1/*f* Noise Floor of Solid-State Voltage Standards

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Abstract

To characterize the performance of solid-state voltage standards (Zener standards) the 1/*f* nature of Zener noise must be considered. At the Bureau International des Poids et Mesures (BIPM) this is done by using the Allan variance to take into account correlation in serial measurements by specifying the limit of precision quantified by the constant value called the 1/*f* noise floor taken on by the Allan variance. This paper discusses two methods to determine the noise floor of Zener standards of the National Institute of Standards and Technology (NIST) using either a Josephson voltage standard to measure a single Zener or by comparing two Zener standards. Results for two types of Zener standard are presented.

1. Introduction

In metrology, almost all observations are correlated in time, either deterministically or randomly. Deterministic correlations, such as a steady drift in time, are routinely modelled so that, for example, the value of an artifact standard such as the resistance of a wirewound standard resistor can be predicted with acceptable accuracy. Another kind of correlation, usually less familiar, is random correlation in time or random serial correlation. An everyday example of random serial correlation is the fluctuation of share prices on stock markets. A share price on any trading day is clearly correlated with the price of the previous trading day and, of course, in spite of this correlation, the price at the end of today's trading day cannot be precisely predicted.

Random fluctuations in a quantity are usually called noise. These fluctuations may be serially independent (uncorrelated) or they may be correlated. In metrology we usually assume, and there are ways of testing this assumption, that observations are identically distributed, meaning that they have the same population mean and variance. Random independent and identically distributed observations result in white noise. Metrologists often wrongly assume that all noise is white noise and this can lead to wasted time and, sometimes, serious underestimations of measurement uncertainties. Consider the example of the calculation of the standard deviation of the mean $\bar{x}$ of N measurements of a quantity x, $\bar{x} = (\sum_{i=1}^{N} x_i)/N$,

$$\mathrm{var}(\bar{x}) = \left[\sum\nolimits_i \mathrm{var}(x_i) + 2\sum\nolimits_{j=1}^{N-1}\sum\nolimits_{k>j}^{N} \mathrm{cov}(x_j, x_k)\right] \Big/ N^2 , \tag{1}$$

where, in general, covariance terms $\mathrm{cov}(x_j, x_k)$ are non-zero.

If the observations are taken from a white noise process,

$$\mathrm{var}(x_1) = \mathrm{var}(x_2) = ... = \mathrm{var}(x_N) = \mathrm{var}(x) , \quad \text{[white noise]} \tag{2}$$

because the observations are identically distributed, and

$$\mathrm{cov}(x_j, x_k) = 0 \text{, for } j \neq k \quad \text{[white noise]} \tag{3}$$

because the observations are independent. Then

$$\mathrm{var}(\bar{x}) = \sum\nolimits_i \mathrm{var}(x_i) \Big/ N^2 = \mathrm{var}(x)/N \quad \text{[white noise]} \tag{4}$$

or, in terms of standard deviations, σ,

$$\sigma(\bar{x}) = \sigma(x)/\sqrt{N} \quad \text{[white noise].} \tag{5}$$

2. Analytical methods: the power spectral density and the Allan variance

Modern measurement methods make use of automated data acquisition and the value of N typically exceeds 100 or more. (For example, for the measurements reported here $N = 8192$ so that applying (1) directly is impractical but, fortunately, unnecessary.) The basis of the data analysis outlined here is to assume that the stream of observations makes up a time series of values obtained at constant intervals of time between successive measurements. One method of analysis is then to estimate the Fourier transform of the time series using the fast Fourier transform techniques incorporated in most data analysis software. This allows the power spectral density (PSD) of the original time series to be estimated; this is our basic analysis tool. The PSD describes the distribution of energy of the original signal as a function of frequency. White noise is characterized by a PSD that is constant over the entire frequency range and the term "white noise" follows from this property, by analogy with the decomposition of white light into its component frequencies. If the observations result in a constant value of PSD with frequency, f, we know that the noise is white, that the covariance terms in (1) are negligible and that (5) can be used to calculate correctly the standard deviation of the mean of the measurements. On the other hand, if the PSD at low frequencies $\propto f^{-1}$, the noise is termed "$1/f$ noise".

