

## A Bridge for Scaling to Higher Resistance from the QHR

Speaker: Randolph E. Elmquist  
National Institute of Standards and Technology  
100 Bureau Drive, MS 8112  
Gaithersburg, MD 20899-8112  
Phone (301)-975-6591 Fax (301)-975-2115  
Email: [relmquist@nist.gov](mailto:relmquist@nist.gov)  
Authors: Randolph E. Elmquist and Dean G. Jarrett  
National Institute of Standards and Technology  
100 Bureau Drive, MS 8112, Gaithersburg, MD 20899-8112

### Abstract

A two-terminal bridge for resistance scaling directly from the quantum Hall resistance (QHR) to higher-resistance values now provides a secondary starting point in decade scaling at NIST, beginning at the 1 M $\Omega$  resistance level. This cryogenic bridge has better repeatability and lower uncertainty than Hamon transfer standard scaling. Better scaling will support the high-resistance measurement service at 10 M $\Omega$  and above for which NIST has constructed improved standards with low voltage coefficients. 1 M $\Omega$  standard resistors calibrated using this bridge have helped NIST provide better uncertainty for the ongoing NCSLI inter-laboratory comparison using air-type and oil-type 1 M $\Omega$  transfer standards.

### 1. Introduction

The U.S. representation of the ohm ( $\Omega_{\text{NIST}}$ ) is based on the quantized Hall resistance (QHR) standard. Before the introduction of the QHR standard on Jan 1, 1990,  $\Omega_{\text{NIST}}$  was maintained using five Thomas-type reference resistors at the 1  $\Omega$  level and most transfers of resistance between different levels relied on Hamon device transfer standards. Most resistance values in the intermediate resistance range now are transferred using cryogenic current comparator (CCC) bridges. This scaling method is used to quickly and easily compare the QHR standard against standard resistors in the range between 1  $\Omega$  and 1 M $\Omega$ .

In the early 1990's, five well-characterized reference standards at the 100  $\Omega$  level were selected to form the NIST 100  $\Omega$  reference group. These five 100  $\Omega$  standards are used for scaling between the 1  $\Omega$  and 10 k $\Omega$  decade levels. The five resistors in the 100  $\Omega$  reference group also are periodically compared directly with the QHR standard, and thus are the basis for transferring  $\Omega_{\text{NIST}}$  to the 1  $\Omega$  and 10 k $\Omega$  working standards at NIST. Recently, a second starting point in decade scaling has been introduced. We can now compare 1 M $\Omega$  standards directly with the QHR at the 12 906.4035  $\Omega$  plateau using a CCC two-terminal bridge method. A diagram of the entire scaling process in use at NIST is shown in Fig. 1. On the right-hand side of the figure, NIST calibration test numbers are shown in parentheses.

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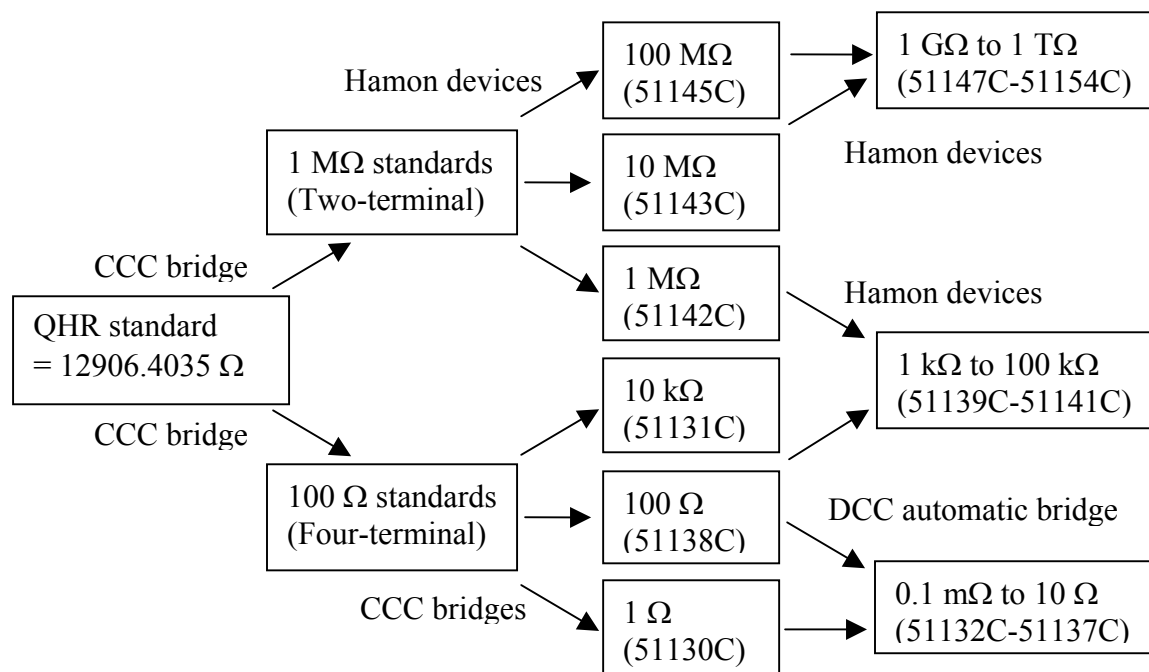


