REVIEW OF SCIENTIFIC INSTRUMENTS VOLUME 74, NUMBER 9 SEPTEMBER 2003

Frequency dependence of capacitance standards

Yicheng Wang[a]
Electricity Division, National Institute of Standards and Technology, Gaithersburg, Maryland 20899

(Received 18 March 2003; accepted 3 June 2003)

The frequency dependence of a 10 pF transportable fused-silica capacitance standard was measured from 50 Hz to 20 kHz. Its capacitance shows a broad minimum from 1592 Hz to 4 kHz. Due to dielectric relaxation, the capacitance at 50 Hz is fractionally 1.2×10^{-6} over the minimum; due to residual inductance, the capacitance at 20 kHz is 0.68×10^{-6} over the minimum. The results have a relative standard uncertainty of 0.61×10^{-6}, 0.32×10^{-6}, 0.15×10^{-6}, and 0.37×10^{-6} at 50 Hz, 100 Hz, 400 Hz, and 20 kHz, respectively. This will reduce the expanded standard uncertainties for the capacitance calibrations at NIST by a factor of three or four. For this work, we used two reference capacitors. The first reference was a 1 pF cross capacitor chosen for its negligible frequency dependence at low frequencies. The second reference was a 10 pF nitrogen-filled capacitor chosen because it has a very small inductance, thereby reducing problems at higher frequencies. [DOI: 10.1063/1.1599063]

The scope of the capacitance calibration services available at the National Institute of Standards and Technology (NIST) is largely determined by industrial needs. Recently, ultraprecision multifrequency (from 50 Hz to 20 kHz) capacitance bridges have become commercially available and secondary calibration laboratories have started using this new type of bridge for their impedance calibrations; thus, NIST is compelled to reexamine the frequency dependence of reference capacitors in the audio frequency range. The primary maintenance standard for NIST capacitance calibrations consists of a bank of four 10 pF fused-silica standards (referred to as the Farad Bank) which are maintained in an oil bath at 25 °C. The Farad Bank is very stable, drifting only fractionally 0.02×10^{-6} per year, and the standards are calibrated twice a year indirectly against the calculable capacitor[1] at NIST at a frequency of $\approx 1592\,\mathrm{Hz}$ ($\omega = 10^4$ rad/s), using a 10 pF transportable fused-silica capacitor, C_{112}. This frequency was chosen for a convenient link between the farad and the ohm.

The NIST calculable capacitor is based on the Thompson–Lampard theorem.[2] It is composed of four parallel, uniform electrodes with very small gaps between them as shown in Fig. 1(a). The capacitances per unit length, C_a and C_b, between the opposing pairs of electrodes obey the relationship:

$$\exp(-\pi C_a/\varepsilon_o)+\exp(-\pi C_b/\varepsilon_o)=1,$$

where ε_o is the permittivity of vacuum. When $C_a=C_b=C$, this reduces to

$$C=\frac{\varepsilon_o \ln 2}{\pi},$$

which is a constant that directly links to the defined speed of light. The calculable capacitor provides an absolute determination of capacitance in terms of length only and is the ultimate reference for all impedance measurements in the U.S. However, the calculable capacitor system is optimized for operation at 1592 Hz; modifications and corrections required to operate at other frequencies are nontrivial.

Today, capacitance calibrations in the audio frequency range are available from NIST only at frequencies of 1000, 400, and 100 Hz.[3] In the 1960's, the frequency dependence of the Farad Bank between 1592 Hz and these lower frequencies was determined by comparison to a 10 pF guarded cylindrical capacitor filled with nitrogen, C_{10}. The frequency dependence of this nitrogen capacitor was estimated to be small; however, it had not been experimentally determined. Therefore, large relative uncertainties were assigned to the frequency dependent capacitance calibrations: 1.5×10^{-6}, 2.5×10^{-6}, and 4.0×10^{-6} $(k=2)$ for 1000, 400, and 100 Hz, respectively.

Recently, Jeffery and Koffman[3] have recharacterized the frequency dependence of the Farad Bank from 1592 to 1000 Hz using the calculable capacitor system. Thus, NIST was able to reduce the overall expanded uncertainty for 1000 Hz calibrations of 10 and 100 pF fused-silica capacitors from 1.5×10^{-6} to 0.5×10^{-6}. However, this approach is very tedious; calibration of the bridge transformer used for the calculable capacitor alone involves multiple steps at each frequency. The method cannot be simply copied to reduce the uncertainties at very low frequencies such as 100 Hz. The reported type A uncertainty (the variability of repeated observations) of 0.05×10^{-6} at 1000 Hz is expected to increase by a factor of 100 at 100 Hz. The impedance of a capacitor increases with decreasing frequency and the operating voltage has to be reduced linearly with frequency below 1000 Hz to avoid saturating the ac bridge for the calculable capacitor system. These two effects lead to severe reduction in signal with no corresponding reduction in noise.