This paper deals with precise measurements of the voltages of Zener-diode-based electronic voltage standards using a digital nanovoltmeter to compare a Zener voltage with that of a Josephson voltage standard or a second Zener. Observations of $1/f$ noise in Zeners have recently been reported [1, 2]. One key result of the presence of $1/f$ noise is that the statistical or type-A uncertainty is underestimated if (5) is used to calculate the standard deviation of the mean of N measurements. How then can the type-A uncertainty of such serially correlated data be specified?

For over two generations now, time and frequency metrologists have been characterizing correlated noise (or "stability") of frequency standards in terms of a statistic called the Allan variance [3, 4]. Assuming an infinite number of voltage measurement results, y_i, taken at regular time intervals τ_0, the basic approach is to average the y_i over successive groups of n, corresponding to the sampling time $\tau = n\tau_0$. Beginning with $n = 1$, the value of n is increased in some regular series such as in the present paper, where $n = 2^k$, and k is a non-negative integer. The Allan variance is defined as

$$\sigma_y^2(\tau) = \left\langle (\bar{y}_{j+1}(\tau) - \bar{y}_j(\tau))^2 \right\rangle / 2, \tag{6}$$

where, for voltage measurements, $\bar{y}_j(\tau)$ is the average voltage of the j-th group of n successive voltage readings and the angular brackets indicate an infinite time average. The Allan variance generally *varies* with sampling time and is not constant. An important consequence of the definition (6) is that for white noise the Allan variance is just the variance of the mean as expressed in (4). To emphasize the τ–dependence in (4), it is sufficient to replace n by τ / τ_0 to show that $\mathrm{var}(\bar{x}) \propto \tau^{-1}$. Practically, the Allan variance of M measurements is estimated from

$$s_y^2(\tau) = (2P)^{-1} \sum\nolimits_{j=1}^{P} [\bar{y}_{j+1}(\tau) - \bar{y}_j(\tau)]^2, \tag{7}$$

where $P = \mathrm{Int}(M/n) - 1$ is the number of available first differences, $\bar{y}_{j+1}(\tau) - \bar{y}_j(\tau)$, corresponding to $\tau = n\tau_0$.

In this paper we take the point of view of time and frequency metrologists: the Allan variance is used to specify the type-A uncertainty of the measurement results.

The Allan variance can be calculated from the PSD of the noise process by integrating over frequency the product of the PSD, the power transfer function (the product of the complex conjugate of the voltage transfer function of the measuring instrument times the voltage transfer function itself), and a window function. In time and frequency metrology it has been found that the PSD, $S_y(f)$, of the relevant noise processes can be expressed as a simple power series in frequency with the powers ranging over the integers between –2 and 2. In low-frequency electrical measurements, only the powers –1 and 0 have been observed [1, 5] and $S_y(f) = h_{-1}f^{-1} + h_0 f^0$, where h_{-1} and h_0 are sensitivity coefficients for $1/f$ noise and white noise, respectively. The measurements described in this paper were made using a digital nanovoltmeter with the analog and digital filters switched off so that, to a close approximation, the transfer function of the instrument is unity up to a cut-off frequency and zero for higher frequencies. For such a sharp cut-off filter, the Allan variances for $1/f$ noise and white noise are $2\,h_{-1}\ln 2$ and $h_0/2\tau$, respectively. Finally, when presenting results, it is usual practice to give the Allan deviation, the square root of the Allan variance. Table 1 summarizes the forms of the PSD and the Allan deviation that will be needed in the present paper.

Table 1. Characteristics of types of noise observed in voltage measurements made using a voltmeter having a sharp cut-off filter.

