A digitally programmable capacitance standard

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We constructed a digitally programmable capacitance standard by modifying a commercial temperature-stabilized 100 pF capacitance standard which consists of 23 binary-weighted capacitor elements on a single fused-silica disk. The variable capacitor can be programmed through a computer from 0.000 115 4 to 113.046 496 4 pF with a resolution smaller than 0.000 115 4 pF over the entire range. Using selected combinations, we also demonstrate a method to calibrate the 23 capacitor elements against a 10 pF capacitance standard. [DOI: 10.1063/1.1687044]

Capacitance standards utilizing fused silica as the dielectric were thoroughly investigated by Cutkosky and Lee[1] in the 1960s at then National Bureau of Standards. A set of twelve 10 pF fused-silica capacitors that they built in 1964 contained several standards which were exceptionally stable. A bank of four of these 10 pF fused-silica standards (referred to as the Farad Bank), which are maintained in an oil bath at 25 °C, have been used ever since as the primary maintenance standard for capacitance calibrations at the National Institute of Standards and Technology (NIST). The Farad Bank has a drift rate of 0.02×10^{-6} per year, and the standards are calibrated twice a year indirectly against the calculable capacitor[2] at NIST at a frequency of $\approx$1592 Hz ($\omega=10^4$ rad/s). This frequency was chosen for a convenient link between the farad and the ohm through a quad bridge.[3] The calibrations are performed using a 10 pF transportable fused-silica capacitance standard, C_{112}. The calculable capacitor provides an absolute determination of capacitance in terms of length only and is the ultimate reference for all impedance measurements in the USA.

Temperature-stabilized fused-silica capacitors are now commercially available and the most commonly used capacitance standards have nominal values at decades (1, 10, and 100 pF, etc.). Such standards can be calibrated against the Farad Bank using a coaxial ac bridge. A typical coaxial ac bridge consists of a 10:1 ratio transformer with a multitap secondary to provide precise N:1 (N is an integer ranging from 1 to 10) voltage ratios; additional secondary windings are used to provide a small adjustable voltage controlled by multidial inductive voltage dividers.[4] To minimize loading errors and maintain bridge accuracy as high as parts per billion, the range of the adjustable voltage is often kept small resulting in a narrow dynamic range for the bridge system, typically $\pm$500 parts per million (ppm). Linking of an "odd value" capacitor (say, 10.35 pF) to the primary capacitance standards at sub-ppm levels presents a challenge.

The difficulty to precisely link a capacitor with an arbitrary value to the Farad Bank hinders progress in several research areas of precision measurements. For example, a new capacitance standard based on single electron tunneling (SET) has been demonstrated with repeatability on the order of 1 part in 10^7 at NIST-Boulder.[5] However, the SET-based capacitance standard requires a cryogenic capacitor that is difficult to construct with, and maintain at, a precise value. In order to confirm the accuracy of the standard, it must be compared to the SI Farad represented by the Farad Bank at NIST-Gaithersburg. The available method now is to use a set of transportable capacitance standards to calibrate a commercial AH 2500A bridge made by Andeen-Hagerling Inc.[6] over a narrow range of capacitance where the value of the cryogenic capacitor lies; the accuracy of the measured value of the cryogenic capacitor is then determined by the stability and the local linearity of the bridge. Although this approach allows us to reduce the measurement uncertainty by a factor of 10 compared to the specified uncertainty of the bridge, this uncertainty is expected to be the dominant source of uncertainties for the comparison. Another example that requires precise measurement of an "odd value" capacitor is a new pressure standard based on measurements of the dielectric constant of helium. Moldover[7] has proposed to use cross capacitors to achieve stability for accurate measurements of the dielectric constant; however, it is difficult to construct a cross capacitor with a precise value.

This article reports a method to construct a digitally programmable capacitance standard by modifying a commercial fused-silica capacitor which consists of 23 approximately binary-weighted capacitors on a single fused-silica disk. The resultant variable capacitor can be programmed through a computer from 0.000 115 4 to 113.046 496 4 pF with a resolution smaller than 0.000 115 4 pF. The article also demonstrates a calibration method that links all 23 capacitor elements to the Farad Bank.