Figure 1. Resistance scaling from the QHR to working standards

The QHR device is characterized based on international guidelines when it is used in resistance comparisons [1]. An NCSL RISP [2] describes processes in use at NIST for CCC measurements and the criteria for selecting QHR devices for use as a QHR standard. In particular, CCC scaling to the 100  $\Omega$  reference group involves comparing each resistor several times over a two-week or longer period to the QHR at the 12 906.4035  $\Omega$  plateau. Primary scaling to 10 k $\Omega$  and 1  $\Omega$  working standards using CCC bridge methods and to the 1 M $\Omega$  level using the two-terminal CCC method also occurs within this time period.

## 2. CCC and Hamon Device Ratios

The NIST CCC devices are of the overlapped-tube type with a commercial superconducting quantum interference device (SQUID) sensor used to detect the ampere-turn condition of the comparator. Due to the Meissner effect of total flux exclusion, the superconducting overlapped-tube acts as a shield for the magnetic flux produced by small currents in the windings. This eliminates any dependence on the position of the windings, while the SQUID and absence of a magnetic core greatly reduce noise compared to conventional direct current comparator systems.

In the four-terminal bridge, a commercial nanovolt detector D (see Fig. 2) senses the voltage difference across the resistors, and provides a feedback current through  $R_F$  and  $N_F$ . This feedback current is monitored by the voltage drop across  $R_F$  using an optical isolator and digital voltmeter. This output signal is a measure of the difference of the resistor corrections. In order to eliminate leakage currents, the primary and secondary current sources are floating and isolated from one another [3]. The 100/1 and 129.06/1 winding-ratio CCC bridges in use at NIST have a combined relative uncertainty of less than  $1 \times 10^{-8}$  for typical measurement conditions [4].

