser beams which are reflected from corner cubes attached to the induction coil. Horizontal displacements and velocities of the induction coil become horizontal displacements and velocities of the reflected laser beams. The velocity in the direction parallel to the knife edge (rotation axis of the balance wheel), $\Delta\nu_x$, is minimized by adjusting the tilt of the wheel about the y axis in Fig. 1. The horizontal velocity perpendicular to the knife edge, $\Delta\nu_y$, is adjusted by shifting the position of the knife edge vertically with respect to the center of the wheel. This adjustment is cumbersome and limits our ability to zero the horizontal velocity in that direction. The resulting relative standard uncertainties $\Delta\nu_x/|\vec{v}|$ and $\Delta\nu_y/|\vec{v}|$ are 8×10^{-5} and 1.7×10^{-4}, respectively.

A nonzero ΔF_x can be caused by the axis of either the superconducting solenoid or the induction coil being misaligned from vertical. A nonzero $\Delta\tau_x$ is caused by the electrical center of the induction coil being horizontally displaced from the center of mass of the induction coil, or by the electrical center of the induction coil being horizontally displaced from the electrical center of the superconducting solenoid. The angle of the superconducting solenoid axis with respect to vertical, and the displacement between the electrical centers of the superconducting solenoid and the induction coil can be measured independently. The measurements of the other two quantities, the angle of the induction coil axis with respect to vertical, and the displacement between the electrical and mass centers of the induction coil, are coupled to each other and to the first two quantities. Therefore, ΔF_x and $\Delta\tau_x$ are minimized by first aligning the superconducting solenoid, both vertically and with respect to the induction coil, and then aligning the axis and center of mass of the induction coil.

The magnetic flux density is aligned to vertical using a pickup coil whose axis has been separately aligned to vertical using a precision solenoid. The pickup coil is placed in the region of the superconducting solenoid's magnetic flux density that is normally occupied by the induction coil during a watt measurement. The angle of the superconducting solenoid is adjusted to minimize the mutual inductance between it and the pickup coil. This is done at several different azimuthal positions. The resulting uncertainty of the angle is 1.2×10^{-4} rad.

The relative positions of the electrical center of the induction coil and the center of the radial magnetic flux density are determined using a pickup coil which measures the radial magnetic flux density at the position of a particular segment of the induction coil. Attached to this pickup coil is a second set of pickup coils, acting like a linear differential transducer, in which the mutual inductance between the induction coil and this second set is used to locate the radial position of a segment of the induction coil. From the position of several different segments of the induction coil, the position of the electrical center of the induction coil relative to the magnetic flux density is calculated. The standard uncertainty of this procedure is 0.12 mm.

Neither the angle of the induction coil axis nor the position of its center of mass relative to its electrical center could be independently measured. If either of these quantities is misaligned, then a current through the induction coil produces a torque and horizontal force on the induction coil which can be measured as angular and lateral deflections of the induction coil. The induction coil is suspended as a pendulum which constrains its horizontal angular motion; therefore the lateral and angular motions in response to torques and forces are mixed. By causing known changes in the center of mass and angle of the induction coil axis and measuring the lateral and angular response of the induction coil to an electric current, the matrix describing the mixing is determined. Then the center of mass and induction coil angle are adjusted to minimize the deflections, resulting in uncertainties of 0.17 mm for the relative distance between the mass and electrical centers and 6×10^{-5} rad for the static angle of the coil.

It is important to note that the horizontal torques and forces can be the result of not only misalignments in the center of mass and axis of the induction coil, but also misalignments in the electrical center and axis of the superconducting solenoid. Therefore, the above procedure to minimize those torques and forces does not necessarily zero the offsets of the induction coil's center of mass and angle, but rather adjusts them to cancel any residual misalignments of the superconducting solenoid. For the watt measurement, the absolute alignment of these four quantities is less important than ensuring that a current in the induction coil does not induce any horizontal forces or torques and, therefore, that any residual horizontal velocities or tilts do not result in a voltage. However, having the superconducting solenoid nearly aligned is important in that the alignment must be maintained at all points of the velocity measurement, which comprise 70 mm of vertical travel. Thus with the uncertainty of 1.2×10^{-4} rad in the angle of the superconducting solenoid axis, if the electrical centers of the superconducting solenoid and the induction coil were perfectly aligned at one point, they would be misaligned by 0.008 mm at a point 70 mm away. Such an uncertainty is small compared to our ability to align the respective centers, but if the axis of the superconducting solenoid is different from vertical by several milliradians, then such dynamic misalignments will become significant. Furthermore, having the electrical and mass centers of the induction coil aligned is important in our determination of Δd, as discussed below. With our alignment of the superconducting solenoid, the uncertainties in the horizontal forces and torques are well described by a measurement at one point. The resulting relative standard uncertainties for $\Delta F_x/|\vec{F}|$ and $\Delta\tau_x/|\vec{F}|$ are 6×10^{-5} and 0.17 mm, respectively.