Figure 1 show a schematic diagram of the programmable capacitor. Its basic element is a model AH 11A, three-terminal fused-silica capacitance standard purchased from Andeen-Hagerling Inc. The standard came in a temperature-regulated housing. According to the manufacturer, the standard uses a single fused-silica disk to form a set of 23 capacitors, $C_1, C_2, C_3, \ldots, C_{23}$, whose values decrease from C_1 to C_{23} in a roughly binary-weighted fashion; all capacitor elements share the same low terminal. The original capacitor was preset by the manufacturer at a nominal value of 100 pF

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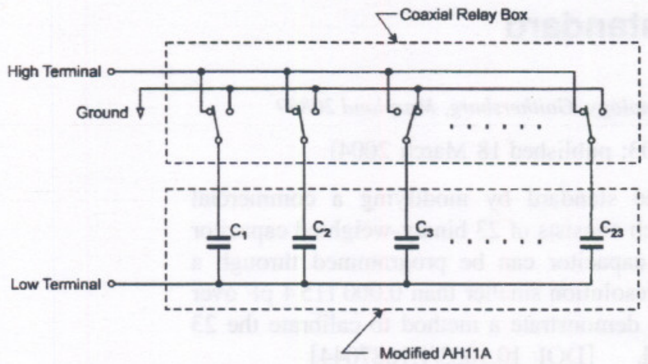

FIG. 1. Schematic diagram of a programmable capacitance standard.

by hard wiring the high terminals of the following capacitor elements from the set in parallel:

$$C_1+C_2+C_3+C_6+C_7+C_{10}+C_{13}+C_{14}+C_{15}+C_{18}+C_{19} +C_{20}+C_{23}=100(1+\gamma_1), \tag{1}$$

where γ_1 is a small number indicating the deviation from the nominal value. The high terminals of all other capacitor elements were shorted to the ground terminal. We modified the original capacitor assembly by first removing all internal permanent connections to the high terminals of the capacitor elements and then connecting them to an external coaxial relay box made by Matrix Systems.[6] Coaxial cables were used for connections between the capacitor housing and the relay box. The relay box setting can be controlled by TTL signals from a computer, allowing an arbitrary selection of capacitor elements to be connected in parallel in real time with the rest shorted to the ground.

Calibration of the capacitors ($C_1, C_2, C_3, \ldots, C_{23}$) turns out to be more challenging than assembling the programmable capacitor. We cannot directly calibrate these "odd value" capacitors individually. However, if we form 23 independent combinations that can be linked to the Farad Bank, we can mathematically find the solution to the 23 unknown capacitances. We start with the combination of capacitors indicated by Eq. (1) (referred to as the combination 1 hereafter). This 100 pF combination is linked to the Farad bank at 1592 Hz via the transportable C_{112} using a 10:1 four-terminal pair bridge. This measurement configuration allows one to quickly determine the values of small capacitors from C_{12} to C_{23} because the dynamic range (±500 ppm) of the bridge is sufficient to allow rebalance of the bridge after adding or removing a small capacitor in this range to the combination 1. The measured results are listed in Table I. The standard uncertainty of these measurements is 0.2 aF, dominated by the short-term (a few minutes) stability of the combination 1.

There are other combinations that form capacitance close to 100 pF, including the following:

$$C_1+C_2+C_3+C_6+C_7+C_{10}+C_{12}+C_{15}=100(1+\gamma_2), \tag{2}$$

$$C_1+C_2+C_3+C_6+C_7+C_{10}+C_{11}+C_{15}=100(1+\gamma_3), \tag{3}$$

$$C_1+C_2+C_3+C_6+C_7+C_{11}+C_{12}+C_{13}+C_{14}+C_{15} =100(1+\gamma_4), \tag{4}$$

TABLE I. Calibration results with the associated standard uncertainties.