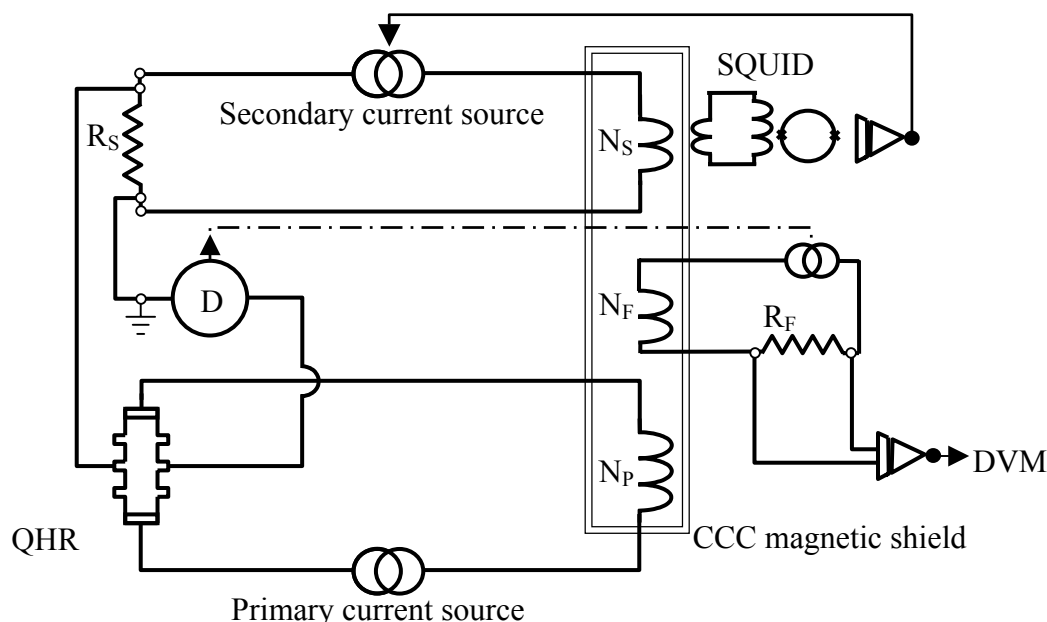


Figure 2. NIST 4130:32 winding-ratio CCC bridge for QHR/100  $\Omega$  measurements

The resistors that comprise the 100  $\Omega$  reference group have small first-order temperature coefficients of resistance (TCR) [5]. Internal power dissipation between zero current and the 10 mA (10 mW) calibration level has a negligible effect for the group average resistance. These resistors can be calibrated against the QHR using a CCC at a variety of different current levels, from about 2 mA up to about 8 mA.

Current comparator bridges apply equal voltage to the resistance standards, such that the standard of lower resistance dissipates higher power internally. Scaling between the 100  $\Omega$  reference group and the 1  $\Omega$  working group uses a 100 mA current level for the 1  $\Omega$  standards, as does the DCC potentiometer in 1  $\Omega$  Thomas-type customer-calibrations. In scaling to the 10 k $\Omega$  level, 10 k $\Omega$  standards are used at different loading levels than in calibrations. Uncertainty due to the effect of loading can be determined by comparison with Hamon device scaling at equal loading, and by using intermediate 10 k $\Omega$  standards with positive and negative TCRs.

The two-terminal CCC bridge used for the 1 M $\Omega$  resistance level makes use of a unique property of the quantum Hall effect. Physics related to the direction of the magnetic flux through the device determines exactly how much current flows through each device lead and contact when more than two contacts provide distinct paths for the measurement current entering or leaving a QHR device [6,7]. Surprisingly, this property of multiple connections can result in an ideal two-terminal resistor of value 12906.4035  $\Omega$  that does not depend significantly on the resistance of the leads, when two sets of three or more leads are used to connect the QHR device to the measurement circuit (see Fig. 3).

