Broadband Impedance Parameters of Asymmetric Coupled CMOS Interconnects: New Closed-Form Expressions and Comparison with Measurements**

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Abstract

New broadband closed-form expressions for the frequency-dependent impedance parameters of asymmetric coupled coplanar interconnects on a conductive silicon substrate are presented. The closed-form expressions are based on a complex image approach that accounts for eddy-current loss in the substrate and incorporates the effect of substrate contacts in the ground lines. The closed-form expressions for asymmetric coupled interconnects fabricated in a CMOS process generally agree well with measurements. To further investigate small differences between the model and measurement results, we also examined the importance of skin- and proximity-effects in the closely spaced conductors.

Introduction

The nonideal transmission characteristics of on-chip interconnects are becoming a significant factor in the overall performance of silicon-based integrated circuits operating at gigahertz frequencies. In particular, the frequency-dependent substrate loss mechanisms can have a significant impact on the characteristics of on-chip interconnects for heavily doped silicon substrates, as has been shown by extensive full-wave and quasi-static electromagnetic simulation (e.g., [1] – [5]), as well as by experimental characterization [6].

It is desirable for computer-aided design (CAD) of silicon-based integrated circuits to have available fast and accurate models for the transmission-line parameters of on-chip interconnects. Recently, Weisshaar et al. developed simple physics-based closed-form expressions for the frequency-dependent line parameters of microstrip-type on-chip interconnects that are suitable for CAD ([7, 8]). In [9], this modeling methodology was extended to single coplanar interconnects. The general case of asymmetric coupled coplanar interconnects has recently been studied experimentally [6]; however, closed-form solutions for the line parameters are not available.

In this paper, new broadband closed-form expressions for the frequency-dependent impedance parameters of asymmetric coupled coplanar CMOS lines are developed and compared with measurement and EM simulation. The closed-form expressions for the $R(\omega)$ and $L(\omega)$ parameters are derived using a complex image approach that takes into account the effects of eddy-current loss in the substrate. The new expressions are directly given in terms of the interconnect dimensions and material parameters. The modeling approach is applied to the structure that was studied experimentally [6] and compared to measurement results as well as to a quasi-analytical solution developed at the University of Hannover [3]. The substrate contacts in the ground lines can have a significant impact on the return current distribution and are included in the closed-form solution. The closed-form and quasi-analytic solutions both neglect conductor skin and proximity effects. The significance of these effects in the closely spaced asymmetric coupled interconnects is further studied with an EM solver based on the simultaneous discretization of conductors and substrate [10], and by comparison with measurements.

Modeling Approach

Figure 1(a) shows the cross section of the asymmetric coupled coplanar CMOS lines of this study [6]. The asymmetric coupled lines are built in a 0.25 μm CMOS technology. The two coupled conductors are fabricated in metal 2 with widths of 1 μm and 10 μm, respectively, and are separated by a gap of 1 μm. The thickness of metal 2 is 0.7 μm, and the conductivity is 27.8×10^6 S/m. The coupled lines are surrounded by 20 μm wide ground lines composed of via stacks that connect all six metalization layers together to the substrate. The silicon substrate is heavily doped with a resistivity of about 0.01 Ω·cm.

The time-varying magnetic fields in the heavily doped substrate give rise to frequency-dependent eddy currents, which are nonuniform and a function of operating frequency and substrate resistivity. With increasing frequency, the eddy-current density in the substrate becomes larger and more concentrated near the interconnects. This can result in a significant reduction in series inductance and a substantial increase in series resistance.

The frequency-dependent complex inductance per-unit-length (p.u.l.) $L(\omega) + R(\omega)/(j\omega)$ can be efficiently determined by applying a complex image theory [11] combined with closed-form expressions for partial inductance [12]. In the complex image theory, the complicated conductive substrate is replaced by a conducting image plane located at a *complex* distance h_{eff} from the coupled lines, as illustrated in Fig. 1(b).

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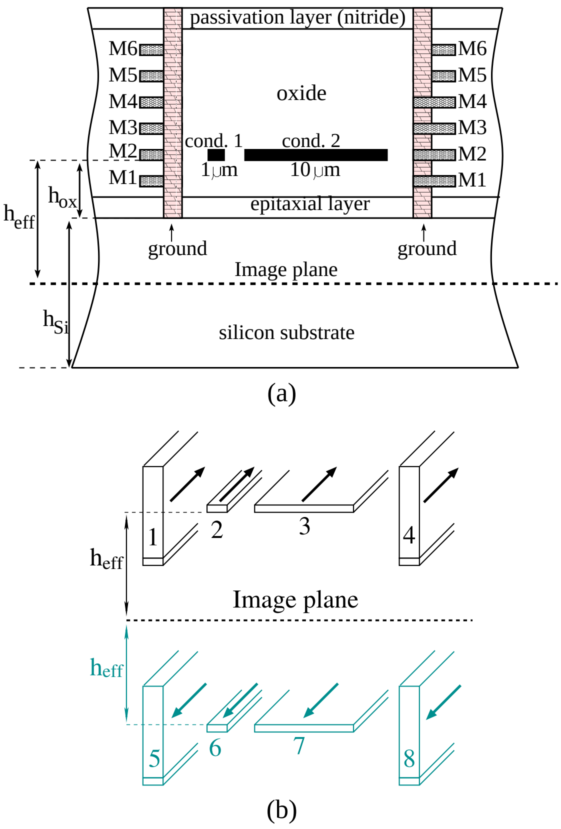