Capacitor element	Capacitance (pF)	Standard uncertainty (aF)
C_1	54.054 514 1	1.2
C_2	27.745 001 1	0.7
C_3	14.612 300 1	1.0
C_4	7.772 716 1	0.6
C_5	4.140 175 8	1.8
C_6	2.170 942 8	0.5
C_7	1.155 698 2	0.5
C_8	0.616 622 6	1.4
C_9	0.350 394 0	0.8
C_{10}	0.192 486 2	0.4
C_{11}	0.102 325 0	0.3
C_{12}	0.054 079 8	0.2
C_{13}	0.034 625 0	0.2
C_{14}	0.018 809 8	0.2
C_{15}	0.010 953 8	0.2
C_{16}	0.006 342 6	0.2
C_{17}	0.003 750 4	0.2
C_{18}	0.002 196 6	0.2
C_{19}	0.001 183 4	0.2
C_{20}	0.000 671 6	0.2
C_{21}	0.000 383 6	0.2
C_{22}	0.000 208 4	0.2
C_{23}	0.000 115 4	0.2

where γ_2, γ_3, and γ_4 are all within ±500 ppm. Subtracting Eq. (2) from Eq. (3) yields

$$C_{11}=C_{12}+\Delta_{3,2}, \tag{5}$$

where the notation $\Delta_{i,j}=100(\gamma_i-\gamma_j)$ is introduced to simply equations for the following discussions. The standard uncertainty of $\Delta_{i,j}$ is 0.2 aF. Similarly, subtracting Eq. (4) from Eq. (3) yields

$$C_{10}=C_{12}+C_{13}+C_{14}+\Delta_{3,4}. \tag{6}$$

The results for C_{10} and C_{11} are shown in Table I, together with the standard uncertainties. The uncertainties increase because of the accumulative effects.

Using calibrated auxiliary standards of two 1 pF (C_{912} and C_{913}) and three 10 pF (C_{172}, C_{173}, and C_{174}) fused-silica capacitors, we can measure the following combinations against C_{112}:

$$C_1+C_2+C_3+C_6+C_{10}+C_{11}+C_{12}+C_{13}+C_{14}+C_{15}+C_{912} =100(1+\gamma_5), \tag{7}$$

$$C_1+C_2+C_3+C_7+C_{10}+C_{11}+C_{12}+C_{13}+C_{14}+C_{15}+C_{912} +C_{913}=100(1+\gamma_6), \tag{8}$$

$$C_1+C_2+C_4+C_{10}+C_{11}+C_{12}+C_{13}+C_{14}+C_{15}+C_{172} =100(1+\gamma_7), \tag{9}$$

$$C_1+C_2+C_4+C_9+C_{12}+C_{14}+C_{172}=100(1+\gamma_8), \tag{10}$$

$$C_1+C_2+C_5+C_6+C_7+C_9+C_{10}+C_{11}+C_{12}+C_{14}+C_{15} +C_{172}=100(1+\gamma_9), \tag{11}$$

$$C_1+C_2+C_5+C_6+C_7+C_8+C_{11}+C_{172}=100(1+\gamma_{10}), \tag{12}$$

$$C_1+C_3+C_4+C_6+C_7+C_{11}+C_{12}+C_{13}+C_{14}+C_{15}+C_{172}+C_{173}=100(1+\gamma_{11}), \tag{13}$$

$$C_1+C_3+C_7+C_{11}+C_{13}+C_{14}+C_{15}+C_{172}+C_{173}+C_{174}=100(1+\gamma_{12}), \tag{14}$$

where all γ_i's with i ranging from 5 to 12 are within ± 500 ppm. During the measurement period (a few hours), all the auxiliary capacitance standards are stable within 0.1 aF.

The values of C_i with i ranging from 1 to 9 are determined by comparing measurements of the above combinations in a bootstrapping manner. Subtracting Eq. (7) from Eq. (3) yields

$$C_7=C_{912}+C_{12}+C_{13}+C_{14}+\Delta_{3,5}. \tag{15}$$

Subtracting Eq. (8) from Eq. (3) yields

$$C_6=C_{912}+C_{913}+C_{12}+C_{13}+C_{14}+\Delta_{3,6}. \tag{16}$$

Subtracting Eq. (9) from Eq. (10) yields

$$C_9=C_{10}+C_{11}+C_{13}+C_{15}+\Delta_{8,7}=2C_{12}+2C_{13}+C_{14}+C_{15}+\Delta_{3,2}+\Delta_{3,4}+\Delta_{8,7}. \tag{17}$$