There are two dominant sources of capacitance change with frequency. Residual inductance tends to increase the apparent capacitance of a simple capacitor. Consider a three-terminal capacitor with the residual series inductance equal

[a]Electronic mail: yicheng.wang@nist.gov

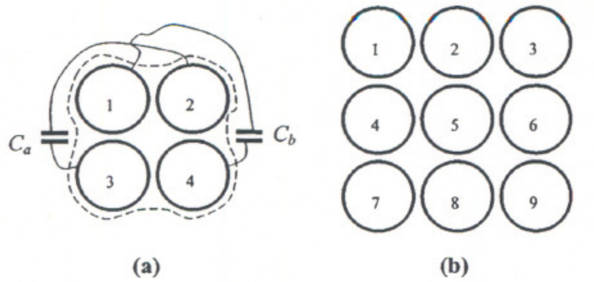

FIG. 1. (a) 4-rod cross capacitor: dashed line represents a ground shield; (b) 9-rod cross capacitor.

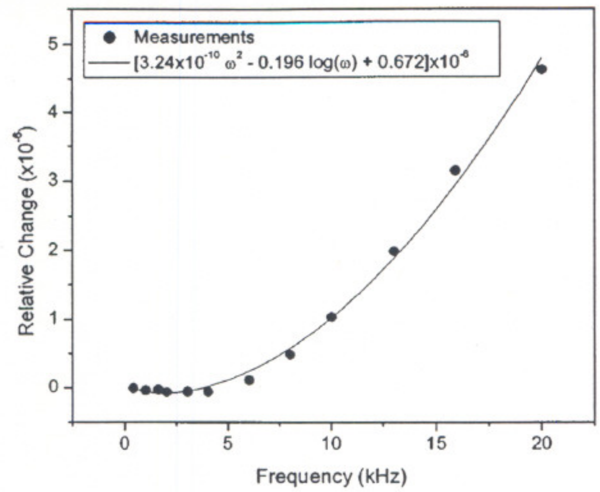

FIG. 2. Measured ratio, C_{10}/C_1, expressed as relative change from the ratio at 400 Hz, together with a least-square fit of $a\omega^2+b\log\omega+c$, as a function of frequency.

to L, the capacitance between the two active electrodes C_o, and the stray capacitance from leads and electrodes to the ground terminal C_g. A simplified equivalent circuit for the capacitor consists of L in series with C_g+C_o. The fractional increase in capacitance is approximately $\Delta C/C_o \approx \omega^2 L(C_g+C_o)$. This increase can be significant when the frequency is high, but as it will become clear later, it is negligible below 1592 Hz compared to other possible frequency dependencies.

The other dominant source of frequency dependence results from dielectric relaxation, which tends to increase the capacitance at low frequencies. This can be understood by considering a set of noninteracting dipoles in an ac field. At low enough frequencies, the orientation of the dipoles can keep up with the instantaneous value of the electric field, therefore the resultant dielectric constant is at maximum. As the frequency increases, the dipoles start to lag behind the field, decreasing the effective dielectric constant.

Dielectric relaxation manifested in the audio frequency range may not be solely caused by bulk properties of the dielectric in a capacitor. Cutkosky and Lee[4] have shown that the relative frequency dependence of a set of 10 pF fused-silica capacitors, which were fabricated identically, changes greatly from one capacitor to another, suggesting that some defects may be responsible for the observed frequency dependence. When nitrogen gas fills the vacuum gap in a capacitor, the dielectric constant and therefore the capacitance increase; however, the increase is frequency independent in the audio frequency range because the frequency associated with the dielectric relaxation time of the gas molecules is in the microwave region. Dielectric films on electrode surfaces are the main cause of the frequency dependence of gas dielectric and vacuum gap capacitors. Inglis[5] performed a thorough study of electrode surface film effects on the frequency dependence of parallel-plate capacitors. He modeled a parallel-plate capacitor as an ideal capacitor C_o, representing the vacuum gap, in series with a lossy capacitor C_s, lumping capacitances associated with the electrode surface films. The apparent capacitance is then $C \approx C_o(1-C_o/C_s)$, and the component that causes frequency dependence is quadratically proportional to $C_o: \Delta C = -C_o^2/C_s$. By putting two variable parallel-plate capacitors in parallel and varying the separations in such a way that the total capacitance was kept constant at a chosen frequency, he found that the frequency dependence from 50 Hz to 50 kHz due to the surface films on plain brass electrodes or Rh plated electrodes approximately follows the empirical function: $\Delta C/C = b\log\omega + c$, where b and c are constants.