Figure 1: (a) Test structure cross section and (b) equivalent configuration using complex image method.

The complex effective height for a coplanar structure on a single conductive substrate layer of thickness $h_{\rm si}$ is given by [9], [8]

$$h_{\rm eff} = h_{ox} + (1-j)\delta/2 \coth[(1+j)h_{\rm si}/\delta], \qquad (1)$$

where $\delta = 1/\sqrt{\pi f \mu_0 \sigma_{\rm si}}$ is the skin depth of the bulk silicon. The height of the complex image plane can be derived from the Taylor-series expansion of the corresponding Green's function, similar to the derivation for the microstrip case with ground plane given in [8]. Extensions of the complex height formula for multilayer conductive substrates are also available.

Using the complex image approach, the asymmetric coupled coplanar CMOS lines are represented in terms of the interconnect conductors and their images located in free space at a complex distance $2h_{\rm eff}$, as illustrated in Fig. 1(b). The resulting complex 4×4 inductance matrix for the coplanar asymmetric coupled line system may be expressed as

$$[L_{cf}]_{4\times4} = \begin{bmatrix} L^*_{11} & L^*_{12} & L^*_{13} & L^*_{14} \\ L^*_{12} & L^*_{22} & L^*_{23} & L^*_{24} \\ L^*_{13} & L^*_{23} & L^*_{33} & L^*_{34} \\ L^*_{14} & L^*_{24} & L^*_{34} & L^*_{44} \end{bmatrix} \qquad (2)$$

$$= \begin{bmatrix} L_{11}-L_{15} & L_{12}-L_{16} & L_{13}-L_{17} & L_{14}-L_{18} \\ L_{21}-L_{25} & L_{22}-L_{26} & L_{23}-L_{27} & L_{24}-L_{28} \\ L_{31}-L_{35} & L_{32}-L_{36} & L_{33}-L_{37} & L_{34}-L_{38} \\ L_{41}-L_{45} & L_{42}-L_{36} & L_{43}-L_{37} & L_{44}-L_{48} \end{bmatrix},$$

where the effect of the image conductors at complex distance $2h_{\rm eff}$ has been incorporated into the inductance elements corresponding to the four interconnect conductors. In (2) the complex inductance elements $L^*_{ij}(h_{\rm eff}) = L^*_{ji}(h_{\rm eff}) = L_{ij} - L_{i,j+4}(h_{\rm eff})$, $(i,j = 1,\ldots,4)$ can be readily expressed in closed form by extrapolating the closed-form formulas for partial self and mutual inductances of finite-length conductors (e.g. Grover's formulas [12]). Each complex inductance L^*_{ij} represents a real inductance ${\rm Re}\{L^*_{ij}\}$ and resistance $-\omega{\rm Im}\{L^*_{ij}\}$. Note that each partial inductance divided by length is divergent with increasing length; i.e., p.u.l. partial inductance does not exist. However, because of the subtraction of two partial inductances in (2), p.u.l. inductance elements are obtained.