Noise type	PSD, $S_y(f)$	Allan deviation, $\sigma_y(\tau)$
White noise	h_0	$(h_0 / 2\tau)^{1/2}$
$1/f$ noise	$h_{-1} f^{-1}$	$(2h_{-1} \ln 2)^{1/2}$

3. Experimental methods

The noise of the 10 V output of a Zener voltage standard was studied by two methods: (1) by connecting the Zener in series-opposition to the output of the NIST 10 V standard based on an array of Josephson junctions and measuring the voltage difference with a digital nanovoltmeter or (2) by connecting it in series-opposition to a second Zener and measuring the voltage difference with the same nanovoltmeter. In method (1), it is well known that with arrays of unbiased Josephson junctions as were used here the stability of the selected voltage step is an issue. The measurement software was designed to take this into account. If the array step number changes to another stable value, the new step number is calculated and the measurement continues uninterrupted. There is, of course, a limit to the numbers of these incidences beyond which measurement data must be abandoned. It was not uncommon to find that the step stability was at times good enough to produce a steady time series of observations for periods of sixteen hours. Measurement results were acquired by a computer at a maximum rate of about 0.06 s per point. Measurements were grouped into “frames” of 8192 pairs of voltages and times. Frames were repeated overnight or over even longer periods and the Allan *variances* for a set of frames were averaged and presented in the form of Allan deviations by taking the root-sum-square of the Allan variances. The measurement data and, of course, the resulting Allan deviations, include the noise of the entire measurement process, including the noise characteristics of the digital voltmeter (DVM) itself, possible drift, electromagnetic interference and temperature-related effects. It is therefore important first to establish that the noise characteristics of the DVM are sufficiently small to avoid masking the noise of the Zeners, the main quantity of interest. Since the source resistance of the circuit containing the 10 V Zener, the array, and the filters is some tens of ohms, it is sufficient to study the PSD and Allan deviation of the results of DVM measurements made with the input short-circuited. The measurements were carried out in an air-conditioned screened room in which the peak-to-peak overnight temperature variations were of the order of 0.75 K.

4. Results

4.1 Zeners measured with a Josephson standard

Figure 1 shows the sample root-mean square (rms) Allan deviation (circles), in µV, as a function of sampling time, in seconds, for the measurement of the short-circuit voltage measured with the DVM connected to the array of Josephson junctions biased on the zero-voltage step. The rms Allan deviation was calculated by averaging 61 Allan variances determined for 12 separate sampling times $2^k\tau_0$, $k = 0, 1, 2, \ldots 11$ and $\tau_0 = 0.1025$ s. The error bars represent the confidence intervals containing 68 % of the Allan deviations for each sampling time from the 61 measurement frames or repetitions. The dashed line represents the Allan deviation one would expect if the noise were completely white, in which case $\sigma_y(\tau) = (h_0 / 2\tau)^{1/2}$. The agreement with the white noise model is rather good although some systematic increase in the noise is seen at the longer sampling times. Notice that for this digital nanovoltmeter the Allan deviation (standard deviation of the mean) attains a value of 10^{-3} µV, or 1 nV, for a sampling time of about 25 s.