This work was started with a 9-rod, 1 pF nitrogen cross capacitor, C_1, as shown in Fig. 1(b). This type of 9-rod cross capacitor can be considered as four conventional 4-rod cross capacitors in parallel. It was designed at NIST to increase the capacitance while keeping the capacitor within a manageable length. When rod 5 is powered, rods 1, 3, 7, and 9 are connected to the detector while all other rods are grounded. Conversely, when rods 2 and 8 are powered together, rods 4 and 6 are connected to the detector while all other rods are grounded. All detector rods have guard ends to reduce the fringe effects. This cross capacitor was first chosen for frequency dependence study because it has been shown by Lampard and Cutkosky[6] that thin dielectric films uniformly deposited on the electrodes of a cross capacitor do not alter its individual cross capacitances to the first order. However, it was soon realized that the required lead corrections and the associated uncertainties at high frequencies could be significant, because extra wires were required to connect the capacitors in parallel. To determine the lead corrections, a comparison was made between C_1 and C_{10} which has a very small residual inductance, using the 4-terminal pair bridge at NIST.[7] At each frequency, the transformer ratio of this bridge was evaluated using our usual method of the permutation of eleven 10 pF capacitors.[8] The measured ratio, C_{10}/C_1, expressed as relative change from the ratio at 400 Hz is shown in Fig. 2, as a function of frequency from 400 Hz to 20 kHz. Also shown in the figure is a least-square fit of the formula $a\omega^2+b\log\omega+c$ to the experimental results, where the first term accounts for the inductance of the 1 pF cross capacitor, while the second term represents the surface film effects of the 10 pF nitrogen capacitor. The results indicate that the cross capacitor has small lead corrections at low frequencies, about 0.03×10^{-6} at 1592 Hz, and thus it can serve as a good reference for frequency dependence in the low frequency region. The results also show that the frequency dependence due to the electrode surface films of the 10 pF nitrogen ca-

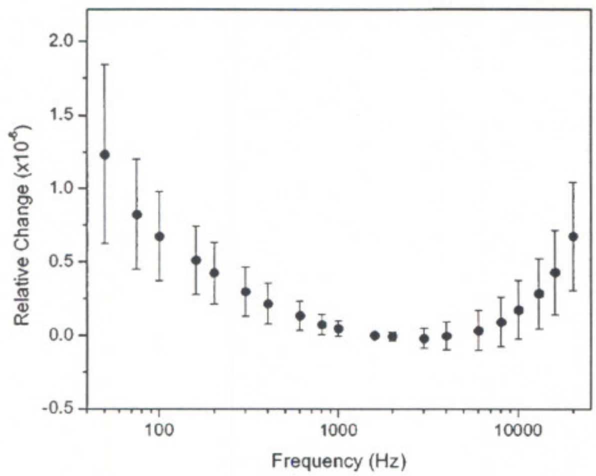

FIG. 3. Frequency dependence of C_{112} expressed as fractional change from the capacitance at 1592 Hz, with 1σ uncertainty bars.

pacitor is about 0.2×10^{-6} per decade change in frequency, which is much smaller than the uncertainties previously assigned to this source.

As a first step to improve the capacitance calibrations at NIST, we decided to extend the frequency range previously explored for the fused-silica reference capacitors and re-evaluate the uncertainties. Shown in Fig. 3 is the measured frequency dependence of the transfer standard C_{112}, expressed as the fractional change from the capacitance at 1592 Hz, using C_{10} as the reference standard. The measurements were performed using the 10:1 4-terminal pair bridge from 400 Hz to 20 kHz and a conventional 10:1 2-terminal pair bridge[9] below 400 Hz; the conductance injection scheme for the latter has been described by Jeffery and Koffman.[3] For the entire frequency range from 50 Hz to 20 kHz, an auxiliary temperature-stabilized 100 pF fused-silica capacitor, C_{272}, was used on the bridge low potential arm, and a simple substitution method for the two 10 pF capacitors on the high potential arm was employed to minimize the sources of uncertainties.