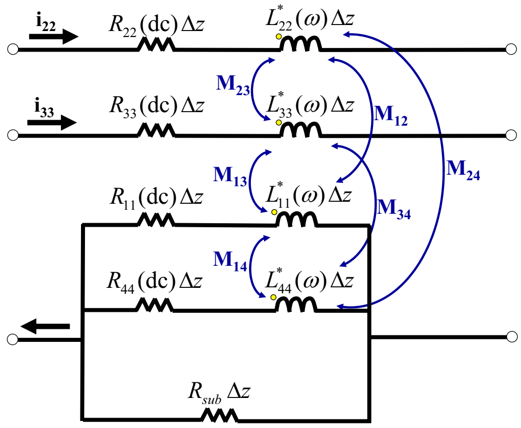


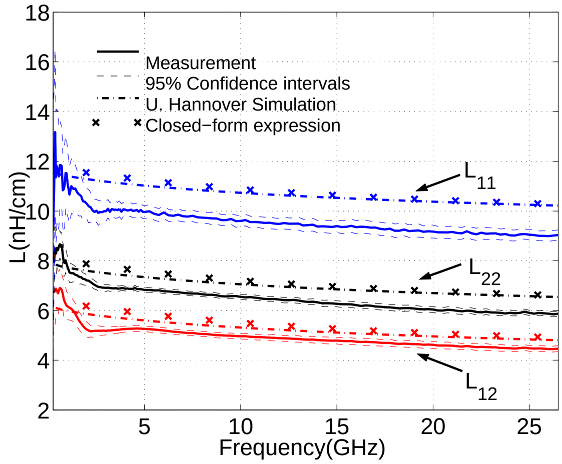
Figure 2: Equivalent circuit model for the series impedance of asymmetric coupled coplanar lines on silicon, where $M_{12} = L^*_{12}(\omega)\Delta z$, $M_{13} = L^*_{13}(\omega)\Delta z$, $M_{14} = L^*_{14}(\omega)\Delta z$, $M_{23} = L^*_{23}(\omega)\Delta z$, $M_{24} = L^*_{24}(\omega)\Delta z$, $M_{34} = L^*_{34}(\omega)\Delta z$.

To derive the final 2×2 inductance matrix for the asymmetric coupled CMOS lines, an equivalent circuit representation is used. The effects of the substrate contacts in the ground lines, which can significantly affect the return current distribution, are also included in the model. The equivalent circuit model for the series impedance of the asymmetric coupled coplanar CMOS lines with the ground lines directly connected to the substrate is illustrated in Fig. 2. Here, $R_{ii}({\rm dc})\Delta z$ $(i = 1,\ldots,4)$ is the dc resistance of conductor i, and L^*_{ii} is the corresponding complex self inductance of conductor i including its image. L^*_{ij} is the complex mutual inductance between line i and line j including images (refer to Fig. 1(b)). $R_{\rm sub}$ is a substrate DC resistance that models the effects of the substrate contacts in the ground lines.

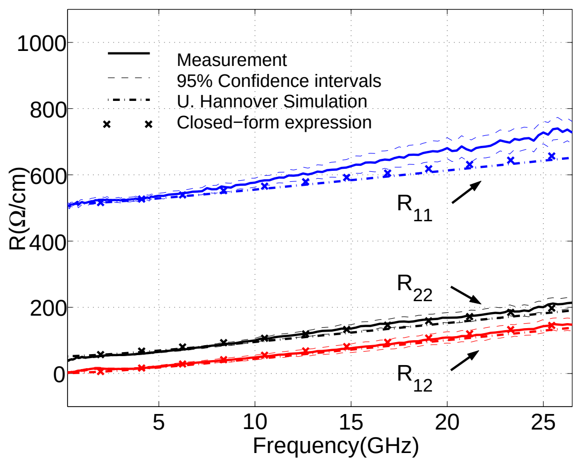
By solving the equivalent circuit model for the complex loop inductances of lines 2 and 3, the 2×2 p.u.l. inductance matrix $[L_{cf}(\omega)]_{2\times2}$ and 2×2 p.u.l. resistance matrix $[R_{cf}(\omega)]_{2\times2}$ are obtained in closed form. Note that the resistance matrix includes the DC resistances for all metal conductors and the substrate.

Results

The new closed-form expressions for $[R(\omega)]$ and $[L(\omega)]$ have been applied to the asymmetric coupled CMOS lines shown in Fig. 1(a), which were fabricated on a heavily-doped silicon sub-

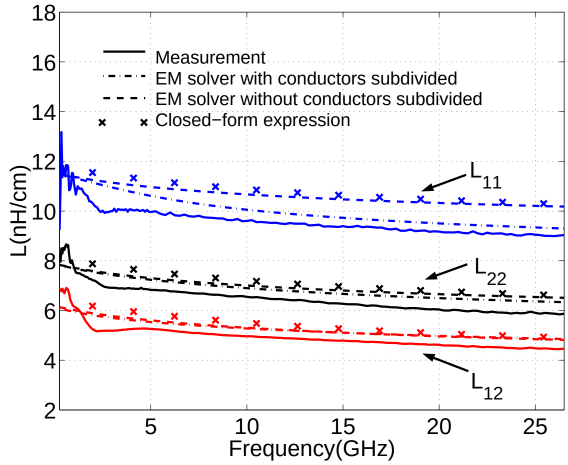


(a)

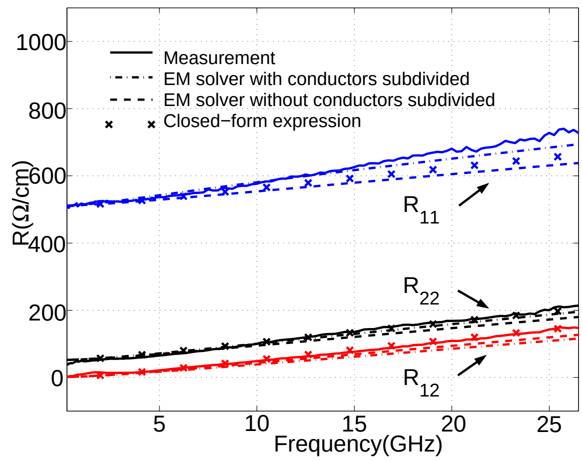


(b)

Figure 3: Frequency dependence of (a) distributed series inductance and (b) series resistance of asymmetric coupled coplanar CMOS lines.