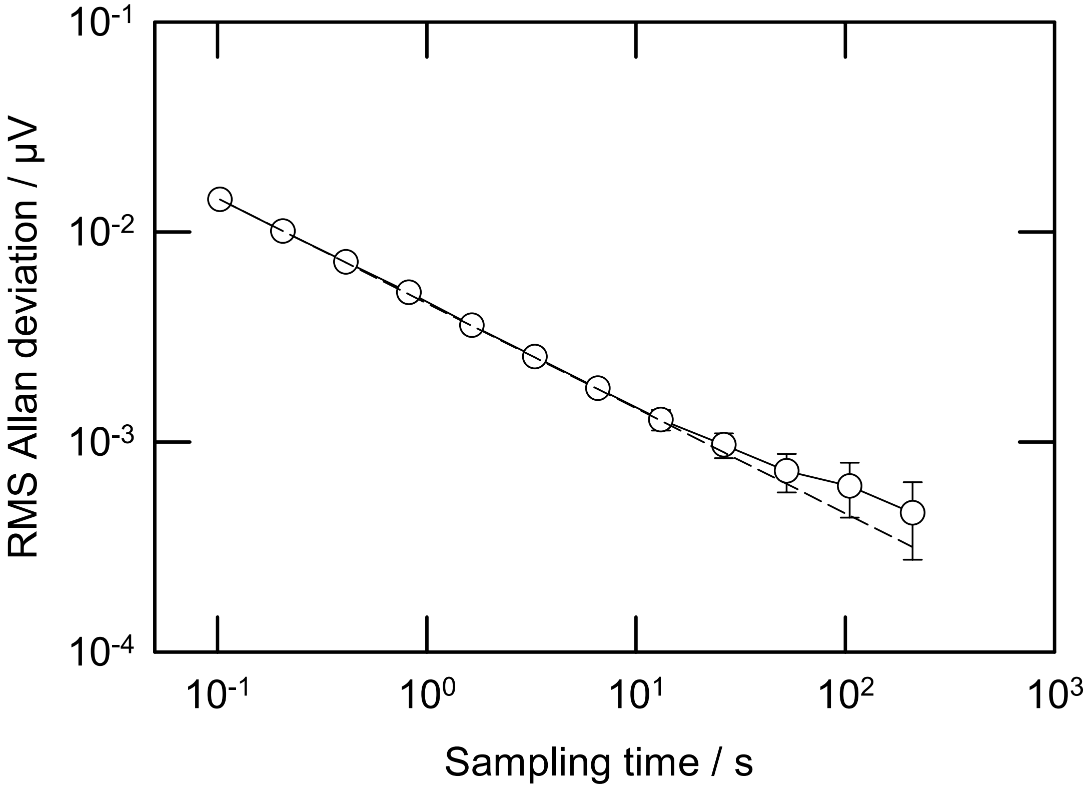

Figure 1. Allan deviation as a function of sampling time for the measurement of the zero-voltage step of the Josephson junction array with a DVM; $\tau_0 = 0.1025$ s.

Figure 2 shows the sample rms Allan deviation (circles) as a function sampling time calculated from measurements of the voltage difference between a 10 V Zener and the NIST Josephson standard made with the same DVM. In this case $\tau_0 = 0.0619$ s. As the

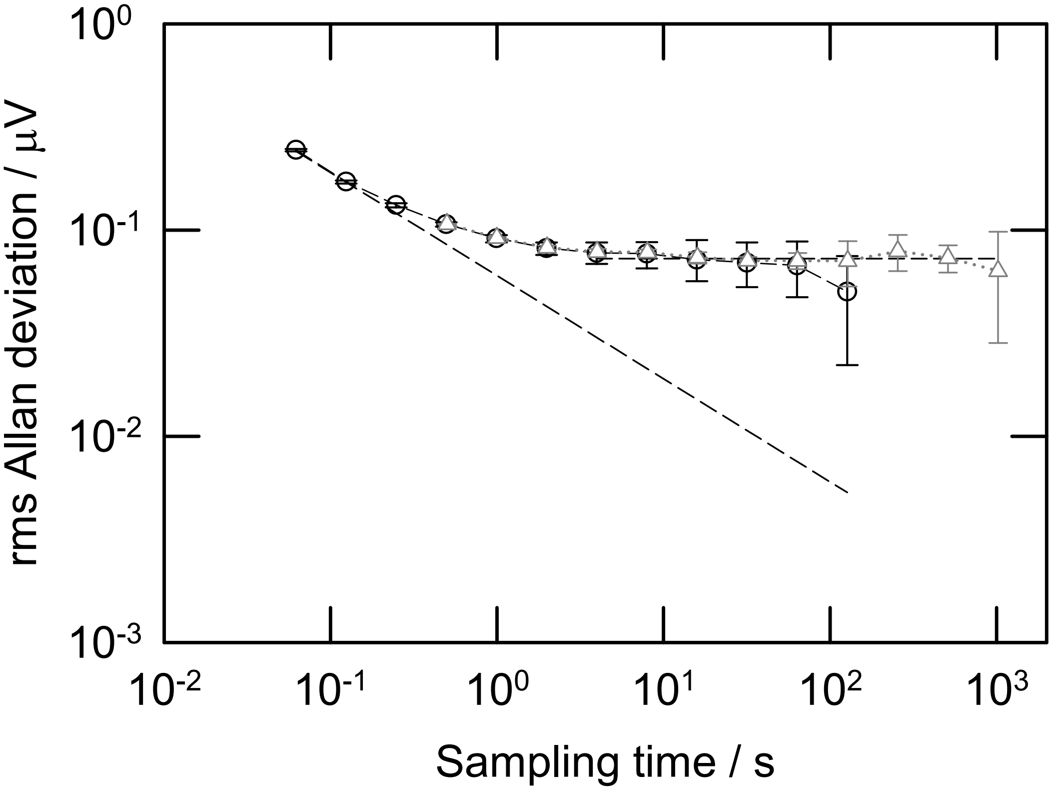

Figure 2. Circles: Allan deviation as a function of sampling time for 61 frames of measurements of a Zener with the Josephson standard with $\tau_0 = 0.0619$ s; triangles: same, with data regrouped into 7 frames of 8192 points obtained by averaging original data by successive groups of eight points; $\tau_0 = 0.4991$ s.