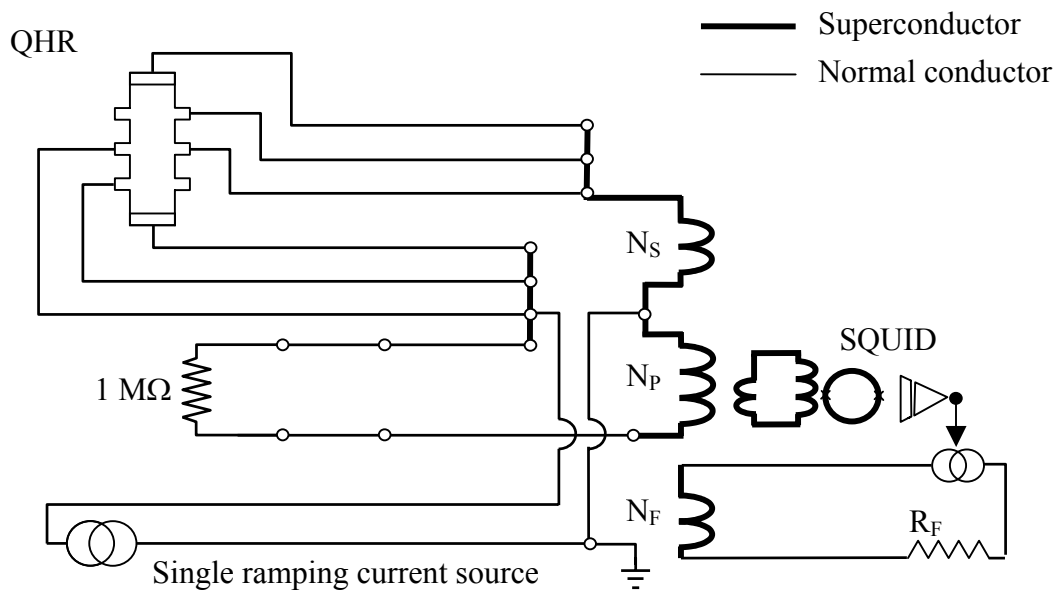


Figure 3. Basic schematic diagram of two-terminal CCC bridge

With the correct magnitude and direction of magnetic flux through the QHR device, the current in the set of leads nearest the middle of the QHR device shown in Fig. 3 is less than  $10^{-6}$  of the total device current. Leads connected to the second set of QHR terminals, placed diagonally across the device, carry about 0.1 % of the total current. The remaining 99.9 % of the device current flows through leads connected at the top and bottom. The windings and connections within the two-terminal CCC bridge are superconducting, and do not affect the bridge ratio. However, lead resistance in the bridge connections to the  $1 \text{ M}\Omega$  standard must be measured separately and subtracted from the measured resistance on the primary side of the bridge.

### 2.1. $1 \text{ M}\Omega$ CCC Bridge

A bridge as pictured in Fig. 3 with  $N_P = 1937$  and  $N_S = 25$  was constructed and has been used to compare room temperature  $1 \text{ M}\Omega$  resistance standards directly against the QHR  $i = 2$  plateau, at measurement voltage of 0.74 V. The relative Type A uncertainty for measurements of 30 min duration is below  $1 \times 10^{-8}$  and is at least a factor of four better than typical room-temperature resistance comparisons at the  $1 \text{ M}\Omega$  level. The bridge supplies a guard voltage for coaxial leads and connectors, and eliminates leakage errors that sometimes exist in Hamon transfer devices.

Low-TCR phosphor-bronze wiring is used in the cryostat leads of the  $1 \text{ M}\Omega$  CCC bridge, because the lead resistance in the primary arm is significant compared to  $1 \text{ M}\Omega$ . If the measurement uncertainty is to be below  $1 \times 10^{-8}$  it is necessary to know these lead resistances to within  $0.01 \text{ }\Omega$ . This is accomplished on one side using the multiple leads connecting to the QHR device in Fig. 3. Similarly, the other  $1 \text{ M}\Omega$  lead resistance inside the CCC cryostat can be measured through the lead connections of the superconducting CCC windings.

The leads inside the CCC cryostat were measured at different liquid helium levels to better than  $0.005 \Omega$ . Because the phosphor-bronze wire has a low TCR, changes in the lead resistance were less than  $0.01 \Omega$  over 40 % range of liquid helium level in the CCC cryostat, and were repeatable after transfer of liquid helium. The CCC results agree very well with scaling from  $10 \text{ k}\Omega$  using the best available guarded oil-type  $1 \text{ M}\Omega$  series-resistance Hamon device, as the relative scaling difference is better than  $2 \times 10^{-8}$ . This is the approximate limit of accuracy for the Hamon device method, due to the effects of internal power dissipation (loading) and leakage resistance.

## 2.2 Transfer Standards

NIST Hamon devices can provide accurate ratios of 10/1 and 100/1. In a series of measurements using such transfer standards, the  $\Omega_{\text{NIST}}$  can be extended in multiple decade values up to at least  $10^{10} \Omega$ . This scaling technique can be extended to even higher levels using newer hermetically sealed Hamon devices [8] with low TCR and low voltage coefficient of resistance (VCR).

## 3. Resistance Standards at $1 \text{ M}\Omega$ and Above

All NIST calibrations for resistance levels above  $1 \text{ M}\Omega$  are made in air, with the resistors maintained in environmental chambers that control temperature and humidity. Mineral oil readily absorbs humidity from the air, possibly causing changes in leakage resistance. Oil baths produce an extremely stable temperature environment, but leakage resistance can be significant and most resistors in the higher ranges are designed to be measured in air. NIST is developing a new Warshawsky bridge system well suited to measuring air-type  $1 \text{ M}\Omega$  level resistors [9].