The uncertainty in the angular velocity about a horizontal axis $\Delta\omega_x$ is determined by examining the difference in vertical displacements measured by each of the three laser interferometers during the time which the induction coil travels 70 mm. $\Delta\omega_x$ is a property of the suspending band and is not adjustable. The suspending band was designed with a cross-flexure so that horizontal torques cannot be effectively transferred through the band to the induction coil. The uncertainty in the residual angle of horizontal tilt during the 70 mm vertical travel of the induction coil is 3×10^{-7} rad.

The uncertainty in the angular velocity about the vertical axis $\Delta\omega_z$ is determined using a procedure similar to the one used to determine $\Delta\nu_x$, i.e., by measuring the change in position of two points on the induction coil as it moves vertically using light reflected by attached corner cubes. Like $\Delta\omega_x, \Delta\omega_z$ is a property of the suspending band and cannot be easily adjusted. Unlike $\Delta\omega_x$, there is no flexure arrangement to reduce $\Delta\omega_z$. It is reduced using a control system with electrostatic actuators. The control system is designed such that the electrodes do not exert any vertical force; the uncertainty in the vertical force due to the electrosatic actuators is 4×10^{-9} N, which gives a relative standard uncertainty in $\Delta W/W$ of 4×10^{-10}. Hence the control system does not couple significantly to the watt measurement of the balance and can be operated during measurements. The uncertainty in the residual rotation about the vertical axis during the 70 mm vertical travel of the induction coil is 1.7×10^{-5} rad.

The uncertainty in the torque about the vertical axis $\Delta\tau_z$ is determined by monitoring the change in the equilibrium position about the vertical axis of the induction coil upon a current reversal in the induction coil. The contribution to the uncertainty of the watt from this term is small due to the axial symmetry of the superconducting solenoid and induction coil, but is included for completeness. After axial alignment of the solenoid and coil, the resulting relative standard uncertainty for $\Delta\tau_z/|\vec{F}|$ is 1.6×10^{-4} mm.

The distance between the optical and electrical centers Δd is determined by swinging the induction coil as a pendulum and measuring the vertical displacement with the three laser interferometers. The measurement of each individual interferometer is weighted to form an average motion in which the first-order coupling between the swinging motion and the vertical motion is minimized. This procedure minimizes the distance between the optical and mass centers, with an uncertainty of 0.12 mm. The uncertainty in the difference between the mass and electrical centers of the coil is 0.17 mm (see above), giving an uncertainty in Δd of 0.2 mm.

$\Delta\alpha$ is measured by reflecting the laser beam from a mirror, the surface of which is in the horizontal (i.e., x-y) plane and making the beam return on itself. To do this, the incident beam is passed through a beamsplitter, with one path going to the interferometer, reflecting back on itself from the horizontal mirror, and the other path going to a corner cube, which reflects light back on a parallel path. The returning beams then pass back through the beamsplitter and then through a telescope which has been focused at infinity. The telescope arrangement has the property that parallel incoming beams are focused to a point, so that when the interferometer laser beam is vertical, the two return beams overlap. The uncertainty of this procedure is 1.7×10^{-5} rad. However week-to-week drifts in the alignment due to our optics are about twice that much, so the uncertainty in $\Delta\alpha$ is 4×10^{-5} rad.

IV. Uncertainty in the Watt

The key to our improved alignment procedure is our understanding of the importance of including the possible difference between the electrical center of the induction coil and its center of mass. Because of this extra quantity, alignment procedures that only minimize the forces and torques are inadequate. It is essential to have independent electrical means to determine the angle of the magnetic flux density and the displacement between the magnetic flux density and the induction coil.

The components of uncertainty of the watt due to alignment uncertainties are summarized in Table I. The table gives each component and evaluates its uncertainty. The table also notes if the components are two-dimensional and gives its net contribution to $\Delta W/W$, including both dimensions where appropriate. Lastly, the combined uncertainty in the watt measurement due to alignment uncertainties is given.

From Table I the component of uncertainty in $\Delta W/W$ due to alignment is 20 nW/W. This uncertainty is within our immediate goal of a relative combined standard uncertainty of less than 100 nW/W, but greater than our long term-goal of a relative combined standard uncertainty from all sources of less than 10 nW/W. From Table I, steps can be identified to reach the long-term uncertainty goal. The largest components of uncertainty are those dependent on $\Delta\nu_y/|\bar{\nu}|$. Any substantial reduction in the alignment uncertainty requires redesigning the knife edge so that its position can be easily adjusted within the balance wheel. The next largest components of uncertainty are due to terms with $\Delta\nu_x/|\bar{\nu}|$ and $\Delta\omega_z$. The uncertainty of these two quantities is limited by the optics system which is used to measure them, and that has room for substantial improvement. For example, a system similar to the one used to align the interferometer lasers could be implemented. The other quantities need only incremental reduction in value in order to reduce the relative standard uncertainty of the watt due to alignment uncertainties to well below 10 nW/W.

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