sampling time decreases the Allan deviation asymptotically approaches the dashed line representing the Allan deviation expected in the case of purely white noise. For sampling times exceeding one second, the Allan deviation tends to flatten out, taking on a nearly constant value; this is characteristic of 1/*f* noise. The dashed horizontal line represents the mean Allan deviation for the longer sampling times; this is termed the 1/*f* noise floor. In this case, it is estimated to be equal to about 0.073 μV = 73 nV. By comparison with Figure 1, it can be seen that the 1/*f* floor is not caused by a limitation of the DVM; it is a characteristic of the Zener itself. Increasing the sampling time will not result in a significant decrease of the Allan deviation because of the correlated nature of 1/*f* noise. Since measurements are made at equal time intervals, increasing the number of measurements, which is the same as increasing the total measurement time, will not result in a lower Allan variance. This is illustrated in Figure 2 by the Allan deviations represented by grey triangles. The same data used to calculate the Allan deviations represented by circles were averaged by successive groups of eight points and used to construct seven frames of 8192 points each with a value of $\tau_0 = 8 \times 0.0619 \text{ s} = 0.4991$ s. As seen in the plot, averaging in this way extends the maximum sampling time to about 1000 s but the Allan deviation remains constant.

The analysis presented in the description of the first plot of Figure 2 was carried out on some of the electronic voltage standards used by the NIST as secondary standards for voltage calibrations and standards maintenance purposes. These Zener standards are of two types. The first, designated as Type-1 for the present purposes, is a model widely used in metrology laboratories throughout the world both as a secondary standard and a travelling standard by laboratories using a Josephson primary standard and as the primary voltage standard in many laboratories not possessing a Josephson standard. The second model, designated Type-2, is a more compact electronic voltage standard that has become available in the last few years. The results of determinations of the 1/*f* noise floor of six Type-1 Zeners, obtained from repeated frames of DVM measurements of each Zener with respect to the NIST Josephson standard, are shown in grey in the histogram in Figure 3. Figure 3 also includes the results reported by the BIPM [1] for other Type-1 Zeners. It can be seen that the results are in close agreement. It is worth pointing out that the BIPM results were calculated from measurements using a manually operated Josephson standard and with an analog nanovoltmeter as the detector.