At present, NIST uses an unbalanced-bridge system to calibrate most customer standards between  $1 \text{ k}\Omega$  and  $1 \text{ M}\Omega$ . Primarily of the oil-type Rosa design, the  $1 \text{ M}\Omega$  level resistors are measured in mineral oil at  $(25.00 \pm 0.01) \text{ }^\circ\text{C}$ . However, we have found that certain commercial air-type  $1 \text{ M}\Omega$  resistors have generally better predictability and transport properties than well-aged Rosa type resistors. These standards are supplied with data on the TCR, and typically the first-order TCR is  $\leq 0.2 \text{ ppm}/^\circ\text{C}$ . They can be measured with reasonable accuracy in the laboratory air environment in most cases. NIST also has available several temperature/humidity controlled air boxes for use with the air-type  $1 \text{ M}\Omega$  and higher-value air-type resistors.

In March 2002, NIST calibrated four  $1 \text{ M}\Omega$  standard resistors (two oil type and two air type) for an ongoing NCSLI inter-laboratory comparison (ILC). Each of the four resistors was measured 12 times over approximately 30 days using a NIST unbalanced bridge system [10]. This NIST system uses oil type  $1 \text{ M}\Omega$  reference standards selected for good stability and calibrated both with 100:1 Hamon transfer (from  $10 \text{ k}\Omega$ ) and with the new  $1 \text{ M}\Omega$  CCC bridge. While comparable statistical uncertainty was seen within the individual runs for all the  $1 \text{ M}\Omega$  resistors used in the ILC, the changes between runs over the 30 days were larger by a factor of five or more for the two NCSLI Rosa type resistors measured in oil at  $25.00 \text{ }^\circ\text{C}$ . The standard deviation of the 12 results was  $1.5 \times 10^{-7}$  to  $2.0 \times 10^{-7}$  for the oil type standards and  $2.2 \times 10^{-8}$  to  $3.5 \times 10^{-8}$  for the air type resistors. The two air type standards were measured in open air without correction for air temperature.

Demand for better high-resistance measurement capability has driven substantial improvement in the measurement instrumentation and techniques [8, 11]. However, the TCR and VCR characteristics of many film-type high-resistance standards (10 M $\Omega$  and above) often do not allow electrical metrology laboratories to achieve the full potential of improved instrumentation. A recent NIST-led international comparison of high-resistance measurement capability (CCEM-K2) [12] clearly demonstrated the potential for improved metrology with improved standards.

NIST has constructed and delivered new high-resistance standards based on commercially available film-type resistance elements to the Primary Standards Laboratories of the Army and Air Force. The new standards with nominal resistances of 1 G $\Omega$  and 10 G $\Omega$  have characteristics that are better than have been demonstrated previously at these resistance levels. A substantial improvement in stability results from heat treatment of the resistance elements. Sets of the treated elements were carefully selected with net resistance very close to the nominal value, and then hermetically sealed in brass cylinders to further improve their long-term stability.

These standards were characterized for their drift with time and for their dependence on temperature and voltage. The four 1 G $\Omega$  resistance standards were found to typically exhibit temperature coefficients of  $5 \times 10^{-6} / ^\circ\text{C}$ , voltage coefficients of  $0.003 \times 10^{-6} / \text{V}$ , and drift rates of less than  $10 \times 10^{-6} / \text{year}$ . The four 10 G $\Omega$  resistance standards were found to typically exhibit temperature coefficients of  $-3 \times 10^{-6} / ^\circ\text{C}$ , voltage coefficients of  $0.003 \times 10^{-6} / \text{V}$ , and drift rates of less than  $20 \times 10^{-6} / \text{year}$ . For comparison, high-resistance standards that are presently available commercially in this resistance range have thermal coefficients that are up to 10 times larger and voltage coefficients that are sometimes larger by a factor of 100 or more.

#### 4. Conclusion

The design principles of the 1 M $\Omega$  CCC bridge are based on work done in the 1970's at NIST [12, 13]. Special properties of QHR devices allow us to eliminate lead resistance effects in the low-resistance arm of the bridge, so that the QHR standard provides an ideal two-terminal resistance. In addition to 1 M $\Omega$ , the CCC design can be used for direct scaling to 10 M $\Omega$  and 100 M $\Omega$ , and a QHR/100 M $\Omega$  CCC bridge is being tested at the time of this paper's submission.

While other laboratories have described CCC bridges [14, 15] that are used for high resistance, the most important result of this work is that it ties a wide range of high resistance levels directly to the QHR through CCC measurements, with only one step of measurement. This allows testing of other scaling methods including Hamon transfer standards and voltage-arm high resistance bridges against the CCC method.

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