The main sources of uncertainties for the measurements shown in Fig. 3 are listed in Table I for five representative frequencies. All measurements are relative to C_{112} at 1592 Hz, where it is linked to the calculable capacitor with a relative standard uncertainty of 0.019×10^{-6}. At other frequencies, three sources of uncertainties dominate. The type A uncertainty, which is directly linked to the signal-to-noise ratio of the ac bridge systems, is the largest at 50 Hz, about 0.5×10^{-6}. It decreases quadratically with increasing frequency and reaches a minimum at a few kHz; it then increases slightly at the highest audio frequencies owing to a lower output voltage from the source and a bandwidth rolloff of the preamplifier. The inductances of the leads and electrodes of C_{10} were estimated using a resonance method similar to the one described by Free and Jones.[10] The uncertainty due to the residual inductances and stray capacitances is the largest at 20 kHz, about 0.27×10^{-6}; this uncertainty decreases quadratically with decreasing frequency. The fitting results in Fig. 2 indicate that the frequency dependence due to electrode surface films of C_{10} follows the equation $x\equiv b\log\omega+c$. Here $2x$ is taken as the upper bound to estimate the uncertainty due to the surface film effects, leading to a standard deviation of 0.23×10^{-6} in the capacitance per decade change in frequency from 1592 Hz. All other sources of uncertainties, including the stabilities of the capacitors during the experiments and the bridge linearity errors, make very small contributions to the final relative combined standard uncertainties which are shown in Fig. 3, together with the frequency dependence data.

The new measurements of the frequency dependence of C_{112} are consistent with all the previous frequency dependence data released from NIST. The present result of 0.05×10^{-6} at 1000 Hz relative to the capacitance at 1592 Hz agrees well with the result (0.04×10^{-6}) determined by Jeffery and Koffman,[3] using the calculable capacitor chain directly. It is also interesting to note that the relative standard uncertainty (0.06×10^{-6}) at 1000 Hz determined using the present method agrees with the uncertainty (0.055×10^{-6}) assigned earlier for the calculable capacitor measurement of C_{112} at 1000 Hz; however, this agreement may be fortuitous because the dominant source of uncertainties is the electrode surface films of C_{10} for the former method while it is the type A uncertainty for the latter. In addition, the present data overlap, within the experimental uncertainties, with the fre-

TABLE I. Contribution of component uncertainties to the total uncertainty at six representative frequencies for C_{112}.

Source of uncertainty	Relative standard uncertainty ($\times10^{-6}$)					
	50 Hz	100 Hz	400 Hz	1000 Hz	4 kHz	20 kHz
Type A	0.5	0.13	0.01	0.001	0.001	0.01
Lead inductance effects of C_{10}	0	0	0	0.002	0.012	0.27
Surface film effects of C_{10}	0.34	0.28	0.14	0.05	0.09	0.25
Stability of C_{10}	0.03	0.03	0.03	0.03	0.03	0.03
Stability of C_{112}	0.002	0.002	0.002	0.002	0.002	0.002
Stability of C_{272}	0.01	0.01	0.01	0.01	0.01	0.01
Bridge linearity errors	0.05	0.05	0.005	0.005	0.005	0.005
Relative combined standard uncertainty	0.61	0.32	0.15	0.06	0.1	0.37

quency dependence data obtained earlier for 159 and 15 900 Hz by Cutkosky and Lee.[4] The relative standard uncertainties determined at 400 and 100 Hz (0.15×10^{-6} and 0.32×10^{-6}, respectively) are smaller than the uncertainties previously assigned to frequency dependence at these frequencies by a factor of 5. This will lead to improvements in the capacitance calibration services at NIST by a factor three or four. Next, we plan to transfer the frequency dependence data of C_{112} to the Farad Bank and other reference capacitance standards, so that improved capacitance calibrations will be available for the entire audio frequency range from NIST in the near future.

The author acknowledges the contribution of S. Shields for the construction of a low-frequency transformer and L. Lee for the design and construction of an air bath.

[1] R. D. Cutkosky, IEEE Trans. Instrum. Meas. **IM-23**, 305 (1974).
[2] A. M. Thompson and D. G. Lampard, Nature (London) **177**, 888 (1956).
[3] A.-M. Jeffery and A. D. Koffman, Proceedings of the 2002 Inst. Meas. Technol. Conference (Anchorage, AK, 21–23 May 2002), p. 73.
[4] R. D. Cutkosky and L. H. Lee, J. Res. Natl. Bur. Stand. **69C**, 173 (1965).
[5] B. D. Inglis, IEEE Trans. Instrum. Meas. **IM-24**, 133 (1975).
[6] D. G. Lampard and R. D. Cutkosky, Proc. Inst. Elect. Eng. (London) Part C: Monographs, Monograph No. 351 M, Vol. 107, p. 112, 1960.
[7] R. D. Cutkosky, J. Res. Natl. Bur. Stand. **74C**, 63 (1970).
[8] R. D. Cutkosky and J. Q. Shields, IRE Trans. Instrum. **I-9**, 243 (1960).
[9] A. M. Thompson, Metrologia **4**, 1 (1968).
[10] G. M. Free and R. N. Jones, NIST Tech. Note **1348** (1992).