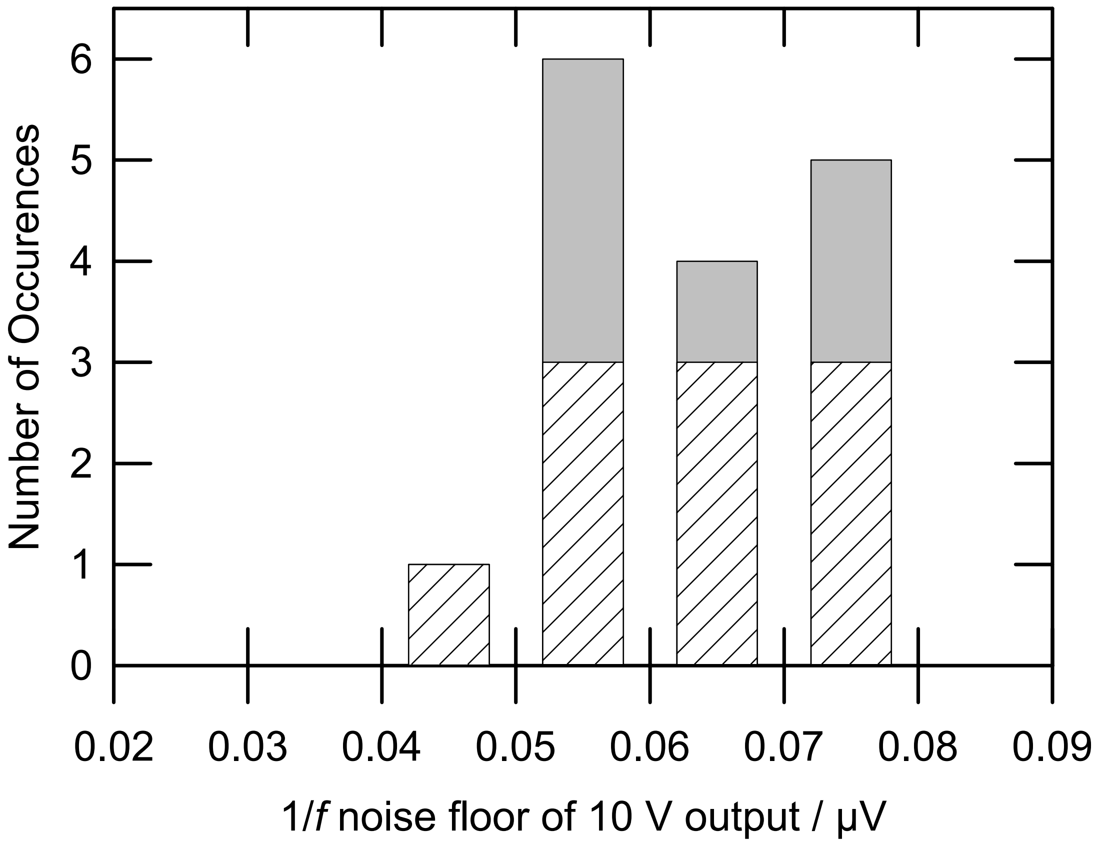

Figure 3. Histogram of values of 1/*f* noise floor for sixteen Type-1 Zeners; grey shading: six NIST instruments; hached shading; ten BIPM instruments.

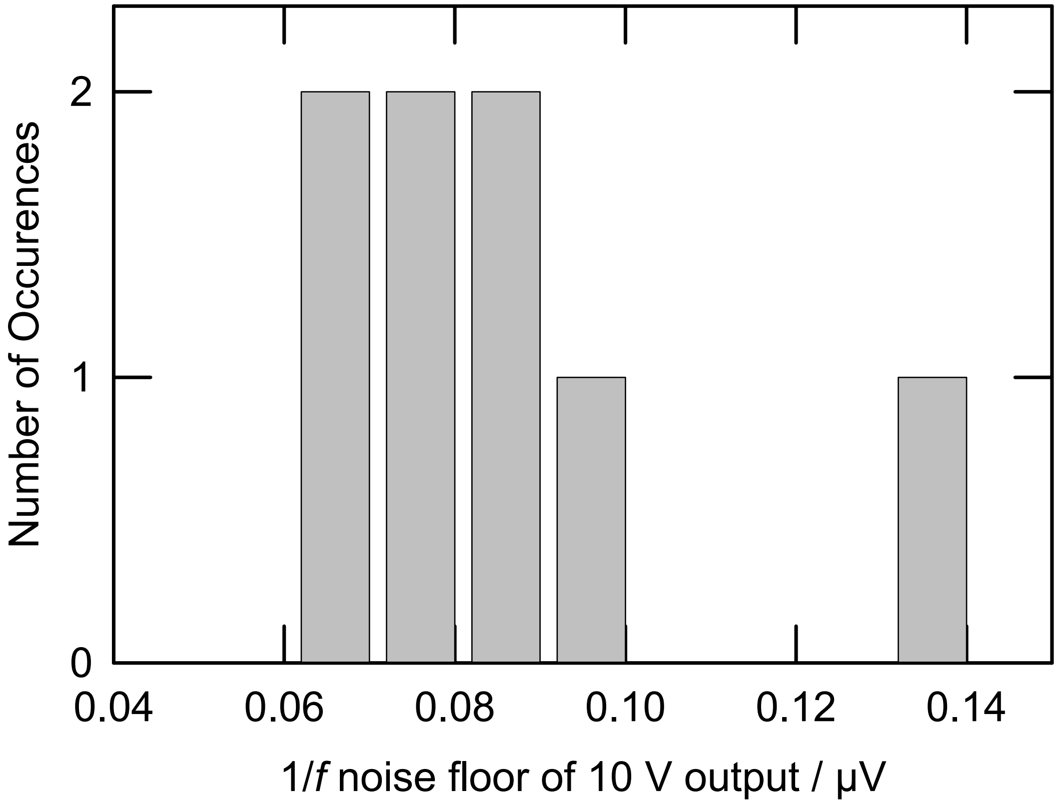

Figure 4. Histogram of 1/*f* noise floor for eight Type-2 NIST Zeners.