In the derivation of C_9 above, it is noted that the measurements of C_{10} and C_{11} are correlated. These correlated variables are further decomposed such that C_9 is expressed in terms of uncorrelated quantities to simply uncertainty analysis. The same strategy is employed below. Subtracting Eq. (11) from Eq. (12) yields

$$C_8=4C_{12}+3C_{13}+3C_{14}+2C_{15}+\Delta_{3,2}+2\Delta_{3,4}+\Delta_{8,7}+\Delta_{10,9}. \tag{18}$$

Subtracting Eq. (14) from Eq. (13) yields

$$C_4=C_{174}-C_{912}-C_{913}-2C_{12}-C_{13}-C_{14}-\Delta_{3,6}+\Delta_{11,12}. \tag{19}$$

Subtracting Eq. (10) from Eq. (11) yields

$$C_5=C_{174}-3C_{912}-2C_{913}-6C_{12}-4C_{13}-4C_{14}-C_{15}-\Delta_{3,2}-\Delta_{3,4}-\Delta_{3,5}-2\Delta_{3,6}+\Delta_{9,8}+\Delta_{11,12}. \tag{20}$$

Subtracting Eq. (9) from Eq. (8) yields

$$C_3=C_{172}+C_{174}-3C_{912}-2C_{913}-3C_{12}-2C_{13}-2C_{14}-\Delta_{3,5}-\Delta_{3,6}+\Delta_{6,7}+\Delta_{11,12}. \tag{21}$$

With all the capacitors smaller than C_2 now determined, Eq. (14) gives

$$C_1=100(1+\gamma_{12})-2C_{172}-C_{173}-2C_{174}+2C_{912}+2C_{913}+C_{12}-C_{15}+\Delta_{2,6}-\Delta_{6,7}-\Delta_{11,12}, \tag{22}$$

where the first term has a stability within 1 aF during the entire measurement period. Finally, subtracting Eq. (14) from Eq. (4) gives

$$C_2=C_{172}+C_{173}+C_{174}-C_{912}-C_{913}-2C_{12}-C_{13}-C_{14}-\Delta_{3,6}+\Delta_{4,12}. \tag{23}$$

The results are shown in Table I, together with the standard uncertainties. The standard uncertainty differs significantly from one capacitor element to another because the unfolded uncertainty for each capacitor strongly depends on its calibration pathway. The contributions of the auxiliary capacitors to the listed uncertainties are small because each of the standards can be directly calibrated against C_{112}.

The variable capacitor shown in Fig. 1 is programmable from C_{23} (0.000 115 4 pF) to

$$\sum_{i=1}^{23} C_i (113.046\,496\,4\,\text{pF}).$$

The resolution is not uniform in the programmable range; however, it is less than C_{23}.

In summary, we have demonstrated a method to construct a digitally programmable capacitance standard utilizing commercial components. Using selected combinations, we have also shown a method to calibrate 23 binary-weighted capacitors against the 10 pF Farad bank. This programmable capacitor can be used to compare a SET-based capacitance standard with a calculable capacitor. It can be used to explore the possibility of a capacitance-based pressure standard. This kind of programmable capacitor could also be employed to simplify a quad bridge for linking ac quantum Hall resistance[8] to capacitance and to test the linearity of commercial capacitance bridges.

[1] R. D. Cutkosky and L. H. Lee, J. Res. Natl. Bur. Stand. **69C**, 173 (1965).
[2] R. D. Cutkosky, IEEE Trans. Instrum. Meas. **IM-23**, 305 (1974).
[3] R. D. Cutkosky, J. Res. Natl. Bur. Stand. **74C**, 63 (1970).
[4] A. M. Thompson, Metrologia **4**, 1 (1968).
[5] M. W. Keller, A. L. Eichenberger, J. M. Martinis, and N. M. Zimmerman, Science **285**, 1706 (1999).
[6] The identification of a specific commercial product does not imply endorsement by NIST, nor does it imply that the product identified is the best available for a particular purpose.
[7] M. R. Moldover, J. Res. NIST **103**, 167 (1998).
[8] B. M. Wood, A. D. Inglis, M. Côté, and R. B. Young, IEEE Trans. Instrum. Meas. **48**, 305 (1999).