(a)



(b)

Figure 4: Results obtained with a modified PEEC-like EM solver [10]: (a) distributed series inductance and (b) series resistance of asymmetric coupled coplanar CMOS lines.

strate with $\rho_{si} = 0.01\ \Omega{\cdot}\text{cm}$ [6]. In order to verify the accuracy of the closed-form expressions, the frequency-dependent R and L line parameters shown in Figs. 3 (a) and (b) are compared with a quasi-analytical simulation approach [3] and the experimental data from [6], including the lower and upper bounds of the $95\ \%$ confidence intervals. Note that in the R and L parameters shown in Fig. 3, index 1 refers to the $1\ \mu$m signal line and index 2 to the $10\ \mu$m signal line. We observe that the closed-form solutions are almost identical to the corresponding quasi-analytical results. Both theoretical results are consistent with the measurement data over the broad frequency range. We see, however, that some of the theoretical values fall outside the $95\ \%$ confidence interval for the measurement data, particularly in the case of the self inductance L_{11} and self resistance R_{11} of the narrower ($1\ \mu$m) signal line. Possible reasons for the small discrepancies include uncertainty in the process parameter data and systematic measurement errors, as well as conductor skin, proximity, and edge effects in the signal conductors.

To further investigate whether the proximity effect is significant in this conductor configuration, we have simulated the asymmetric coupled line structure using a Partial Element Equivalent Circuit (PEEC)-like electromagnetic (EM) solver [10] that has been modified for the coplanar geometry and substrate contacts. The EM solver performs the simultaneous subdivision of the conductor cross section as well as the conductive substrate to capture the nonuniform current distributions in the conductors and substrate. A similar approach has also been used in [13].

The frequency-dependent R, L line parameters of the asymmetric coupled on-chip interconnects obtained from the EM solver are shown in Figs. 4(a) and (b), with and without subdivision of the conductor cross sections. For the case without subdivision, the results are virtually identical with the closed-form solutions and the quasi-analytical solutions. This comparison further demonstrates the accuracy of the closed-form expressions to accurately model the substrate eddy-current loss. With conductor subdivision the EM solution moves closer to the measurement results, in particular for the conductor with smaller

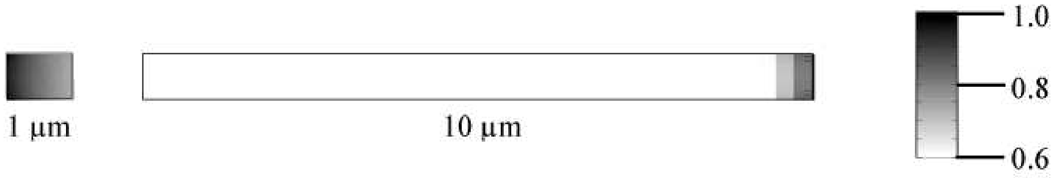


Figure 5: Relative current distribution in the two signal conductors of the asymmetric coupled coplanar CMOS lines at $f = 26.5$ GHz.

width. Figure 5 shows the relative current density distributions in the signal conductors. It can be seen that the current distributions are nonuniform with maximum current density located at the edges farthest apart, as predicted by the proximity effect. These results show that some (but not all) of the discrepancies between model and measurement data can be attributed to conductor skin- and proximity-effect in the signal conductors.

Conclusions

We have developed new broadband closed-form expressions for the frequency-dependent impedance parameters of asymmetric coupled coplanar interconnects on a conductive silicon substrate. We showed that the closed-form expressions accurately account for eddy-current loss in the substrate and incorporate the effect of substrate contacts in the ground lines. We demonstrated excellent agreement with a quasi-analytical solution as well as with the results of a customized EM solver without conductor subdivision. The theoretical results were shown to be close to measurement data. Some of the discrepancies between the theoretical solution and measurement results have been attributed to nonuniform current distributions in the signal conductors due to their close proximity. The fast and accurate closed-form expressions for the frequency-dependent series impedance parameters should be useful for computer-aided design of silicon-based integrated circuits.

Acknowledgments

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