Figure 4 is a histogram of the results of determinations of the 1/*f* noise floor of eight Type-2 Zeners. With one exception, the results are almost comparable to those obtained for the Type-1 Zeners.

4.2 Comparisons of two Zeners

Although Allan variance analyses of noise in electronic voltage standards is best done using a Josephson standard, useful information can be obtained by analyzing measurements of the voltage difference between two Zeners. A simplified approach to the analysis can be applied if the two Zeners have approximately the same noise level. On this assumption, the Allan variance at the 1/*f* noise floor for each unit is just half of the Allan variance of the measured voltage difference, or the Allan deviation of each is $\sqrt{2}/2$ times the Allan deviation of the measured voltage difference. Figure 5 illustrates this method. In Figure 5 (a) we plot the Allan deviation calculated from DVM measurements of the voltage difference between two Type-1 Zeners with $\tau_0 = 0.5$ s. After a few seconds of sampling time, the Allan deviation tends to a constant value combined with a small oscillation. (The oscillation may be due to residual sensitivity of at least one of the Zeners to small variations of the room temperature. This is probably the Zener for which the Allan deviation for the difference between its voltage and that of the Josephson standard is shown in Figure 5 (b).) The mean Allan deviation over the

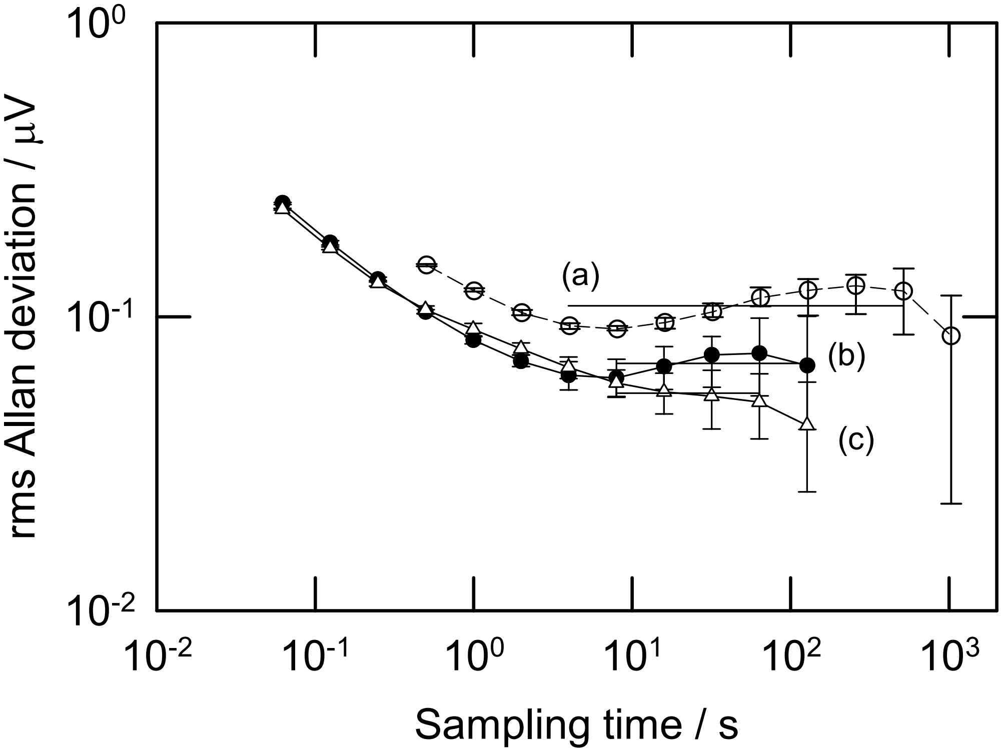

Figure 5. (a): Allan deviation as a function of sampling time for the voltage difference between two Zeners; (b) and (c): Allan deviations for voltage difference between the Josephson standard and each Zener from (a). Horizontal lines indicate 1/*f* noise floors from averaging over the corresponding sampling-time spans.

sampling-time span from about 4 s to 500 s, indicated by the accompanying horizontal line, is 0.109 µV. If we assume that the noise floors of the two Zeners are equal and that the Allan variances of the two Zeners are independent, then each has a noise floor of $\sqrt{2}/2$ of this value or 0.077 µV. In Figure 5, (b) and (c), are plotted the Allan deviations derived from measurements of each Zener with respect to the NIST Josephson standard made using the same DVM with $\tau_0 = 0.062$ s. The Allan deviations of the noise floors are 0.069 µV and 0.055 µV, respectively. The result obtained from the comparison of two Zeners agrees only approximately with those obtained from the individual measurements with the Josephson standard. Nevertheless, the Allan variance analysis of the Zener-Zener measurement gives a much more realistic assessment of the statistical uncertainty than the one obtained by naively using (5).

There is a more elaborate method for estimating Allan variances of individual Zeners without requiring a clearly less noisy standard such as a Josephson junction array. It is known as the three-cornered hat method and requires three Zeners that are compared pairwise in the three possible combinations [1].

5. Discussion

In this section we discuss the interpretation of the Allan deviation plots and how they can provide a powerful way of evaluating instruments and designing experiments; and by the term experiments we include routine measurements of the type commonly encountered in calibration laboratories. In a calibration context, say a series of 10 to 1000 voltage measurements are used to obtain a single calibration point. Such measurements may be carried out a great number of times every day. This justifies thinking about how to achieve an acceptable statistical uncertainty in the least time and the Allan variance is one way of determining this. For example, suppose it is desired to achieve a statistical uncertainty of 1 nV using the DVM that led to the Allan deviation plot in Figure 1. It can be seen from that figure that an Allan deviation of 1 nV (10^{-3} μV) requires a sampling time of about 25 s. As another example, suppose a Zener is to be calibrated using a Josephson standard and it is desired to know over how much time the voltage measurements should be averaged. For a measurement system leading to the Allan deviation plot of Figure 2, the answer is "about 4 s"; this is surprisingly short. Nothing more is gained by making additional measurements. Since each measurement system is unique, each should be analyzed individually and the latter result does not, in general, apply to a "similar" measurement system because of effects such as temperature variations in the laboratory, electromagnetic interference, settling times for thermal emfs, filter functions used with the DVM, and so on.

Another interesting application of the Allan variance is to use it to compare the performance of different voltmeters by connecting them, in turn, to the same stable voltage source, acquiring series of data, calculating the Allan deviations and comparing them on a single graph.

The Allan variance is only one of an increasing number of methods for treating time series of measurements. Others include spectral analysis, autocorrelation functions, auto-regressive moving-average methods and so on [6]. They are particularly effective when applied to large data sets obtained at equal time intervals. These conditions are readily fulfilled by using modern computer-aided data acquisition. For these reasons we believe that these techniques can and should be applied very broadly in electrical metrology and in other areas of metrology.

6. Conclusions

We have considered the problem of how to characterize the scatter of times series of data that may be stochastically correlated by using the Allan variance. We addressed the practical problem of specifying $1/f$ noise in electronic voltage standards using the $1/f$ noise floor. We characterized the noise of the electronic voltage standards that provide the indispensable link between the US Josephson voltage standard maintained at the NIST and the practical dissemination of this standard and confirmed observations in other laboratories. For the 10 V outputs of the two types of Zener standards, the $1/f$ noise floor lies between 40 nV and 100 nV (or between four and ten parts in 10^9 of the output voltage) for 23 of the 24 Type-1 and Type-2 instruments studied at the NIST and the BIPM. We emphasize that this method characterizes the entire specific measurement *system*, by which we mean the Zener(s), the voltage measuring instrument and the environment, by specifying the sampling time required to reach the $1/f$ noise floor.

7. Acknowledgement

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8. Reference

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